Chapter 4 Motion in Two Dimensions (p.78),

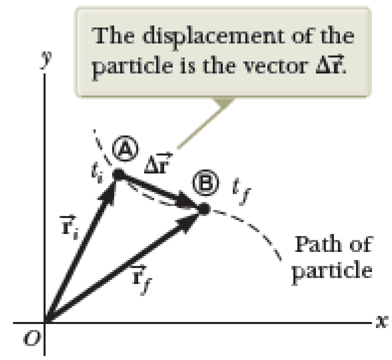
Problems: 1, 5, 9, 15,16,20,21 Sec.4+5: 40, 42, 43 (p.101)

4.1 The Position, Velocity, and Acceleration Vectors

$\vec{r}(t) = \vec{r}_f$ represents the position vector at any time t_f=t

$\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_f - \vec{\mathbf{r}}_i$ is the change in position vector as the particle moves from A to B in the time interval $\mathbf{\Delta t = t_f - t_i}$,

It is the displacement vector between its final position the initial position vectors



The average velocity during the time interval Δ*t*

$$\vec{v}_{avg} = \bar{\vec{\mathbf{v}}} = \frac{\Delta\vec{\mathbf{r}}}{\Delta t}$$

The instantaneous velocity **v** is defined as the limit of the average velocity Δ**r**/Δ*t* as Δ*t* approaches zero:

$$\vec{\mathbf{v}}_x = \lim_{\Delta t \to 0} \frac{\Delta\vec{\mathbf{r}}}{\Delta t} = \frac{d\vec{\mathbf{r}}}{dt}$$

The *speed= |v|, and is* a scalar quantity, is the magnitude of the instantaneous velocity vector ***v.***

The average acceleration during time interval Δt

$$\bar{\vec{\mathbf{a}}} = \frac{\Delta\vec{\mathbf{v}}}{\Delta t} = \frac{\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i}{t_f - t_i}$$

The instantaneous acceleration **a** is defined as

$$\vec{\mathbf{a}} = \lim_{\Delta t \to 0} \frac{\Delta \vec{\mathbf{v}}}{\Delta t} = \frac{d\vec{\mathbf{v}}}{dt}$$

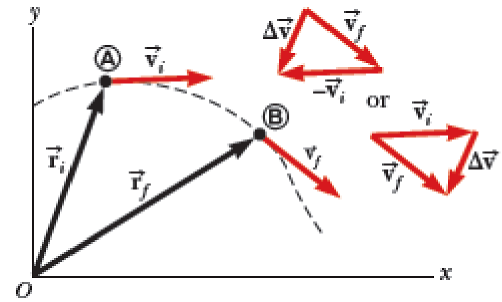


Figure 4.3 A particle moves from position Ⓐ to position Ⓑ. Its velocity vector changes from $\vec{v}_i$ to $\vec{v}_f$. The vector diagrams at the upper right show two ways of determining the vector $\Delta\vec{v}$ from the initial and final velocities.

4.2 Two-Dimensional Motion with Constant Acceleration

The position vector for a particle moving in the *xy* plane can be written

$$\vec{\mathbf{r}} = x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}}$$

the velocity of the particle can be obtained as

$$\vec{\mathbf{v}} = \frac{d\vec{\mathbf{r}}}{dt} = \frac{dx}{dt}\hat{\mathbf{i}} + \frac{dy}{dt}\hat{\mathbf{j}} == v_x\,\hat{\mathbf{i}} + v_y\,\hat{\mathbf{j}}$$

Since **a** is constant, its components a_x and a_y are constant.

$$\mathbf{v}_f = (v_{xi} + a_x t)\,\hat{\mathbf{i}} + (v_{yi} + a_y t)\,\hat{\mathbf{j}}$$

$$= (v_{xi}\,\hat{\mathbf{i}} + v_{yi}\,\hat{\mathbf{j}}) + (a_x t\,\hat{\mathbf{i}} + a_y t\,\hat{\mathbf{j}})$$

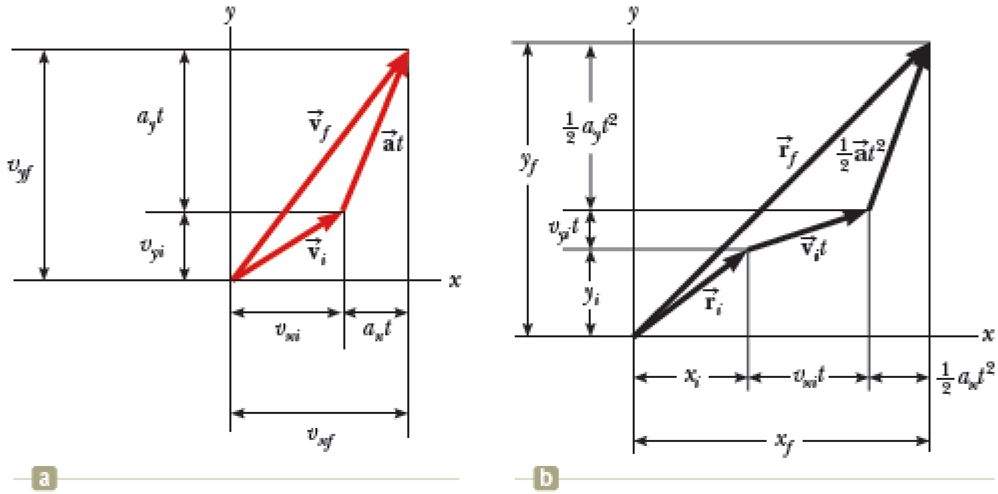
$$\vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \vec{\mathbf{a}}t$$

Similarly

$$\mathbf{r}_f = x_f\,\hat{\mathbf{i}} + y_f\,\hat{\mathbf{j}} = (x_i + v_{xi}t + \tfrac{1}{2}a_x t^2)\,\hat{\mathbf{i}} + (y_i + v_{yi}t + \tfrac{1}{2}a_y t^2)\,\hat{\mathbf{j}}$$

$$= (x_i\,\hat{\mathbf{i}} + y_i\,\hat{\mathbf{j}}) + (v_{xi}\,\hat{\mathbf{i}} + v_{yi}\,\hat{\mathbf{j}})\,t + \frac{1}{2}(a_x\,\hat{\mathbf{i}} + a_y\,\hat{\mathbf{j}})\,t^2$$

$$\mathbf{r}_f = \mathbf{r}_i + \mathbf{v}_i t + \tfrac{1}{2}\mathbf{a}t^2$$



Example 4.1 Motion in a Plane

A particle starts from the **origin at t = 0** with an initial velocity having an ***x* component of 20 m/s** and a ***y* component of – 15 m/s**. The particle moves in the ***xy* plane with an *x* component of acceleration only, given by a_x =4.0 m/s^2.**

Solution

(A) Determine the **total velocity vector** at any **time**.

V_{xi} = 20 m/s,   **v_{yi}** = - 15 m/s, a_x = 4 m/s^2, a_y =0

$$v_{xf} = v_{xi} + a_x t = 20 + 4t \quad m/s$$

$$v_{yf} = v_{yi} + a_y t = -15 + 0 = -15 \quad m/s$$

$$v_f = v(t) = (v_{xi} + a_x t)\hat{i} + (v_{yi} + a_y t)\hat{j} = (20 + 4t)\hat{i} - 15\hat{j}$$

(B) Calculate the velocity and speed of the particle at t = 5.0 s and the angle the velocity vector makes with the x axis.

$$\vec{v}(5) = \vec{v}_f = (20 + 4 \times 5)\hat{i} - 15\hat{j} = 40\hat{i} - 15\hat{j}$$

Speed = $|\vec{v}| = \sqrt{40^2 + 15^2} = 43 \ m/s$, $\theta = tan^{-1}\dfrac{-15}{40} = -21$

(C) Determine the x and y coordinates of the particle at any time t and its position vector at this time.

$$x = x(t) = x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 = 20t - 2t^2$$

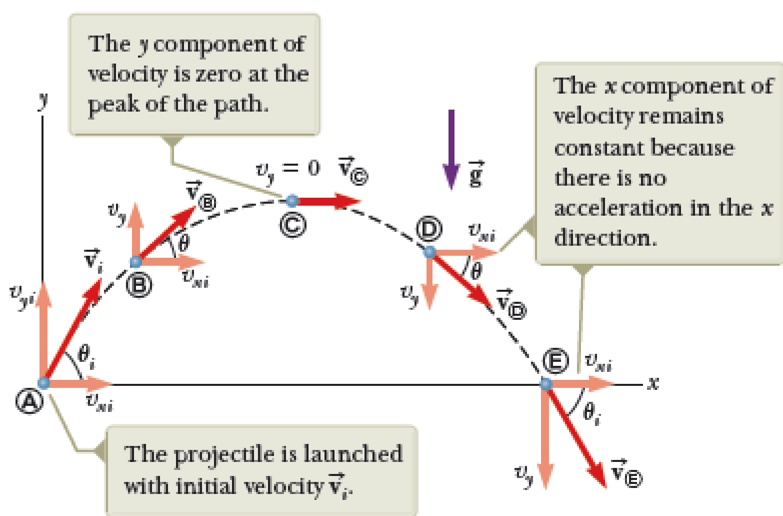
$$x{=}y(t){=}y_f = y_i + v_{yi}t + \tfrac{1}{2}a_y t^2 = -15t + 0 = -15t$$

$$\vec{r} = x\hat{i} + y\hat{j} = (20 - 2t^2)\hat{i} - 15t\,\hat{j}$$

4.3 Projectile Motion

In projectile motion particles moves in a curved path. Assumption:
(1) the free-fall acceleration **g** is constant over the range of motion and is directed downward, and
(2) the effect of air resistance is negligible.

Consider the projectile leaves the origin ($x_i = y_i = 0$) with speed v_i, as shown in Figure . The vector $\mathbf{v_i}$ makes an angle θ_i with the horizontal.



Therefore, the initial x and y components of velocity are

$$v_{xi} = v_i \cos\theta_i$$
$$v_{iy} = v_i \sin\theta_i$$

$$\boxed{\begin{array}{ll} v_{xf} = v_{xi} & \Rightarrow v_{xi} = v_i \cos\theta_i \\ v_{yf} = v_{yi} + a_y t & \Rightarrow v_{yf} = v_i \sin\theta_i - gt \end{array}}$$

$$a_x = 0 \quad \text{always}$$

$$a_y = -g \quad \text{always} \quad \text{at all points } a = a_y = -g$$

$$\boxed{x_f = v_x t = \left(v_i \cos\theta_i\right) t \qquad \Rightarrow t = \frac{x_f}{v_i \cos\theta_i}}$$

$$\boxed{y_f = v_{yi} t - \frac{1}{2} g t^2 = \left(v_i \sin\theta_i\right) t - \frac{1}{2} g t^2}$$

With $\boxed{t = \dfrac{x_f}{v_i \cos\theta_i}}$ $\qquad \boxed{y_f = x_f\left(\tan\theta_i\right) - \dfrac{g}{2 v_i^2 \cos\theta_i^2} x_f^2}$

The equation is of form $\boldsymbol{y = ax + bx^2}$, which **is the equation of a parabola that passes through the origin.**

The vector expression for the position vector of the projectile as a function of time follows directly from Equation with **a =g**:

$$\mathbf{r}_f = \mathbf{r}_i + \mathbf{v}_i t - \tfrac{1}{2} \mathbf{g} t^2$$

Two-dimensional motion with constant acceleration can be analyzed as a combination of two independent motions in the *x* and *y* directions, the superposition of two motions: (1) constant-velocity motion in the horizontal direction and (2) free-fall motion in the vertical direction.

Horizontal Range and Maximum Height of a Projectile

A projectile is launched from the origin at $\boldsymbol{t_i = 0}$ with a positive v_{yi} component, **The distance *R* is called the *horizontal range*** of the projectile, and the **distance *h* is its *maximum height.*** Let us find h and R in terms of v_i, θ_i and g. at the peak (point A) $\mathbf{v_{yA} = 0}$ at max. h

$$v_{yf} = v_{yi} + a_y t \qquad \Rightarrow 0 = v_i \sin\theta_i - g t_A \qquad \Rightarrow t_A = \frac{v_i \sin\theta_i}{g}$$

The maximum height h which is at t_A is

$$\boxed{H = v_i \sin\theta_i \frac{v_i \sin\theta_i}{g} - \frac{1}{2} g \left(\frac{v_i \sin\theta_i}{g}\right)^2 \qquad \Rightarrow H = \frac{v_i^2 \sin^2\theta_i}{2g}}$$

The range R is given by

$$R = v_{xi} t = (v_i \cos\theta_i) \cdot 2t_A$$

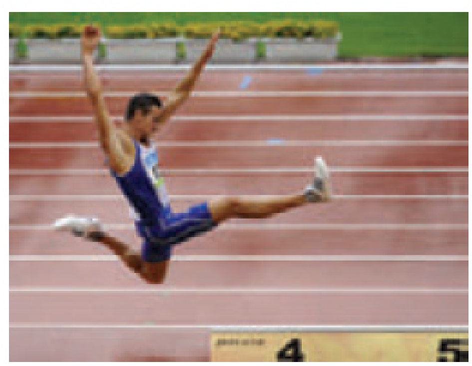
$$\boxed{\Rightarrow R = (v_i \cos\theta_i) \frac{(2 v_i \cos\theta_i)}{g} = \frac{2 v_i^2 \cos\theta_i \sin\theta_i}{g}}$$

$$\boxed{R = \frac{v_i^2 \sin 2\theta_i}{g}} \quad ,$$

R is maximum when $2\theta = 90°$, or $\theta = 45°$.

Example 4.2 The Long Jump

A long-jumper (Fig.) leaves the ground at an angle of 20.0° above the horizontal and at a speed of 11.0 m/s.
(A) How far does he jump in the horizontal direction?
(Assume his motion is equivalent to that of a particle.)



Sipa via AP Images

Figure 4.11 (Example 4.2) Romain Barras of France competes in the men's decathlon long jump at the 2008 Beijing Olympic Games.

(A) How far does he jump in the horizontal direction?

Solution

The takeoff point is the origin of coordinates O=(x_i =0, x_f =0)
Label the peak at A and the landing point as B.
The horizontal distance is the range R:

$$R = \frac{2 v_i^2 \sin 2\theta_i}{g} = \frac{2\times 11^2 \times \sin(2\times 20)}{9.8} = 7.94\,\text{m}$$

(B) What is the maximum height reached?

Solution

$$h = \frac{v_i^2 \sin^2\theta_i}{2g} = \frac{11^2 \times \sin^2(20)}{2\times 9.8} = 0.722\,\text{m}$$

Alternative Solution

Find t_h for the jumper to reach max. height h where v_{yA} = 0

$$v_{yA} = v_{yi} + a_y t_A \quad \Rightarrow 0 = v_i \sin\theta_i - g t_h$$

$$\Rightarrow t_h = \frac{v_i \sin\theta_i}{g} = \frac{11 \sin 20}{9.8} = 0.384\,\text{s}$$

Find R (since a_x =0, x_i = 0) with $\boxed{t_R = 2t_h}$

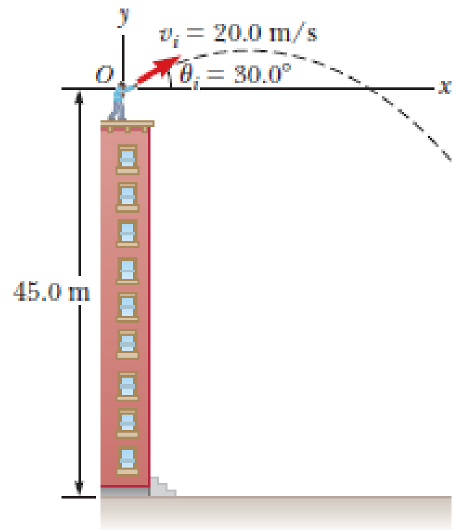
$$x_f - x_i = R = v_{xi} t = \left(v_i \cos\theta_i\right) t = 11\cdot(\cos 20)\times 2\times(0.384) = 7.94\,\text{m}$$

Find h (since a_y =g, y_i = 0)

$$\Delta y = h = \left(v_i \sin\theta_i\right) t - \tfrac{1}{2} g t^2$$
$$= 11\times \sin 20 \times 0.384 - \tfrac{1}{2}\times 9.8\times(0.384)^2 = 0.722\,\text{m}$$

Example 4.4 that's Quite an Arm!

A stone is thrown from the top of a building upward at an angle of 30.0° to the horizontal with an initial speed of 20.0 m/s, as shown in Figure. If the height of the building is 45.0 m,



(A) How long does it take the stone to reach the ground?

Solution

The initial *x* and *y* components of the stone's velocity are

$$v_{xi} = v_i \cos\theta_i = 20\cos 30 = 17.3\,\mathrm{m/s}$$

$$v_{yi} = v_i \sin\theta_i = 20\sin 30 = 10.0\,\mathrm{m/s}$$

When the stone reach the ground then

$y_f = -45\,\mathrm{m}$ but

$$-45 = y_i + (v_i \sin\theta_i)t - \tfrac{1}{2}gt^2 = 0 + 10t - \tfrac{1}{2}(9.8)\times t^2 \quad 4.9t^2 - 10t - 45 = 0$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad t = \frac{-10 \pm \sqrt{10^2 - 4\times 4.9\times 45}}{2\times 4.9} \qquad \boxed{t = 4.22\ \mathrm{s}}$$

(B) What is the speed of the stone just before it strikes the ground?

Solution

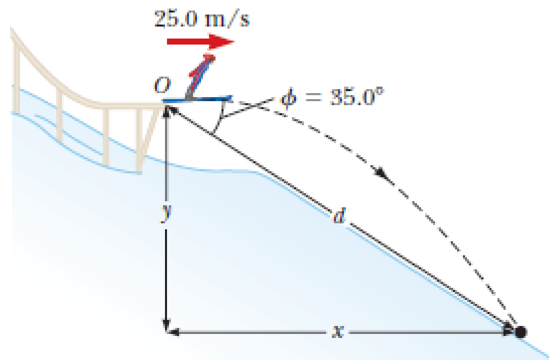
$$v_{yf} = v_{yi} + a_y t \;\Rightarrow\; v_{yf} = v_i \sin\theta_i - gt = 10 - (9.8)(4.22) = -31.3\,\mathrm{m/s}$$

$$v_{xf} = v_{xi} \quad \Rightarrow \quad v_{xf} = v_i \cos\theta_i = 17.3\,\mathrm{m/s}$$

Speed = $v = \sqrt{{v_x}^2 + {v_y}^2} = \sqrt{(-31.3)^2 + (17.3)^2} = 35.8\,\mathrm{m/s}$

Example 4.5 The End of the Ski Jump

A ski-jumper leaves the ski track moving in the horizontal direction with a speed of 25.0 m/s, as shown in Figure. The landing incline below him falls off with a slope of 35.0°. Where does he land on the incline?



Solution

We have to find the value of *d*, the distance traveled along the incline, we note

$$x = d\cos\varphi$$
$$y = d\sin\varphi$$

But we have for the projectile

$$x_f - x_i = x = v_{ix}t = v_i\,t$$
$$y_f - y_i = y = v_{iy}t = 0 + (0)t - \tfrac{1}{2}gt^2$$

$$d\cos 35 = (25)\,t \quad \Rightarrow t = \frac{d\cos 35}{25}$$

$$-d\sin 35 = -\tfrac{1}{2}(9.8)\,t^2$$

$$-d\sin 35 = -\tfrac{1}{2}(9.8)\,(\frac{d\cos 35}{25})^2 \quad \Rightarrow d = 109\,\text{m}$$

$$x_f = d\cos 35 = 109\times\cos 35 = 89.3\,\text{m}$$

$$y_f = -d\sin 35 = 109\times\sin 35 = -62.5\,\text{m}$$

4.4 Analysis Model: Particle in Uniform Circular Motion

When an object is moving on circular path with *constant speed v,* we call this motion uniform **circular motion.**
Velocity changes in direction but its magnitude remains constant, the object has an acceleration called the centripetal **(مركزي) acceleration**

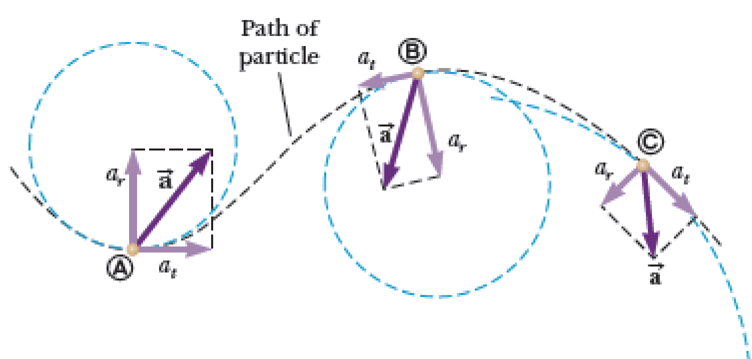
$$a_c = \frac{v^2}{r} \qquad [\text{يشير نحو المركز}]$$

The time to complete one circle is called **period** *T,*

Where T = distance travelled/v = 2π r/ v = المحيط/ السرعة

$$\omega = \frac{2\pi}{T}$$ is the angular velocity

4.5 Tangential and Radial Acceleration



When the of a particle moving in curved path with velocity changes both in direction and in magnitude, the particle has a total acceleration

a = a$_t$ + a$_r$

$$a_t = \frac{dv}{dt}, \qquad a_r = -a_c = \frac{v^2}{r}$$

And a = $\sqrt{a_t^2 + a_r^2}$

Example 4.6 The Centripetal Acceleration of the Earth

(A) What is the centripetal acceleration of the Earth as it moves in its orbit around the Sun?

$$v = \frac{2\pi r}{T},$$

$$a_c = \frac{v^2}{r} = \frac{(\frac{2\pi r}{T})^2}{r} = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2 \times 1.496\times 10^{11}}{(365\times 24\times 60\times 60)^2} = 5.93\times 10^{-3} m/s^2$$

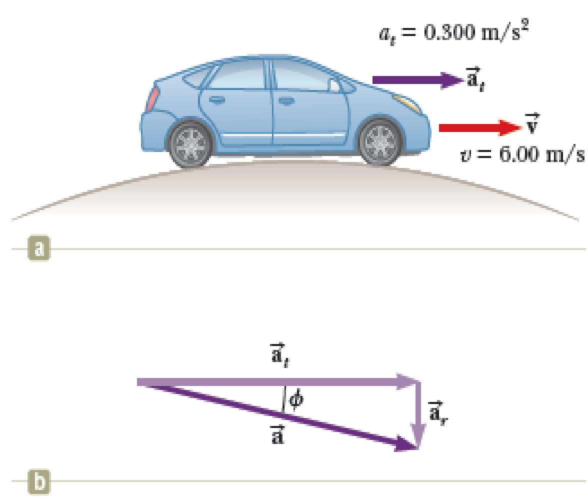
$$1yr = 365\times 24\times 60\times 60 \ \text{sec}$$

(B) What is the angular speed of the Earth in its orbit around the Sun?

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{1yr} = \frac{2\pi}{3.156\times 10^7 s} = 1.99\times 10^7 s^{-1}$$

Example 4.7 Over the Rise

A car leaves **a stop sign** and exhibits **a constant acceleration of 0.300 m/s²** parallel to the roadway. The **car passes over a rise** in the roadway such that **the top of the rise is shaped like an arc** of a circle of **radius 500 m**. At the moment the car is at the top of the rise, its velocity vector is horizontal and has a magnitude of **6.00 m/s**. What are the **magnitude** and **direction** of the total acceleration vector for the car at this instant?



Solution

$$a_r = -a_c = -\frac{v^2}{r} = -\frac{6.00^2}{500} = -0.072 m/s^2$$

$$a = \sqrt{a_t^2 + a_r^2} = \sqrt{(-0.072)^2 + (0.300)^2} = 0.309\ m/s^2$$

$$\theta_a = \tan^{-1}(\frac{-0.072}{0.300}) = -13.5^o$$

Section 4.5 Tangential and Radial Acceleration

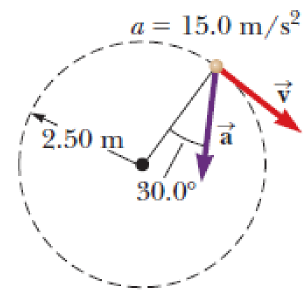


Figure P4.40

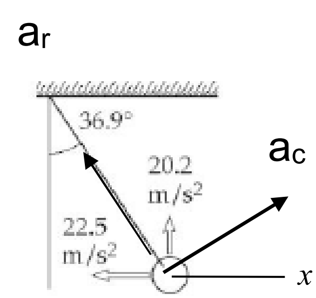
4-40. Figure P4.40 represents the total acceleration of a particle moving clockwise in a circle of radius 2.50 m at a certain instant of time. For that instant,

find (a) the radial acceleration of the particle,
(b) the speed of the particle, and (c) its tangential acceleration.

$a_r = - a_c = - a \cos 30 = - 15.0\ (0.866) = - 13\ m/s^2$

$v^2 = a_c r = 13 \times 2.5 = 32.5 \Rightarrow v = \sqrt{32.5} = 5.7 m/s$

$a_t = a \sin 30 = 15.0\ (0.5) = 7.5\ m/s^2$



ANS. FIG. P4.42

4-42. A ball swings (**يتأرجح**) counterclockwise in a vertical circle at the end of a rope 1.50 m long. When the ball is 36.9° past the lowespoint (ادنى نقطة) on its way up, its total acceleration is $(-22.5\hat{i} + 20.2\hat{j})$ m/s². For that instant, (a) sketch a vector diagram showing the components of its acceleration, (b) determine the magnitude of its radial acceleration, and (c) determine the speed and velocity of the ball.

(a) The diagram is shown in the figure.

(b) The magnitude of the radial acceleration is the sum of the components of -22.5 and 20.2 along the rope and in the inwards direction,

$$a_r = 20.2\cos(36.9) + 22.5\sin(53.1) = 29.7 m/s^2 \ \Omega$$

(c) $a_r = \dfrac{v^2}{r} \quad \Rightarrow v = \sqrt{r\,a_r} = \sqrt{(1.5)(29.7)} = 6.67\ m/s$

at 36.9° above the horizontal.

4-43. (a) Can a particle moving with instantaneous speed 3.00 m/s on a path with radius of curvature 2.00 m have an acceleration of magnitude 6.00 m/s^2? (b) Can it have an acceleration of magnitude 4.00 m/s^2? In each case, if the answer is yes, explain how it can happen; if the answer is no, explain why not.

$$a_r = \frac{v^2}{r} = \frac{3^2}{2} = 4.5 m/s^2$$

$$a = \sqrt{a_t^2 + a_r^2}$$

(a) Yes because **$a_t \neq 0$** tangential acceleration may not zero. And the particle may be speeding or slowing

$$a_t = \sqrt{a^2 - a_r^2} = \sqrt{6^2 - 4.5^2} = 3.97 m/s^2$$

(b) Total **a** cannot be less than **a_r**.

4-47. A police car traveling at 95.0 km/h is traveling west, chasing (يطارد) a motorist traveling at 80.0 km/h.
(a) What is the velocity of the motorist relative to the police car? (b) What is the velocity of the police car relative to the motorist? (c) If they are originally 250 m apart, in what time interval will the police car overtake the motorist?

(a) $v_{MP} = v_{MG} + v_{GP} = v_{MG} - v_{PG} = 80 - 95 = -15\ \mathrm{km/h}$

(b) $v_{PM} = v_{PG} + v_{GM} = v_{PG} - v_{MG} = 95 - 80 = 15\ \mathrm{km/h}$

(c) Relative to motorist (origin) police is approaching at +15 km/h.

$$d = v_{PM} t = v \cdot t$$

$$\Rightarrow t = \frac{d}{v} = \frac{0.250\mathrm{km}}{15\mathrm{km/hr}} = 1.67 \times 10^{-2} \times \frac{3600}{1} = 60\mathrm{s}$$