

Chapter 4 Motion in Two Dimensions (p.78),

Problems: 1, 5, 9, 15,16,20,21 Sec.4.4+4.5: 40, 42, 43 (p.101)

4.4 Analysis Model: Particle in Uniform Circular Motion

When an object is moving on circular path with *constant speed* v , we call this motion uniform **circular motion**.

Velocity changes in direction but its magnitude remains constant, the object has an acceleration called the centripetal (مركزي) **acceleration**

$$a_c = \frac{v^2}{r} \quad [\text{يشير نحو المركز}]$$

The time to complete one circle is called **period** T ,

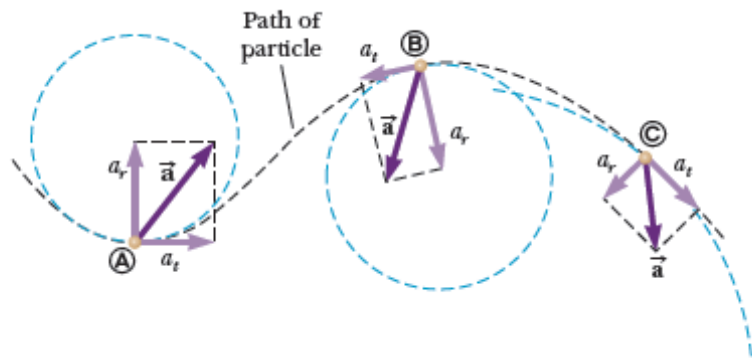
Where $T = \text{distance travelled}/v = 2\pi r/v = \text{السرعة}/\text{المحيط}$

$\omega = \frac{2\pi}{T}$ is the angular velocity

4.5 Tangential and Radial Acceleration

When the of a particle moving in curved path with velocity changes both in direction and in magnitude, the particle has a total acceleration

$$\mathbf{a} = \mathbf{a}_t + \mathbf{a}_r$$



$$a_t = \frac{dv}{dt}, \quad a_r = -a_c = \frac{v^2}{r}$$

$$\text{And } a = \sqrt{a_t^2 + a_r^2}$$

Example 4.6 The Centripetal Acceleration of the Earth

(A) What is the centripetal acceleration of the Earth as it moves in its orbit around the Sun?

$$v = \frac{2\pi r}{T},$$

$$a_c = \frac{v^2}{r} = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2 \times 1.496 \times 10^{11}}{(365 \times 24 \times 60 \times 60)^2} = 5.93 \times 10^{-3} \text{ m/s}^2$$

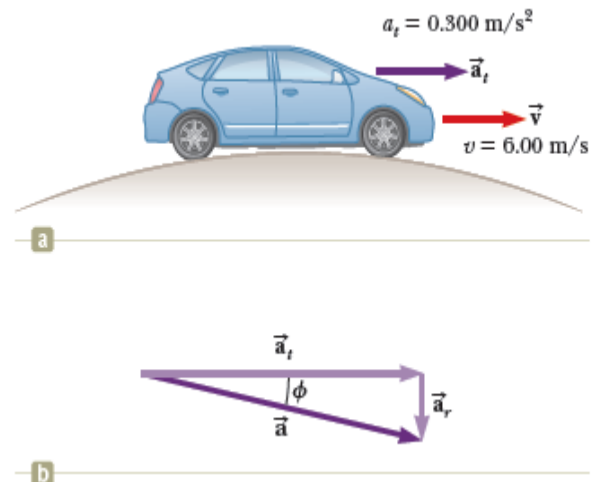
$$1\text{yr} = 365 \times 24 \times 60 \times 60 \text{ sec}$$

(B) What is the angular speed of the Earth in its orbit around the Sun?

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{1\text{yr}} = \frac{2\pi}{3.156 \times 10^7 \text{s}} = 1.99 \times 10^{-7} \text{ s}^{-1}$$

Example 4.7 Over the Rise

A car leaves a stop sign and exhibits a constant acceleration of 0.300 m/s^2 parallel to the roadway. The car passes over a rise in the roadway such that the top of the rise is shaped like an arc of a circle of radius 500 m . At the moment the car is at the



top of the rise, its velocity vector is horizontal and has a magnitude of **6.00 m/s**. What are the **magnitude** and **direction** of the total acceleration vector for the car at this instant?

Solution

$$a_r = -a_c = -\frac{v^2}{r} = -\frac{6.00^2}{500} = -0.072 \text{ m/s}^2$$

$$a = \sqrt{a_t^2 + a_r^2} = \sqrt{(-0.072)^2 + (0.300)^2} = 0.309 \text{ m/s}^2$$

$$\theta_a = \tan^{-1}\left(\frac{-0.072}{0.300}\right) = -13.5^\circ$$

Section 4.5 Tangential and Radial Acceleration

4-40. Figure P4.40 represents the total acceleration of a particle moving clockwise in a circle of radius 2.50 m at a certain instant of time. For that instant,

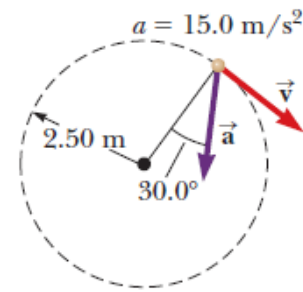


Figure P4.40

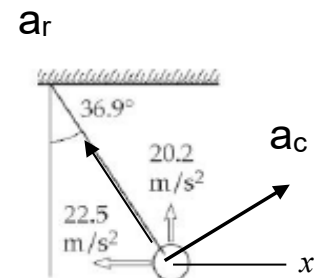
find (a) the radial acceleration of the particle, (b) the speed of the particle, and (c) its tangential acceleration.

$$a_r = -a_c = -a \cos 30 = -15.0 (0.866) = -13 \text{ m/s}^2$$

$$v^2 = a_c r = 13 \times 2.5 = 32.5 \Rightarrow v = \sqrt{32.5} = 5.7 \text{ m/s}$$

$$a_t = a \sin 30 = 15.0 (0.5) = 7.5 \text{ m/s}^2$$

4-42. A ball swings (يتأرجح) counterclockwise in a vertical circle at the end of a rope 1.50 m long. When the ball is 36.9° past the lowest point (ادنى نقطة) on its way up, its total acceleration is $(-22.5\hat{i} + 20.2\hat{j}) \text{ m/s}^2$. For that



ANS. FIG. P4.42

instant, (a) sketch a vector diagram showing the components of its acceleration, (b) determine the magnitude of its radial acceleration, and (c) determine the speed and velocity of the ball.

(a) The diagram is shown in the figure.

(b) The magnitude of the radial acceleration is the sum of the components of -22.5 and 20.2 along the rope and in the inwards direction,

$$a_r = 20.2\cos(36.9) + 22.5\sin(53.1) = 29.7\text{m/s}^2$$

$$\text{(c) } a_r = \frac{v^2}{r} \Rightarrow v = \sqrt{r a_r} = \sqrt{(1.5)(29.7)} = 6.67 \text{ m/s}$$

at 36.9° above the horizontal.

4-43. (a) Can a particle moving with instantaneous speed 3.00 m/s on a path with radius of curvature 2.00 m have an acceleration of magnitude 6.00 m/s²? (b) Can it have an acceleration of magnitude 4.00 m/s²? In each case, if the answer is yes, explain how it can happen; if the answer is no, explain why not.

$$a_r = \frac{v^2}{r} = \frac{3^2}{2} = 4.5\text{m/s}^2$$

$$a = \sqrt{a_t^2 + a_r^2}$$

(a) Yes because $\mathbf{a}_t \neq \mathbf{0}$ tangential acceleration may not zero. And the particle may be speeding or slowing

$$a_t = \sqrt{a^2 - a_r^2} = \sqrt{6^2 - 4.5^2} = 3.97\text{m/s}^2$$

(b) Total \mathbf{a} cannot be less than \mathbf{a}_r .

Sec 4.6 غير مطلوب

4-47. A police car traveling at 95.0 km/h is traveling west, chasing (يطارد) a motorist traveling at 80.0 km/h.

(a) What is the velocity of the motorist relative to the police car? (b) What is the velocity of the police car relative to the motorist? (c) If they are originally 250 m apart, in what time interval will the police car overtake the motorist?

$$(a) v_{MP} = v_{MG} + v_{GP} = v_{MG} - v_{PG} = 80 - 95 = -15 \text{ km/h}$$

$$(b) v_{PM} = v_{PG} + v_{GM} = v_{PG} - v_{MG} = 95 - 80 = 15 \text{ km/h}$$

(c) Relative to motorist (origin) police is approaching at +15 km/h.

$$d = v_{PM} t = v \cdot t$$

$$\Rightarrow t = \frac{d}{v} = \frac{0.250 \text{ km}}{15 \text{ km/hr}} = 1.67 \times 10^{-2} \times \frac{3600}{1} = 60 \text{ s}$$