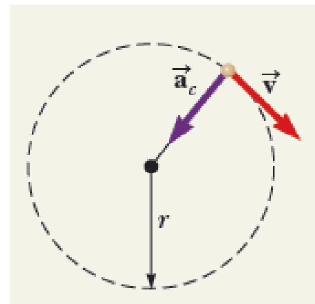
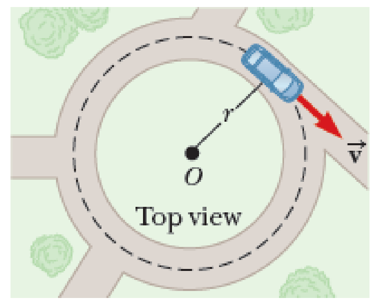
Ch6. Circular Motion page 150

CH6	Circular	4 hrs.	Discussion ch6: 1, 6, 8, 10, 16, 18, 65

4.4 Analysis Model: Particle in Uniform Circular Motion page 91

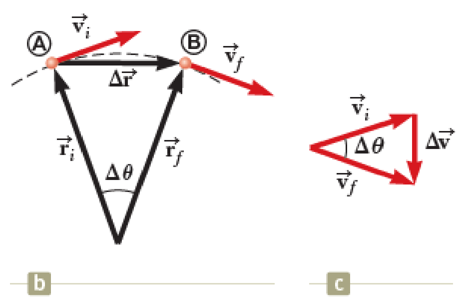


When an object **is moving on circular path** with ***constant speed v***, we call **this motion uniform circular motion.**
Velocity changes in direction but its magnitude remains constant, the object has an acceleration called the centripetal **(مركزي) acceleration**



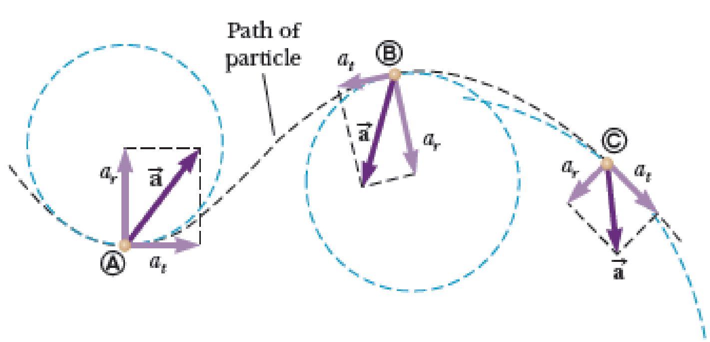
$$\frac{|\Delta\vec{v}|}{v}=\frac{|\Delta\vec{r}|}{r} \quad , |\vec{a}_{avg}|=\frac{|\Delta\vec{v}|}{|\Delta t|}=\frac{v|\Delta\vec{r}|}{r\Delta t}$$

in the limit $\Delta t \to 0$ $a_c=\frac{v^2}{r}$



$a_c=\frac{v^2}{r}$ [يشير نحو المركز]

The time to complete one circle is called **period** *T*, Where T = distance travelled/v
= 2π r/ v = المحيط/ السرعة



$\omega=\frac{2\pi}{T}$ is the angular velocity

4.5 Tangential and Radial Acceleration page 94

When the of a particle

moving in curved path with velocity changes **both in direction and in magnitude, the particle has a total acceleration**

a = a_t + a_r

$$a_t = \frac{dv}{dt}, \qquad a_r = -a_c = -\frac{v^2}{r}$$

$$a = \sqrt{a_t^2 + a_r^2}$$

Example 4.6 The Centripetal Acceleration of the Earth

(A) What is the centripetal acceleration of the Earth as it moves in its orbit around the Sun?

$$v = \frac{2\pi r}{T},$$

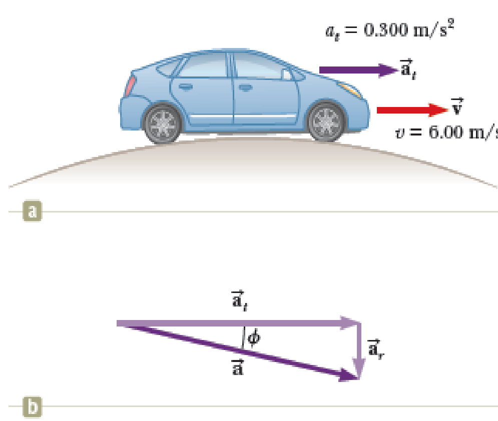
$$a_c = \frac{v^2}{r} = \frac{(\frac{2\pi r}{T})^2}{r} = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2 \times 1.496 \times 10^{11}}{(365 \times 24 \times 60 \times 60)^2} = 5.93 \times 10^{-3} m/s^2$$

$$1yr = 365 \times 24 \times 60 \times 60 \ \ sec$$

(B) What is the angular speed of the Earth in its orbit around the Sun?

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{1yr} = \frac{2\pi}{3.156 \times 10^7 s} = 1.99 \times 10^7 s^{-1}$$

Example 4.7 Over the Rise



A car leaves a stop sign and exhibits **a constant acceleration of 0.300 m/s² parallel to the roadway**. The car passes over a rise in the roadway such that the top of the rise is shaped like an **arc of a circle of radius 500 m**. At the moment the car is at the top of the rise, **its velocity vector is horizontal** and has a **magnitude of 6.00 m/s**. What are the magnitude and direction of **the total acceleration vector** for the car at this instant?

Solution

$$a_r = -a_c = -\frac{v^2}{r} = -\frac{6.00^2}{500} = -0.072 m/s^2$$

$$a = \sqrt{a_t^2 + a_r^2} = \sqrt{(-0.072)^2 + (0.300)^2} = 0.309\ m/s^2$$

$$\theta_a = \tan^{-1}\left(\frac{-0.072}{0.300}\right) = -13.5^\circ$$

Section 4.5 Tangential and Radial Acceleration

4-40. Figure P4.40 represents the total acceleration of a particle moving clockwise in a circle of radius 2.50 m at a certain instant of time. For that instant,

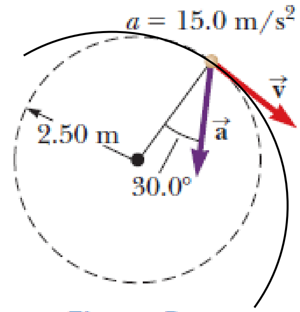


Figure P4.40

find (a) the radial acceleration of the particle, (b) the speed of the particle, and (c) its tangential acceleration.

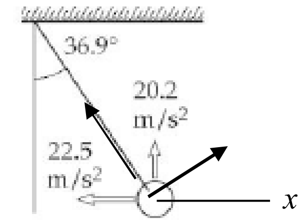
$a_r = -a_c = -a \cos 30 = -15.0\ (0.866) = -13\ m/s^2$

$v^2 = a_c r = 13 \times 2.5 = 32.5 \Rightarrow v = \sqrt{32.5} = 5.7 m/s$

$a_t = a \sin 30 = 15.0\ (0.5) = 7.5\ m/s^2$

4-42. A ball swings **(يتأرجح)** counterclockwise in a vertical circle at the end of a rope 1.50 m long. When the ball is 36.9° past the lowest point (ادنى نقطة) on its way up, its total acceleration is $(-22.5\hat{i} + 20.2\hat{j})$ m/s^2 .

For that instant, (a) sketch a vector diagram showing the components of its acceleration, (b) determine the magnitude of its radial acceleration, and (c) determine the speed and velocity of the ball.



ANS. FIG. P4.42

(a) The diagram is shown in the figure.
(b) The magnitude of the radial acceleration is the sum of the components of -22.5 and 20.2 along the rope and in the inwards direction,

$$a_r = 20.2\cos(36.9) + 22.5\cos(53.1) = 29.7 m/s^2$$

(c) $$a_r = \frac{v^2}{r} \quad \Rightarrow v = \sqrt{r\, a_r} = \sqrt{(1.5)(29.7)} = 6.67\ m/s$$

at 36.9° above the horizontal.

4-43. (a) Can a particle moving with instantaneous speed 3.00 m/s on a path with radius of curvature 2.00 m have an acceleration of magnitude 6.00 m/s^2? (b) Can it have an acceleration of magnitude 4.00 m/s^2? In each case, if the answer is yes, explain how it can happen; if the answer is no, explain why not.

$$a_r = \frac{v^2}{r} = \frac{3^2}{2} = 4.5 m/s^2$$

$$a = \sqrt{a_t^2 + a_r^2}$$

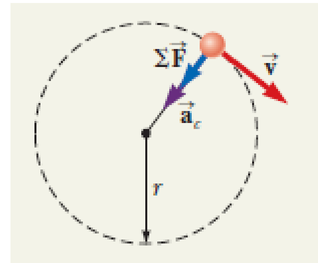
(a) Yes because **$a_t \neq 0$** tangential acceleration may not zero. And the particle may be speeding or slowing

$$a_t = \sqrt{a^2 - a_r^2} = \sqrt{6^2 - 4.5^2} = 3.97 m/s^2$$

(b) Total **a** cannot be less than **a_r**.

Chapter 6

Circular Motion and Other Applications of Newton's Laws



6.1 Extending the Particle in Uniform Circular Motion Model

A particle moving with **constant speed *v*** in a circular path of radius ***r*** experiences an acceleration that has a magnitude

$$a_c = \frac{v^2}{r}$$

a_c is the centripetal acceleration, directed toward the center of the circle and perpendicular to the velocity v.

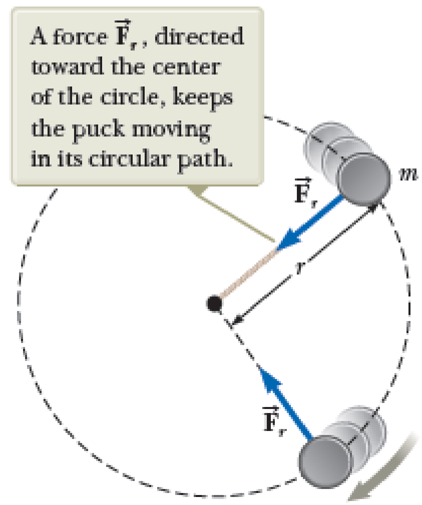


Figure 6.1 An overhead view of a puck moving in a circular path in a horizontal plane.

Consider a puck of mass *m* that is tied to a string of length *r* and moves at constant speed in a horizontal, circular path as illustrated in Figure 6.1. Its weight is supported by a frictionless table, and the string is anchored to a peg at the center of the circular path of the puck.

The radial force F_r directed along the string toward the center of the circle, causing the centripetal acceleration and from Newton's 2nd law:

$$\sum F_r = ma_c = m\frac{v^2}{r},$$

When the string is cut the ball leaves the circle and move in straight line.

Example 6.1 The Conical Pendulum

A small object of mass *m* is suspended from a string of length *L*. The object revolves with constant speed *v* in a horizontal circle of radius *r*, as shown in Figure. (Because the string sweeps out the surface of a cone, the system is known as a *conical pendulum*.) Find an expression for *v*.

Solution

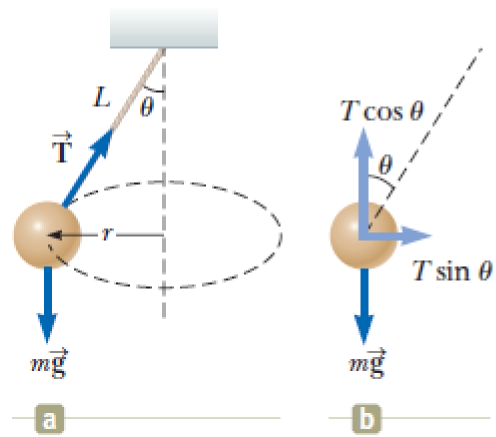


Figure 6.3 (Example 6.1) (a) A conical pendulum. The path of the ball is a horizontal circle. (b) The forces acting on the ball.

From the **free-body diagram** shown, the force T exerted by the string is resolved into two components, such that:

$$\sum F_y = ma_y = 0 = T\cos\ \theta - mg$$

The centripetal force:

$$\sum F = ma_c, \quad or \quad T\sin\theta = m\frac{v^2}{r},$$

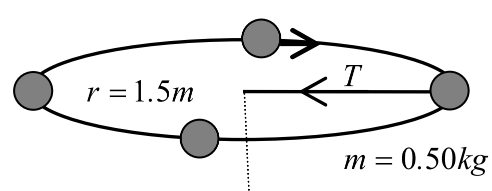
$$T\cos\ \theta = mg$$

$$T\sin\theta = m\frac{v^2}{r} \qquad \Rightarrow \tan\theta = \frac{v^2}{rg}$$

$$v = \sqrt{rg\tan\theta}$$

Further: $r = L\sin\theta$, $v = \sqrt{g\ L\sin\theta\tan\theta}$

Example 6.2 How Fast Can It Spin?



A puck of mass 0.500 kg is attached to the end of a cord 1.50 m long. The puck moves in a horizontal circle as shown in Figure. If the cord can withstand a maximum tension of 50.0 N, what is the maximum speed at which the puck can move before the cord breaks? Assume that the string remains horizontal during the motion.

Solution

$$F_r = T = m\frac{v^2}{r}, \qquad \Rightarrow v = \sqrt{\frac{T\,r}{m}}$$

The maximum speed corresponds to the maximum tension

$$v_{max} = \sqrt{\frac{T_{max}\,r}{m}} = \sqrt{\frac{50\times 1.5}{0.5}} = 12.2\,m/s$$

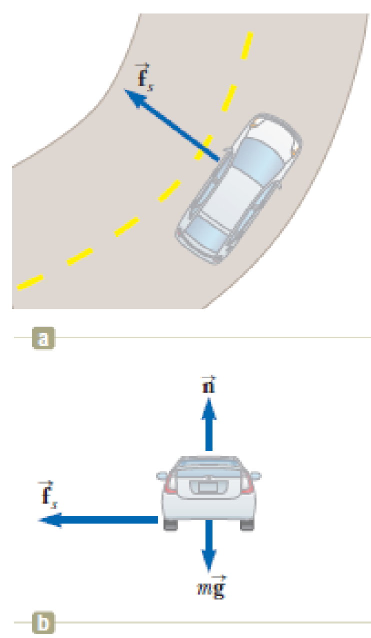
What If? Suppose that the puck moves in a circle of larger radius at the same speed v. Is the cord more likely to break or less likely?

$$T = m\frac{v^2}{r}$$

Same **v** and **large r** means **smaller T**, the string is less likely to break.

Example 6.3 What Is the Maximum Speed of the Car?

A 1 500-kg car moving on a flat, horizontal road negotiates a curve, as shown in Figure. If the radius of the curve is 35.0 m and the coefficient of static friction between the tires and dry pavement is 0.523, find the maximum speed the car can have and still make the turn successfully.



Solution

The force that enables the car to remain in its circular path is **the force of static**

friction f_s. (*Static* because no slipping occurs at the point of contact between road and tires). If $\mathbf{f_s}$ were zero— the car would continue in a straight line and
Slide off the road.)

The maximum speed to round the curve is the speed at which it is on the verge of skidding outward where **f_s is maximum at this moment:**

$$f_{s,max} = m\frac{v^2}{r}, \qquad \text{but} \qquad f_{s,max} = \mu_s n = \mu_s mg = m\frac{v^2}{r}$$

$$\Rightarrow v_{max} = \sqrt{\frac{\mu_s mgr}{m}} = \sqrt{\mu_s gr} = \sqrt{0.5 \times 9.8 \times 35} = 13.1\, m/s$$

What If? Suppose that **a car travels this curve on a <u>wet day</u> and begins to skid on the curve** when its speed reaches only **8.00 m/s. What can we say about the coefficient of static friction in this case?**

μ_s for a wet road is smaller, since when the car skids:

$$f_{s,max} = \mu_s mg = m\frac{v_{max}^{\ 2}}{r} \quad \Rightarrow \mu_s = \frac{v_{max}^2}{gr} = \frac{8^2}{9.8 \times 35} = 0.187$$

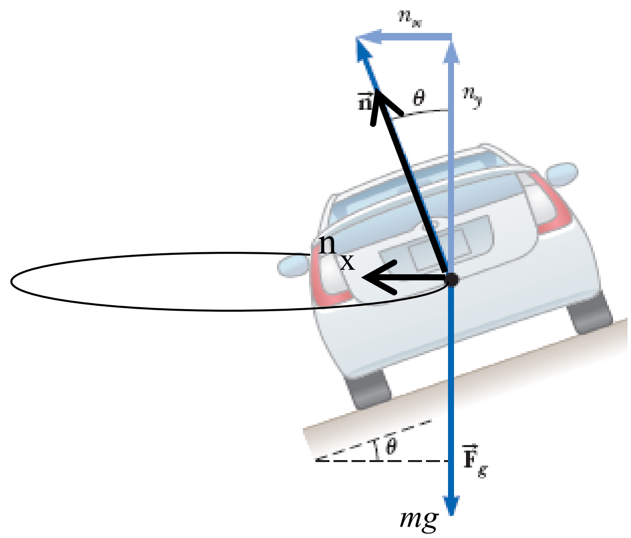
Example 6.4 The Banked Roadway

A civil engineer wishes to design a curved exit ramp for a highway in such a way that a car **will not have to rely on friction** to round the curve without skidding. In other words, a car moving at the designated speed can negotiate the curve even when the road is covered with ice.

Such a ramp is usually *banked*; this means the roadway **is tilted toward the inside of the curve**. Suppose the designated **speed for**

the ramp is to be 13.4 m/s (30.0 mi/h) and the **radius of the curve is 35.0 m**. **At what angle** should the curve be banked?

Solution



On a level (**unbanked**) road static friction between car and the road causes the centripetal acceleration (previous example). **For an icy road, Friction is no longer active. Car will skip out.** Since the road banked at an angle θ to the horizontal, and **static the force of static friction is (neglected) zero** as in Figure, the **only force** that causes the **centripetal acceleration and keeps the car moving in its circular curve** is the horizontal component of the normal force $N \sin\theta$ pointing toward the center of the curve.

$$\sum F_r = N \sin\theta = m\frac{v^2}{r}$$

$$N \cos\theta = mg$$

$$\Rightarrow \tan\theta = \frac{v^2}{rg}, \quad \theta = \tan^{-1}\left(\frac{13.4^2}{35 \times 9.8}\right) = 27.5^o$$

If **v > 13.4** friction is needed to keep the car from sliding up the bank.

Example 6.5 Riding the Ferris Wheel

A child of mass *m* rides on a Ferris wheel as shown in Figure 6.6a. The child moves in a vertical circle of radius 10.0 m at a constant speed of 3.00 m/s.

(A) Determine the force exerted by the seat on the child at the bottom of the ride. Express your answer **in terms** of the weight of the child, ***mg***.

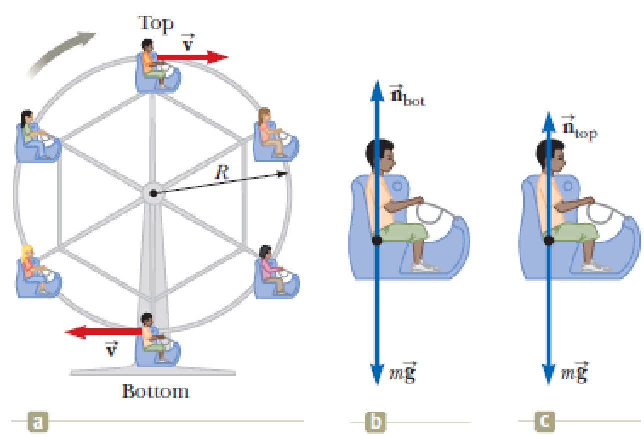
Solution

(A) At the bottom (b): Force exerted on the child are his weight downward, and the upward force by the seat $\mathbf{n_{bot}}$.

$$\sum F_r = n_{bot} - mg = m\frac{v^2}{r}$$

$$n_{bot} = mg + m\frac{v^2}{r} = mg(1 + \frac{v^2}{rg})$$

$$= mg(1 + \frac{3^2}{10\cdot(9.8)}) = 1.09mg$$



(B) Determine the force exerted by the seat on the child at the top of the ride (c).

Solution

At the top: $\sum F_r = mg - n_{top} = m\frac{v^2}{r}$

$$n_{top} = -m\frac{v^2}{r} + mg = mg(1 - \frac{v^2}{rg})$$

$$= mg(1 - \frac{3^2}{(10)(9.8)}) = 0.908\ mg$$

6.2 Nonuniform Circular Motion

If in a circular path the speed **varies**, in addition to the radial acceleration, there is **a tangential acceleration** having magnitude $a_t = dv/dt$.

$$\mathbf{a} = \mathbf{a}_r + \mathbf{a}_t$$

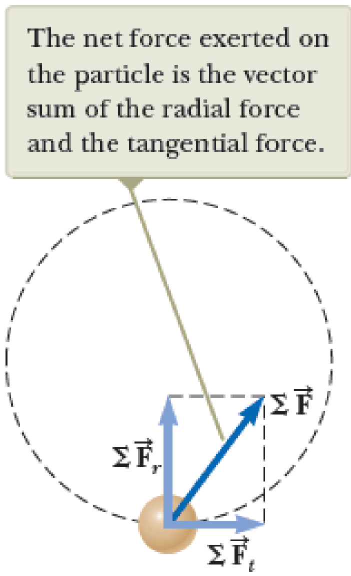


Figure 6.7 When the net force acting on a particle moving in a circular path has a tangential component ΣF_t, the particle's speed changes.

$$\sum \vec{\mathbf{F}} = \sum \vec{\mathbf{F}}_r + \sum \vec{\mathbf{F}}_t$$

$$\mathbf{a} = \mathbf{a}_r + \mathbf{a}_t$$

The net force have a tangential and a radial component.

$$\sum \vec{\mathbf{F}} = \sum \vec{\mathbf{F}}_r + \sum \vec{\mathbf{F}}_t$$

When a force that has a **tangential component** acts on a particle moving in a circular path, the speed of the particle changes.

Example 6.6

Keep Your Eye on the Ball

A small sphere of **mass *m*** is attached to the end of a cord of **length *R*** and set into motion in a ***vertical*** circle about a fixed point *O*, as illustrated in Figure. **Determine the tension** in the **cord at any instant when the speed of the sphere is *v*** and the **cord makes an angle θ with the vertical**.

Solution

At most points along the path the ball is affected by two forces: the gravitational force **Fg** = **mg** exerted by the Earth, and the force **T** exerted by the cord. $\mathbf{F_g}$ **has two components, tangential and radial as shown in figure.**

Tangential force:

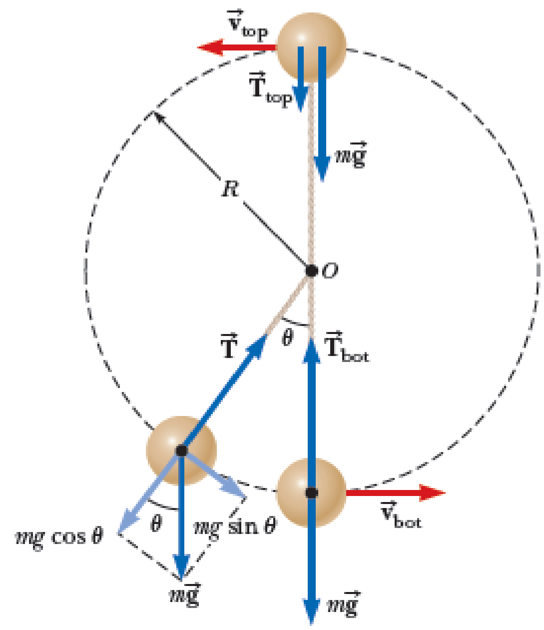
$$\sum F_t = mg\sin\theta = ma_t ,$$

$$a_t = g\sin\theta$$

Radial Force:

$$\sum F_r = T - mg\cos\theta = m\frac{v^2}{R},$$

$$T = m(g\cos\theta + \frac{v^2}{R})$$



What If? What if we set the ball in motion with a slower speed?

(A) What speed would the ball have as it passes over the top of the circle if the tension in the cord goes to zero instantaneously at this point?

Answer Let us set the tension equal to zero in the expression for T_{top}:

At the top of the path, where $\theta = 180^\circ$, we have **cos180º** = -1, and the tension becomes $T_{top} + mg = \frac{v_{top}^2}{R}$

$$T_{top} = m(-g + \frac{v_{top}^2}{R}) = 0, \quad v_{top} = \sqrt{gR}$$

(B) What if the ball is set in motion such that the speed at the top is less than this value? What happens?

Answer In this case, the **ball never reaches the top of the circle**. At some point on the way up, the tension in the string goes to zero and the ball becomes a projectile. It follows a segment of a parabolic path over the top of its motion, rejoining the circular path on the other side when the tension becomes nonzero again.

Example: The Simple Pendulum

Simple pendulum consists of small plumb bob of mass m swinging at the end of a light, inextensible string of length l along a circular arc defined by angle θ. The restoring force is the component of weight mg in the direction of increasing θ.

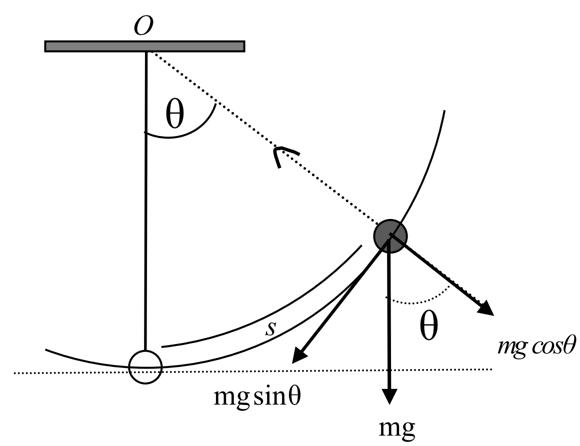
(a) Radial Force:

$$\sum F_r = T - mg\cos\theta = m\frac{v^2}{R},$$

(b) Tangential force:

$$\sum F_t = -mg\sin\theta = m\,a_t,$$

$$a_t = -g\sin\theta$$



Problem 6-18. One end of a cord is fixed and a small 0.500-kg object is attached to the other end, where it swings in a section of a vertical circle of radius 2.00 m as shown in Figure P6.18. When θ = 20°, the speed of the object is 8.00 m/s. **At this instant**, find (a) the **tension** in the string, (b) **the tangential and radial** components of acceleration, and

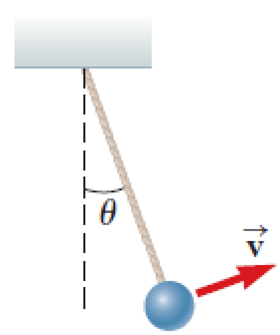
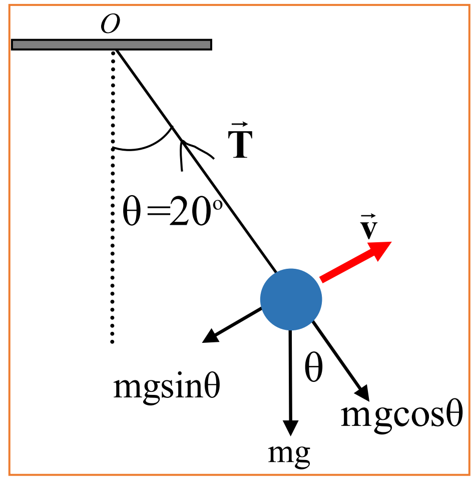


Figure P6.18

(c) **the total acceleration**. (d) **Is your answer changed** if the object is **swinging down** toward its lowest point **instead of swinging up**? (e) Explain your answer to part (d).



(a) Radial Force:

$$\sum F_r = T - mg\cos\theta = m\frac{v^2}{R},$$

$$T = m\left(g\cos\theta + \frac{v^2}{R}\right)$$

$$= 0.5\left(10\cos(20) + \frac{64}{2}\right) = 20.7\ m/s^2$$

(b) Tangential force: $\sum F_t = -mg\sin\theta = m\,a_t,$

$$a_t = -g\sin\theta = -10\sin(20) = -3.42\ m/s^2$$

$$a_r = \frac{v^2}{R} = \frac{8^2}{2} = 32\ m/s^2$$

(c) $a = \sqrt{32^2 + (-3.42)^2} = \sqrt{1024 + 11.7} = 32.18\ m/s^2$

(d) No, it does not changed. $a_t = +3.42\ m/s^2$

(e) a_r Always toward the center,

$a_t = -g\sin\theta = -10\sin(20) = -3.42\ m/s^2$ always perpendicular to v and a_r. but reverse direction in the same as mgsinθ.

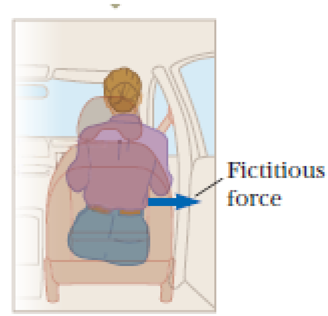
ما بعد هذا الجزء من الفصل (Ch6) ليس مطلوبا في الامتحانات للمطالعة الذاتية
اقصد 6.3 +6.4 غير مطلوبات

6.3 Motion in Accelerated Frames

Newton's laws are valid only in inertial frames of reference. How Newton's second law is applied by an observer in a noninertial frame of reference?

A puck on a hocky table which is in a train moving with constant v, at rest remains at rest, and Newton's first law is obeyed.

An accelerating train is not an inertial frame. For an observer on the train, there appears to be no visible force on the puck, yet it accelerates from rest toward the back of the train, violating Newton's first law.
The force, which causes the puck to accelerate, is an apparent force called the fictitious force, because it is due to an accelerated reference frame.



Another fictitious force is due to the change in the *direction* of the velocity vector (noninertial system) a car traveling along a highway at a high speed and approaching a curved exit ramp, as shown in Figure. As the car takes, the sharp left turn onto the ramp, a person sitting in the passenger seat slides to the right and hits the door. At that point, the force exerted by the door on the passenger keeps her from being ejected from the car.

Incorrect explanation is that force acting toward the right pushes her out. The force often called the "centrifugal force," but it is a fictitious force due to the acceleration associated with the changing direction of the car's velocity vector. (The driver also experiences this effect but wisely holds on to the steering wheel to keep from sliding to the right.)

Correctly, explained as follows. As the car enters the ramp and travels a curved path, the passenger tends to move along the original straight-line path. This is in accordance with Newton's first law: the natural tendency of an object is to continue moving in a straight line.

If the force of friction between her and the car seat is not large enough, she slides to the right as the seat turns to the left under her. Eventually, she encounters the door, which provides a force large enough to enable her to follow the same curved path as the car. She slides toward the door not because of an outward force but because the force of friction is not sufficiently great to allow her to travel along the circular path followed by the car.

Fictitious force commonly called "centrifugal force" is described as a force pulling *outward on* an object moving in a circular path and occurs as a result of the noninertial reference frame.

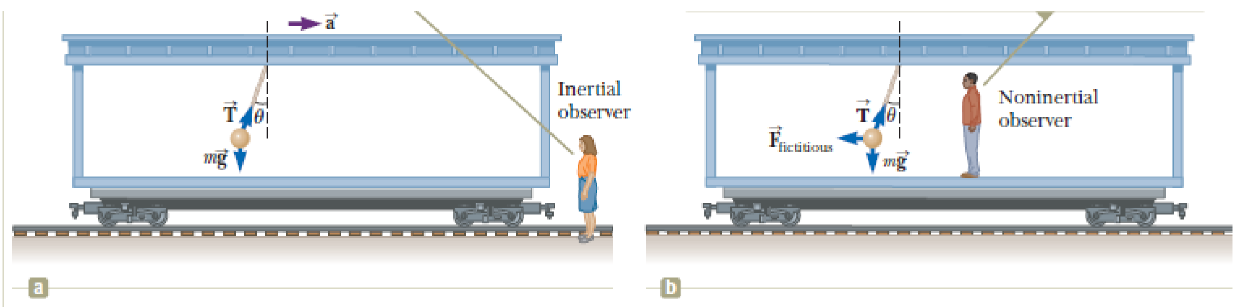
Another interesting fictitious force is the "Coriolis force." This is an apparent force caused by changing the radial position of an object in a rotating coordinate system.

For example, you and your friend sit at the edge of a rotating turntable. Viewed by an observer in an inertial reference frame attached to the Earth, you throw the ball in the direction of your friend. Later time *the* ball arrives at the other side of the turntable, your friend is no longer there to catch it.

According to this observer, the ball followed a straight line path, consistent with Newton's laws. Viewed by your friend, the ball veers to one side during its flight.
Your friend introduces a fictitious force to cause this deviation from the expected path. This fictitious force is called the "Coriolis force."

Example 6.8 Fictitious Forces in Linear Motion

A small sphere of mass *m* is hung by a cord from the ceiling of a boxcar that is accelerating to the right, as shown in Figure. The noninertial observer claims that a force, which we know to be fictitious, must act in order to cause the observed deviation of the cord from the vertical. How is the magnitude of this force related to the acceleration of the boxcar measured by the inertial observer?



The inertial observer concludes that the acceleration of the sphere is the same as that of the boxcar which is provided by the horizontal component of T. Also, the vertical component of T balances the gravitational force because the sphere is in equilibrium in the vertical direction.

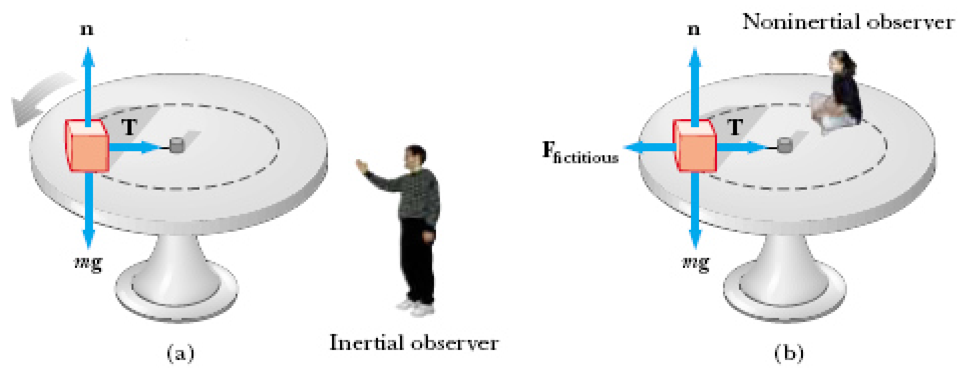
In the inertial frame: $T\sin\theta = ma$ and $T\cos\theta = mg$

According to the noninertial observer the sphere is at rest and so its acceleration is zero. Therefore, he introduces a fictitious force to balance the horizontal component of T

In the noninertial frame: $T\sin\theta - F_{Fictitious} = 0$

and $T\cos\theta = mg$

Example 6.9 Fictitious Force in a Rotating System



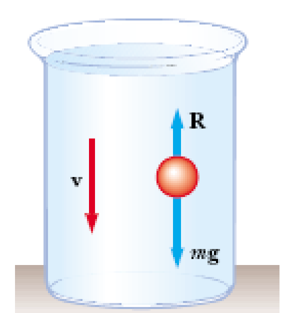
Suppose a block of mass *m* lying on a horizontal, frictionless turntable is connected to a string attached to the center of the turntable, as shown in Figure. How would each of the observers write Newton's second law for the block. In the inertial frame: $T = m\dfrac{v^2}{r}$ and $n = mg$

According to the noninertial observer the block is at rest and so its acceleration is zero. Therefore, he introduces a fictitious force to balance the tension T.

In the noninertial frame:

$$T - F_{Fictitious} = 0 \quad \text{and} \quad n = mg$$

6.4 Motion in the Presence of Resistive Forces



The medium (liquid or gas) exerts a resistive force R on the object moving through it. Some examples are the air resistance called *air drag* and the viscous forces on objects moving through a liquid.

The magnitude of R depends on factors such as the speed of the object increases
with increasing speed.

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Resistive Force Proportional to Object Speed

$R = -bv$, b is a constant.

Consider A small sphere falling through a liquid.

$$mg - R = ma = m\frac{dv}{dt}$$

$$mg - bv = m\frac{dv}{dt} \quad \Rightarrow \frac{dv}{dt} = g - \frac{b}{m}v$$

Solution to the equation is:

$$v(t) = v_T\left(1 - e^{-t/\tau}\right),$$

$\tau = \frac{b}{m}$ is the time constant, e = 2.72 (the Euler number).

Where $\mathbf{v_T}$ is the terminal velocity and it is approached (not reached) after a time where **a = 0**

$$mg - bv_T = 0 \qquad \Rightarrow v_T = \frac{mg}{b}$$