CH. 6 Circular Motion and Other Applications of Newton's Laws:

HWK Key Solution 1, 6, 16, 10, 18

Ch6	Circular Motion	HWK Problems **1, 6, 8, 10, 16, 18, 45, 65** page 101

6-1. A light string **can support** a stationary hanging load of **25.0 kg** before breaking. An object of mass ***m* = 3.00 kg** attached to the string **rotates** on a frictionless, horizontal table **in a circle of radius *r* = 0.800 m**, and the other end of the string is held fixed as in Figure P6.1. What range of speeds can the object have before the string breaks?

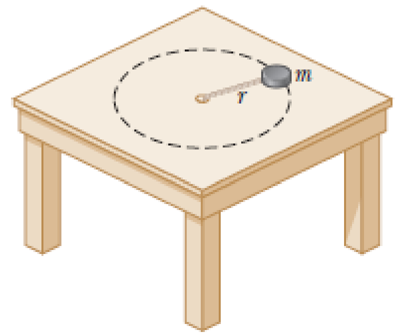
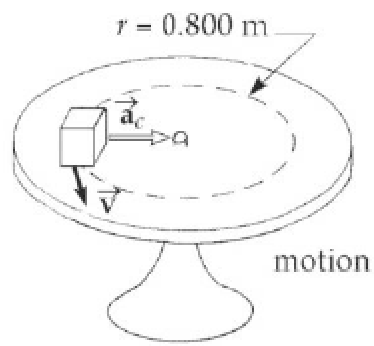


Figure P6.1



The string will break if the tension exceeds the weight corresponding to 25.0 kg, so

$T_{max} = Mg = 25.0 \times 9.80 = 245$ N.

When the 3.00 kg mass rotates in a horizontal circle, the tension causes the centripetal acceleration,

$$T = m\frac{v^2}{r} \Rightarrow v^2 = \frac{rT}{m} = \frac{0.8T}{3} \le \frac{0.8T_{max}}{3} = \frac{0.8 \times 245}{3} = 65.3\, m/s^2$$

$$0 \le v \le \sqrt{65.3} = 8.08\ m/s$$

6-6. A car initially traveling eastward turns north by traveling in a circular path at uniform speed as shown in Figure P6.6. The length of the arc *ABC* is **235 m**, and the car completes the turn in **36.0 s**.

(a) What is the **acceleration when the car is at *B*** located at an angle of **35.0º**? Express your answer in terms of the unit vectors $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$.

Determine (b) the car's average speed and (c) its average acceleration during the **36.0-s interval.**

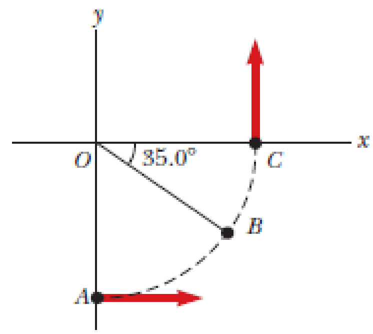


Figure P6.6

(a) The car's speed around the circular path is found from the arc length and time. This is the answer to part (b) of this problem. We calculate the radius of the curve from

$$\frac{1}{4}(2\pi R) = 235 \;\Rightarrow\; R = \frac{2\times 235}{\pi} = 150\ m$$

$$speed = v = \frac{d}{t} = \frac{235}{36} = 6.53\ \ m/s$$

$$|\vec{a}| = a_c = \frac{v^2}{R} = \frac{(6.527)^2}{150} = 0.284\ m/s^2 \quad \texttt{[toward the center]}$$

$$\vec{a} = 0.284\,[-\cos 35\,\hat{\mathbf{i}} + \sin 35\,\hat{\mathbf{j}}] = [-0.233\,\hat{\mathbf{i}} + 0.163\,\hat{\mathbf{j}}\ ]\ \mathrm{m/s^2}$$

(b) $\vec{\mathrm{v}}_{\mathrm{avg}} = |\,\vec{\mathrm{v}}\,| = \ 6.527\ \ \mathrm{m/s^2}$

(c) The speed is constant $|\mathbf{v}| = \mathrm{v}$ velocity has the same magnitude but changes its direction

$$\mathbf{v}_A = \mathrm{v}\,\hat{\mathbf{i}}\ ;\quad \mathbf{v}_C = \mathrm{v}\,\hat{\mathbf{j}} \;\Rightarrow\; \vec{\mathbf{a}}_{\mathbf{avg}} = \frac{\vec{\mathbf{v}}_C - \vec{\mathbf{v}}_A}{36} = \frac{6.527\,\hat{\mathbf{i}} - 6.527\,\hat{\mathbf{j}}}{36}$$

6-8. Consider a conical pendulum (Fig. P6.8) with a bob of mass m = 80.0 kg on a string of length L = 10.0 m that makes an angle of θ = 5.00° with the vertical.
Determine
(a) the horizontal and vertical components of the force exerted by the string on the pendulum and
(b) the radial acceleration of the bob.

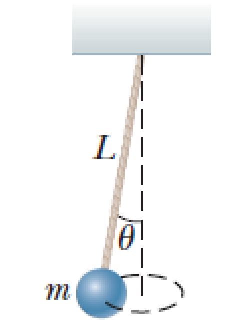
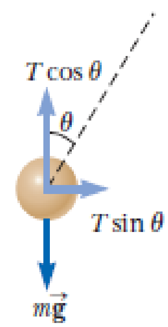


Figure P6.8

[Refer to Example 6-1]



$$\sum F_y = ma_y = 0 = T\cos\theta - mg, \Rightarrow T\cos\theta = mg$$

$$T_y = T\cos\theta = mg = 80 \times 10 = 800\text{N}$$

$$T\cos\theta = 800\text{N} \Rightarrow T = \frac{800}{\cos 5} = 803\text{N}$$

$$T_x = T\sin\theta = 803\ \sin(5) = 70\text{N}$$

The centripetal force:

$$\sum F = ma_c, \text{ or } T_x = T\sin\theta = ma_r = 70, \quad \Rightarrow a_r = \frac{70}{80} = 0.875\text{m/s}^2$$

6-10. *Why is the following situation impossible?* The object of mass $\boldsymbol{m} = \mathbf{4.00\ kg}$ in Figure P6.10 is attached to a vertical rod by two strings of length , $l = \mathbf{2.00\ m}$. The strings are attached to the rod at points a distance $\boldsymbol{d} = \mathbf{3.00\ m}$ apart. The object rotates in a horizontal circle at a constant speed of $\boldsymbol{v} = \mathbf{3.00\ m/s}$, and the strings remain taut. The rod rotates along with the object so that the strings do not wrap onto the rod.

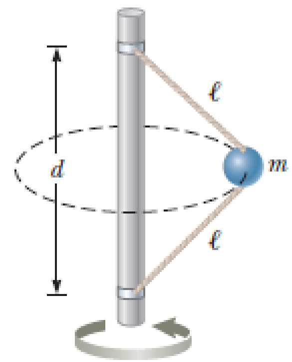


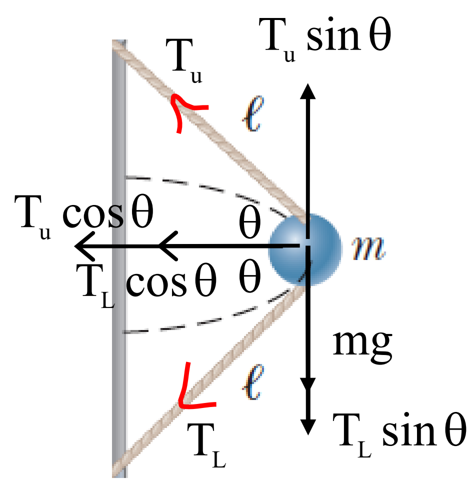
Figure P6.10

What If? Could this situation be possible on another planet?

$$mg = 4.0 \times 9.8 = 39.2\ \text{N}$$

$$\sin\theta = \frac{1.5}{2.0}, \quad \theta = \sin^{-1}\left(\frac{1.5}{2.0}\right) = 48.6^\circ$$

$$r = L\cos\theta = 2.0\cos 48.6 = 1.32\ \text{m}$$



$$\sum F_y = ma_y = 0$$

$$T_u \sin\theta = T_L \sin\theta + mg,$$

$$T_u - T_L = \frac{39.2}{\sin 48.6} = 52.3$$

The centripetal force:

$$\sum F = ma_c; \ ,$$

$$T_L \cos\theta + T_u \cos\theta = m\frac{v^2}{r} = 4.0 \cdot \frac{3^2}{1.32} = 27.27$$

$$T_L + T_u = 41.2$$

$$T_u - T_L = 52.3 \qquad 2T_u = 52.3 + 41.2 = 93.8$$

$$T_u = \frac{93.8}{2} = 46.9, \quad T_L = 41.2 - T_u = 41.2 - 46.9 = -5.7\ \text{N}$$

The situation is impossible because the speed of the object is too small, requiring that the lower string act like a rod and push rather than like a string and pull.

6-16. A roller-coaster car (Fig. P6.16) has a mass of **500 kg** when Fully loaded with passengers. The path of the coaster from its initial point shown in the figure to point **B** involves only up-and-down motion (as seen by the riders), with no motion to the left or right. **(a)** If the vehicle has a speed of **20.0 m/s at point A**, what is the force exerted by the track on the car at this point? **(b)** What is the maximum speed the vehicle can have at point B and still remain on the track? Assume the roller-coaster tracks at points A and B are parts of vertical circles of radius r_1 = **10.0** m and r_2 = **15.0** m, respectively.

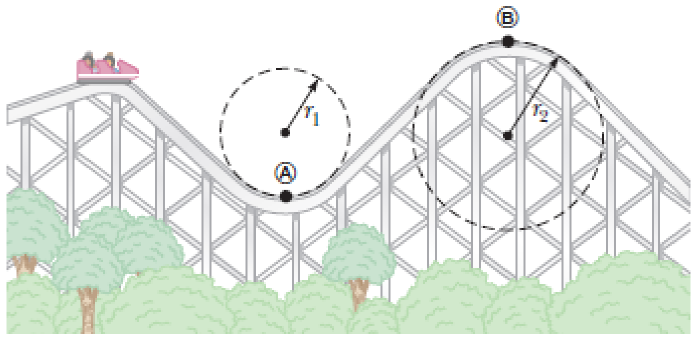
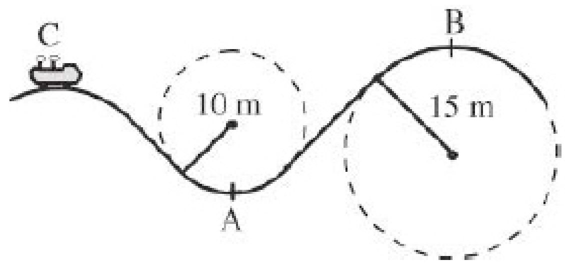


Figure P6.16 Problems 16 and 38.



(a) Coaster at A:

$$N - mg = m\frac{v^2}{r} \Rightarrow N = m\left(\frac{v^2}{r} + g\right) = 500\left(\frac{400}{10} + 10\right) = 25000N$$

(b) Coaster at max speed at B and remain on track, it just starts to leave as $N \to 0$. **Max. speed corresponds to N=0**

$$mg - N = m\frac{v^2}{r}$$

$$mg = m\frac{v^2}{r} \Rightarrow v = \sqrt{rg} = \sqrt{150} = 12.1m/s$$

6-18. One end of a cord is fixed and a small 0.500-kg object is attached to the other end, where it swings in a section of a vertical circle of radius 2.00 m as shown in Figure P6.18. When θ = 20°, the speed of the object is 8.00 m/s. **At this instant**, find (a) the **tension** in the string, (b) **the tangential and radial** components of acceleration, and (c) **the total acceleration**. (d) **Is your answer changed** if the object is **swinging down** toward its lowest point **instead of swinging up**? (e) Explain your answer to part (d).

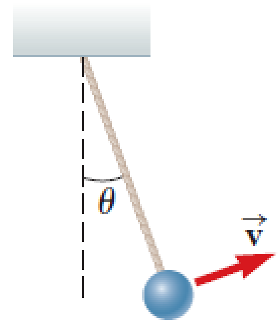
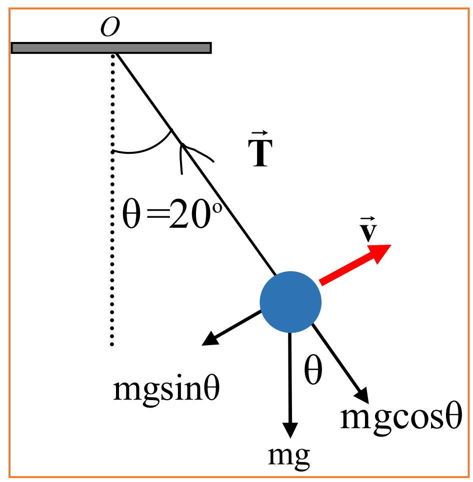


Figure P6.18



(a) Radial Force:

$$\sum F_r = T - mg\cos\theta = m\frac{v^2}{R},$$

$$T = m\left(g\cos\theta + \frac{v^2}{R}\right)$$

$$= 5\left(10\cos(20) + \frac{64}{2}\right) = 20.\ 7\ \text{N}$$

(b) Tangential force: $\sum F_t = -mg\sin\theta = m\,a_t,$

$$a_t = -g\sin\theta = -10\sin(20) = -3.42\ \text{m/s}^2$$

$$a_r = \frac{v^2}{R} = \frac{8^2}{2} = 32\ \text{m/s}^2$$

(c) $a = \sqrt{32^2 + (-3.42)^2} = \sqrt{1024 + 11.7} = 32.18\ \text{m/s}^2$

(d) No, it does not changed.

(e) a_r Always toward the center,

$a_t = -g \sin\theta = -10\sin(20) = -3.42\ \text{m/s}^2$ always parallel to v

and perpendicular to the a_r .

6-45. A ball of mass m = 0.275 kg swings in a vertical circular path on a string L = 0.850 m long as in Figure P6.45. **(a)** what are the forces acting on the ball <u>at any point on the path</u>? **(b)** Draw force diagrams for the ball **when it is at the bottom** of the circle and when it is at the top. **(c)** If its **speed is 5.20 m/s at the top of the circle, what is the tension** in the string there? **(d)** If the **string breaks when its tension exceeds 22.5 N**, what is the **maximum speed the ball** can have <u>**at the bottom**</u> before that happens?

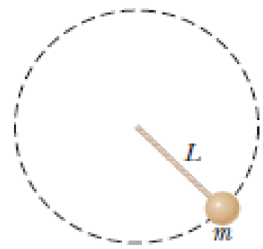
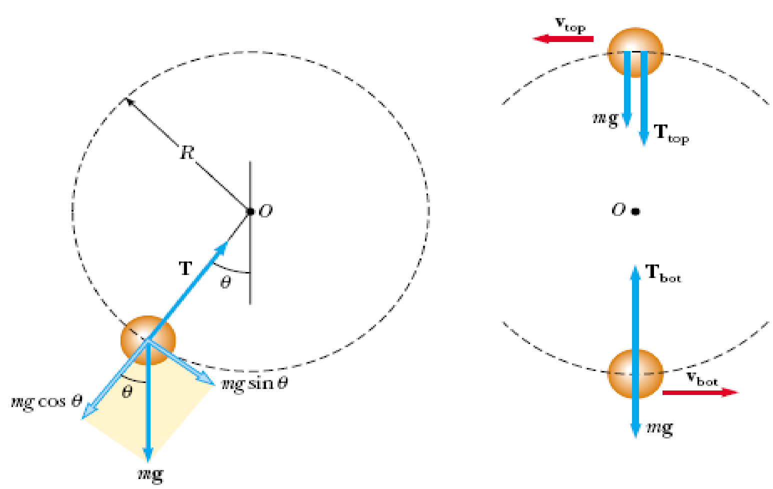


Figure P6.45



(a) Two forces acting on the ball at each point on the vertical circular path,
(1) The downward gravitational force **mg**
(2)The tension force **T** in the string, directed toward the center of the path
(b) Force diagrams for top and bottom cases shown above.
(c) Ball at top:

$$\sum F_r = T + mg = m\frac{v^2}{R}$$

$$T = m(\frac{v^2}{R} - g) = 0.275(\frac{5.2^2}{0.85} - 9.8) = 6.05\ N$$

(d) At bottom:

$$\mathrm{T - mg = m\frac{v^2}{R}},$$

$$v^2 = \frac{R}{m} \cdot (T - mg) = R(\frac{T}{m} - g)$$

$$v_{max} = \sqrt{R(\frac{T_{max}}{m} - g)} = \sqrt{(0.85)(\frac{22.5}{0.275} - 9.8)} = 7.82\ m/s$$

Problem 6-65: On sec. 6.4-6.5 Not required

65. A 9.00-kg object starting **from rest** falls through a viscous medium and experiences a resistive force given by Equation 6.2. The object reaches one half its terminal speed in 5.54 s.
(a) Determine the terminal speed.
(b) At what time is the speed of the object three fourths the terminal speed? (c) How far has the object traveled in the first 5.54 s of motion?

At $t \to \infty$: $\quad v \to v_T = \dfrac{mg}{b}$

At $t = 5.54$ s: $\quad 0.500 v_T = v_T\left[1 - \exp\left(\dfrac{-b(5.54\ \mathrm{s})}{9.00\ \mathrm{kg}}\right)\right]$

Solving,

$$\exp\left(\frac{-b(5.54\ \mathrm{s})}{9.00\ \mathrm{kg}}\right) = 0.500$$

$$\frac{-b(5.54\ \mathrm{s})}{9.00\ \mathrm{kg}} = \ln 0.500 = -0.693$$

$$b = \frac{(9.00\ \mathrm{kg})(0.693)}{5.54\ \mathrm{s}} = 1.13\ \mathrm{kg/s}$$

From $v_T = \dfrac{mg}{b}$, we have

$$v_T = \frac{(9.00\ \mathrm{kg})(9.80\ \mathrm{m/s^2})}{1.13\ \mathrm{kg/s}} = \boxed{78.3\ \mathrm{m/s}}$$