Chapter 7 Energy of a System page 177

HWK: 5, 9, 11, 14, 29, 33, 43, 49, 63

7.2 Work Done by a Constant Force

The work **W** done by a constant force **F** is

$$W = F\,\Delta r\,\cos\theta$$

Prob. 7-5. A block of mass ***m*=2.50 kg** is pushed a distance ***d* = 2.20 m** along **a frictionless**, horizontal table by a constant applied force of magnitude ***F* =16.0** N directed at an angle $\theta = 25.0^\circ$ below the horizontal as shown in Figure P7.5.. Determine the work done on the block by

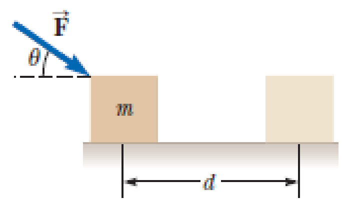


Figure P7.5

(a) the applied force,
(b) the normal force exerted by the table,
(c) the gravitational force, and
(d) the net force on the block.

(a) $W_F = F\,\Delta r\,cos\theta = 2.5\times 2.2\times cos25\ = 31.9\ J$

(b) $W_N = N\ \Delta r\ cos90 =\ 0\ J$

(c) $W_g = mg\Delta r\ cos90 =\ 0\ J$

(d) total work: $W_{tot} = 31.9 + 0 + 0 = 31.9J = W_F$

7.3 The Scalar Product of Two Vectors

The scalar product (often called the **dot product**) of any two vectors and is a scalar quantity:

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$

Work can be expressed as a scalar product:

$$W = F\,\Delta r\cos\theta = \vec{F}\cdot\Delta\vec{r}$$

$$\hat{\mathbf{i}}\cdot\hat{\mathbf{i}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{k}}\cdot\hat{\mathbf{k}} = 1\times 1\times\cos 0 = 1,$$

$$\hat{\mathbf{i}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{k}} = \hat{\mathbf{i}}\cdot\hat{\mathbf{k}} = 1\times 1\times\cos 90 = 0$$

The scalar product of vectors $\vec{A}$ and $\vec{B}$

$$\vec{A} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}, \qquad \vec{B} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}$$

Is reduced to

$$\vec{A}\cdot\vec{B} = A_xB_x\,\hat{\mathbf{i}}\cdot\hat{\mathbf{i}} + A_yB_y\,\hat{\mathbf{j}}\cdot\hat{\mathbf{j}} + A_zB_z\,\hat{\mathbf{k}}\cdot\hat{\mathbf{k}} = A_xB_x + A_yB_y + A_zB_z$$

Example 7.2 The Scalar Product

The vectors $\vec{A}$ and $\vec{B}$ are given by $\vec{\mathbf{A}} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}})$ and $\vec{\mathbf{B}} = (-\hat{\mathbf{i}} + 2\hat{\mathbf{j}})$

Solution

(A) Determine the scalar product of the two vectors.

$$\vec{\mathbf{A}} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}})$$

$$\vec{\mathbf{B}} = (-\hat{\mathbf{i}} + 2\hat{\mathbf{j}})$$

$$\vec{A}\cdot\vec{B} = (2)(-1) + (3)(2) = -2 + 6 = 4,$$ **SINCE**

$$\vec{A}\cdot\vec{B} = -2\hat{\mathbf{i}}\cdot\hat{\mathbf{i}} + 2\hat{\mathbf{i}}\cdot 2\hat{\mathbf{j}} - 3\hat{\mathbf{j}}\cdot\hat{\mathbf{i}} + 3\hat{\mathbf{j}}\cdot 2\hat{\mathbf{j}} = -2(1) + 4(0) - 3(0) + 6(1) = -2 + 6 = 4$$

(B) Find the angle θ between $\vec{A}$ and $\vec{B}$

$$A = \sqrt{A_x^{\,2} + A_y^{\,2}} = \sqrt{2^2 + 3^2} = \sqrt{13}$$

$$B=\sqrt{B_x^{\,2}+B_y^{\,2}}=\sqrt{(-1)^2+(2)^2}=\sqrt{5}$$

$$\cos\theta=\frac{\vec{A}\cdot\vec{B}}{AB}=\frac{4}{\sqrt{13\times 5}}\quad\Rightarrow\theta\ =\ \cos^{-1}\left(\frac{4}{\sqrt{65}}\right)=60.3^{o}$$

Example 7.3 Work Done by a Constant Force

A particle moving in the *xy* plane undergoes a displacement from the **point (-2 m, 0)** to the **point (0,3m)** as a constant force

$\vec{\mathbf{F}}=(5.0\hat{\mathbf{i}}+2.0\hat{\mathbf{j}})$N acts on the particle.

(A) Calculate the magnitudes of the force and the displacement of the particle.

$$\Delta\vec{\mathbf{r}}=\vec{\mathbf{r}}_f-\vec{\mathbf{r}}_i=(x_f\text{-}x_i)\hat{\mathbf{i}}+\ (y_f\text{-}y_i)\hat{\mathbf{j}}\ =0-(-2)\hat{\mathbf{i}}+\ (3.0\text{-}0)\hat{\mathbf{j}}=2\hat{\mathbf{i}}+\ 3\hat{\mathbf{j}}$$

$$F=\sqrt{F_x^{\,2}+F_y^{\,2}}=\sqrt{5^2+2^2}=\sqrt{29}=5.4\,N$$

$$\Delta r=\sqrt{\Delta x^2+\Delta y^2}=\sqrt{(2)^2+(3)^2}=\sqrt{13}=3.6\,m$$

(B) Calculate the work done by F on the particle.

$$\vec{F}\cdot\Delta\vec{r}=5\hat{\mathbf{i}}\cdot 2\hat{\mathbf{i}}+2\hat{\mathbf{j}}\cdot 3\hat{\mathbf{j}}\ =10\ (1)+6(1)=10+6=16\ \ J$$

Prob. 7-9 For $\vec{\mathbf{A}}=\ 3\hat{\mathbf{i}}\ +\hat{\mathbf{j}}\ \text{-}\hat{\mathbf{k}}$, $\vec{\mathbf{B}}=\ \text{-}\hat{\mathbf{i}}\ +2\hat{\mathbf{j}}\ +5\hat{\mathbf{k}}$ and

$\vec{\mathbf{C}}=2\hat{\mathbf{j}}\ -3\hat{\mathbf{k}}$ Find $\vec{\mathbf{C}}\cdot(\vec{\mathbf{A}}-\vec{\mathbf{B}})$

$$\vec{\mathbf{C}}\cdot(\vec{\mathbf{A}}-\vec{\mathbf{B}})=(2\hat{\mathbf{j}}\ -3\hat{\mathbf{k}})\cdot[3\text{-}(\text{-}1)\hat{\mathbf{i}}+(1\text{-}2)\hat{\mathbf{j}}+(\text{-}\hat{\mathbf{k}}\text{-}5\hat{\mathbf{k}})]$$

$$=(2\hat{\mathbf{j}}\ \text{-}3\hat{\mathbf{k}})\cdot[4\hat{\mathbf{i}}\ \text{-}\hat{\mathbf{j}}\ \text{-}6\hat{\mathbf{k}}]=0\times 4+(2)(\text{-}1)+(\text{-}3)(\text{-}6)=16$$

Prob.7-11 A force $\vec{\mathbf{F}}=(6\hat{\mathbf{i}}\text{-}2\hat{\mathbf{j}})\ N$ acts on a particle that undergoes a displacement $\Delta\vec{\mathbf{r}}=(3\hat{\mathbf{i}}+1\hat{\mathbf{j}})\ m$. Find (a) the work done by the force on the particle and (b) the angle between $\vec{\mathbf{F}}$ and $\Delta\vec{\mathbf{r}}$.

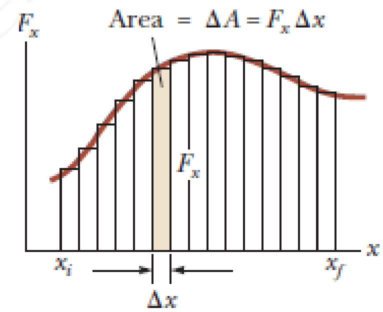
$$\vec{F}\cdot\Delta\vec{r} = 6\times 3\,\hat{\mathbf{i}}\cdot\hat{\mathbf{i}} + (-2)\,\hat{\mathbf{j}}\cdot 1\,\hat{\mathbf{j}} = 18 - 2 = 16\,J$$

$$\cos\theta = \frac{\vec{\mathbf{F}}\cdot\Delta\vec{\mathbf{r}}}{F\,\Delta r} = \frac{16}{\sqrt{40\times 10}} = \frac{16}{10\sqrt{2}},\qquad \theta \approx 37^{0}$$

7.4 Work Done by a Varying Force

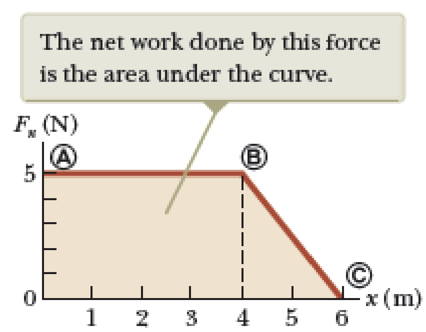
Work $\quad W = \int_{x_i}^{x_f} F_x dx$

is *exactly* equal to the area under this curve.



Example 7.4 Calculating Total Work Done from a Graph

A force acting on a particle varies with x as shown in Figure. **Calculate the work** done by the force on the particle as it moves from $\boldsymbol{x = 0}$ **to** $\boldsymbol{x = 6.0}$ **m**.



Solution

Total work done by the force on the particle = area of the rectangle + area of the triangle

$$W = W_{AB} + W_{BC} = (5)(4) + \tfrac{1}{2}(5)(2) = 25J$$

Prob.7-14. The force acting on a particle varies as shown in Figure P7.14. Find the work done by the force on the particle as it moves (a) from x =0 to x = 8.00 m, (b) from x =8.00 m to x = 10.0 m, and (c) from x = 0 to x = 10.0 m.

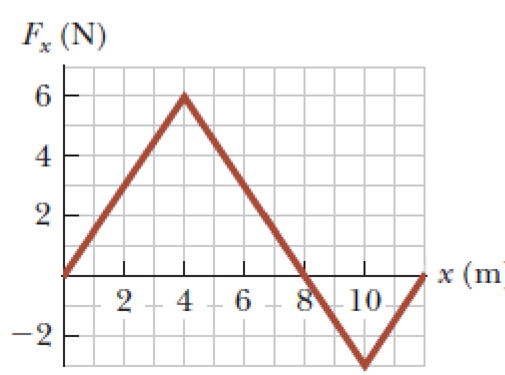


Figure P7.14

$$W = \int_{x_i}^{x_f} F_x dx = \text{area under the curve from } x_i \text{ to } x_f$$

(a) W1 = area of the triangle $\Delta = \frac{1}{2}(8)(6) = 24\ J$

(b) W2 = area of the triangle (from $\mathbf{x_i = 8.0}$ **to** $\mathbf{x_f = 10.0}$)

$$= \frac{1}{2}(2)(-3) = -3\ J$$

(b) W (x_i = 0.0 to x_f = 10.0) = W1 +W2 = 24 – 3 = 21.0 J

Prob.7-15. A particle is subject to a force F_x that varies with Position as shown in Figure P7.15. Find the work done by the force on the particle as it moves (a) from $x = 0$ to $x = 5.00$ m, (d) What is the total work done by the force over the distance $x = 0$ to $x = 15.0$ m?

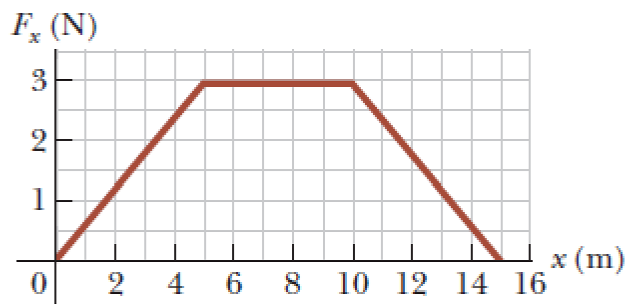
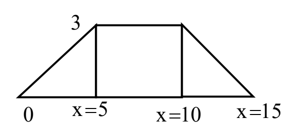


Figure P7.15 Problems 15 and 34.

إيجاد المساحة من شبه المنحرف

$$A = W = \frac{1}{2}[(15-0)+(10-5)]\times 3 = \frac{20}{2}\times 3 = 30J$$



Hooke's law: the force of the spring is proportional to the amount of stretch or compression x:

$$\vec{F}_s = -kx\,\hat{i}$$

The work done by the spring force on the block is

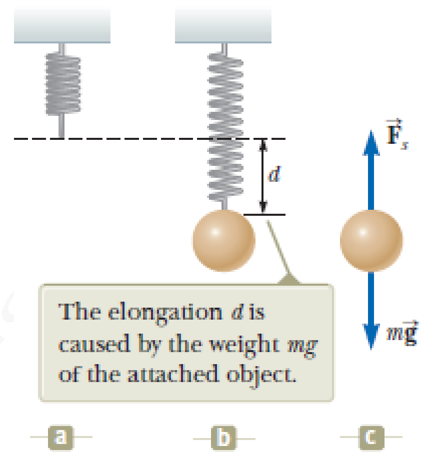
$$W_s = \int_{x_i}^{x_f} (-kx)dx = -\left(\frac{1}{2}k x_f^2 - \frac{1}{2}k x_i^2\right)$$

$$W_{app} = -W_s$$

Example 7.5

Measuring *k* for a Spring

A common technique used to measure the force constant of a spring is to hang the spring vertically (see Fig), and an object of mass *m* is attached to its lower end. Under the action of the "load" *mg*,
the spring stretches a distance *d* from its equilibrium position.



(A) If a spring is stretched 2.0 cm by a suspended object having a mass of 0.55 kg, what is the force constant of the spring?

$$F_s - mg = 0 \qquad (when\ m\ is\ stationary)$$

$$\left|\vec{F}_s\right| = kd \qquad \Rightarrow k = \frac{F_s}{d} = \frac{mg}{d} = \frac{0.55\times 9.8}{0.02} = 2.7\times 10^2\, N/m$$

(B) How much work is done by the spring on the object as it stretches through this distance?

$$W_s = -(\frac{1}{2}k x_f^2 - \frac{1}{2}k x_i^2) = -(\frac{1}{2}kd^2 - 0) = -\frac{1}{2}\times(2.7\times 10^2)\times(0.02)^2$$

$$= -5.4\times 10^{-2} J$$

(C) Evaluate the work done by the gravitational force on the object:

$$W_g = mg(-\hat{j})\cdot(-d\,\hat{j}) \;=\; mgd \;=\; 0.55\times 10\times 0.02 = 1.1\times 10^{-2} J$$

Prob. 7-29. A force, where $\vec{\mathbf{F}} = (4x\,\hat{\mathbf{i}} - 3y\,\hat{\mathbf{j}})\ N$ is in newtons and *x* and *y* are in meters, acts on an object as the object moves **in the *x* direction** from the **origin** to ***x* =5.00 m**.

Find the work $W = \int \vec{F} \cdot d\vec{r}$ done by the force on the object.

$$d\vec{r} = dx\hat{i} + dy\hat{j}, \qquad \vec{F} = F_x\hat{i} + F_y\hat{j}$$

$$dW = \vec{F} \cdot d\vec{r} = F_x dx + F_y dy$$

$$W = \int_0^{x,y} \vec{F} \cdot d\vec{r} = \int_0^x F_x dx + \int_0^y F_y dy = \int_{x=0}^{x=5} 4x\,dx + \int_0^0 3y\,\underbrace{dy}_{0}$$

$$= \int_{x=0}^{x=5} 4x\,dx = 4\left.\frac{x^2}{2}\right|_0^5 + 0 = 2(25) = 50\,J$$

7.5 Kinetic Energy and the Work–Kinetic Energy Theorem

Work–kinetic energy theorem:

$$W = \Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

Prob. 7-31(29)(26). A 3.00-kg object has a velocity $(6.00\hat{\mathbf{i}} + 2.00\hat{\mathbf{j}})$ m/s.

(a) What is its kinetic energy at this moment?

(b) What is the net work done on the object if its velocity changes to $(8.00\hat{\mathbf{i}} + 4.00\hat{\mathbf{j}})$ (m/s).

(*Note:* From the definition of the dot product, $v^2 = \mathbf{v} \cdot \mathbf{v}$.)

(a) $v^2 = \mathbf{v}.\mathbf{v} \Rightarrow \mathbf{v}_\mathbf{i}^2 = 6^2 + 4^2 = 40\ (m/s)^2$

or $v_i^2 = v_x^2 + v_y^2 = 6^2 + 4^2 = 40\ (m/s)^2$

$$K_i = \tfrac{1}{2}mv^2 = \frac{1}{2} \times 3 \times 40 = 60\,J$$

(b) $\mathbf{v}_\mathbf{f}^2 = 8^2 + 4^2 = 80\ (m/s)^2$

$$W = \Delta K = \frac{1}{2}m(v_f^2 - v_i^2) = \frac{1}{2}\times 3(80-60) = 30\,J$$

Prob. 33(25)(24). A 0.300-kg particle has a speed of 2.00 m/s at point A and kinetic energy of 7.50 J at point B. What is (a) its kinetic energy at A? (b) its speed at B (c) the total work done on the particle as it moves from A to B?

(a) $K_A = \frac{1}{2}mv^2 = 0.3\times 2^2 = 1.2\,\text{J}$

(b) $K_B = \frac{1}{2}mv_B^2 \quad \Rightarrow v_B = \sqrt{\frac{2K_B}{m}} = \sqrt{\frac{14}{0.3}} = 5\,\text{m/s}$

(c) $\sum W = K_f - K_i = 7.5 - 1.2 = 6.3\,\text{J}$

[D] Prob. 7-34. A 4.00-kg particle is subject to a net force that varies with position as shown in Figure P7.15. The particle starts from rest at $x = 0$. What is its speed at (a) $x = 5.00$ m,

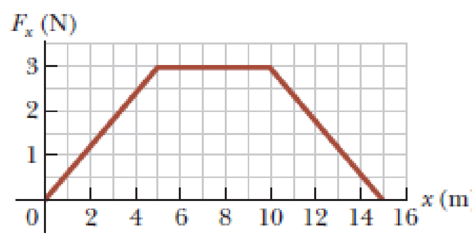
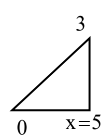


Figure P7.15 Problems 15 and 34.

W = area under the (F_x, x) curve

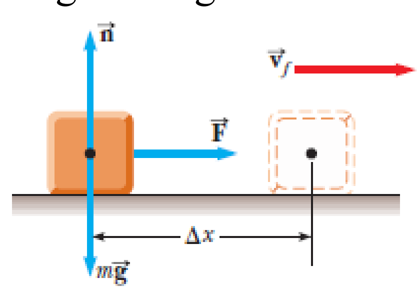
(a) $W = Area\ of\ triangle = \frac{1}{2}(5\times 3) = 7.5J$



$$W = \Delta K = \frac{1}{2}m(v_f^2 - 0) \Rightarrow v_f = \sqrt{\frac{2\Delta K}{m}} = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2\times 7.5}{4}} = 1.94\,\text{m/s}$$

Example 7.6 A Block Pulled on a Frictionless Surface

A **6.0-kg** block initially at rest is pulled to the right along a horizontal, frictionless surface by a constant horizontal force of **12 N**. Find the block's speed after it has moved **3.0 m**.



$W = F\Delta x = 12\times 3 = 36\ \text{J}$

$$W = \frac{1}{2}m\upsilon_f^2 - \frac{1}{2}m\upsilon_i^2 = \frac{1}{2}m\upsilon_f^2 - 0$$

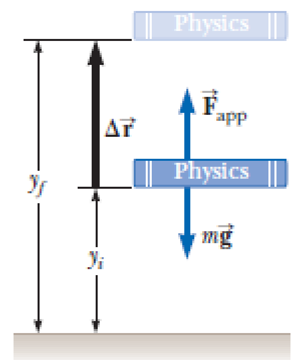
$$\upsilon_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2\times 36}{6}} = 12\,J$$

7.6 Potential Energy of a System

$$W_F = \int_{y_i}^{y_f} \vec{F}\cdot dy\,\hat{j} = -W_g = -\int_{y_i}^{y_f} (-\ mg\,\hat{j})\cdot(dy\,\hat{j}) = mg\,y_f - mgy_i$$

$$W_g = -\Delta U_g \ = \ -\ [U_f\ -\ U_i] \ = -[mg\,y_f - mgy_i]$$

$$U_g \ = \ mg\,y$$



Example 7.8 The Proud Athlete and the Sore Toe

A bowling ball held by a careless bowler slips from the bowler's hands and drops on the bowler's toe. Choosing floor level as the y= 0 point of your coordinate system, estimate the change in gravitational potential energy of the ball–Earth system as the ball falls. Repeat the calculation, using the top of the **bowler's head** as the origin of coordinates.

1) Ball mass is 7 kg, and the top of a person's toe is about 0.03 m above the floor. Assume the ball falls from a height of 0.5 m.

$$\Delta U_g \ = \ U_f\ -\ U_i = mg\,(y_f - y_i) = 7\times 9.8(0.03 - 0.5) = -32.24\ J$$

2) choose reference configuration of the system for zero potential energy at the **bowler's head** and estimate this position to be 1.50 m above the floor.

$$U_i \ = mg\,y_i = 7\times 9.8(\ -1.0) = -68.6\ J$$

$$U_f = mg\,y_f = 7\times 9.8(-\ 1.47)$$
$$= -100.8\ J,\ \Rightarrow \Delta U_g = -100.8 - (-68.6) = -32.2J$$

Elastic Potential Energy

A block attached to a spring, the spring exerts a force

$\vec{F}_s = -kx\ \hat{i}$ **x** distance from its equilibrium position x=0.

$$W_{F_{app}} = -W_s = \int_{x_i}^{x_f} (kx\hat{i})\cdot dx\hat{i} = \frac{1}{2}k\,x_f^2 - \frac{1}{2}k\,x_i^2$$

The **elastic potential energy** of the block-spring system is defined by

$$U_s = \frac{1}{2}k\,x^2$$

Work done by spring $W_s = \frac{1}{2}k\,x_i^2 - \frac{1}{2}k\,x_f^2 = -[\frac{1}{2}k\,x_f^2 - \frac{1}{2}k\,x_i^2] =$

$$W_s = -\Delta U_s = -[\frac{1}{2}k\,x_f^2 - \frac{1}{2}k\,x_i^2]$$

7.7 Conservative and Nonconservative Forces

Conservative Forces

Conservative forces have two equivalent properties:

1. The work done by conservative forces **is independent of the path taken by the particle.**
2. The work done by conservative force through **any closed path is zero.**

Examples $\vec{F}_g = -mg\ \hat{j}$ depends only on the initial and final y

$$\vec{F}_s = -kx\ \hat{i}$$

$$W_g = -[mg\,y_f - mgy_i] = -\Delta U_g$$

$$W_s = -\frac{1}{2}k[x_f^2 - x_i^2] = -\Delta U_s$$

For any conservative force, work done by this force equals

$$W_c = -\Delta U = -[U_f - U_i]$$

Prob. 7- 43. A 4.00-kg particle moves from the origin to Position ©, having coordinates x = 5.00 m and y = 5.00 m (Fig. P7.43). One force on the particle is the gravitational force acting in the negative y direction. Using Equation 7.3, calculate the work done by the gravitational force on the particle as it goes from O to along (a) the purple path, (b) the red path, and (c) the blue path. (d) Your results should all be identical. Why?

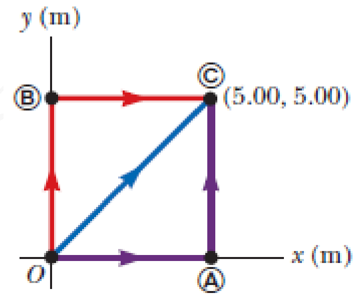


Figure P7.43
Problems 43 through 46.

(a) W (OAC) = W (OA) +W (AC)

= mg (OA) cos 90 +mg (AC) cos 180 = 4×10[0 – (5)1] = -200 J

(b) W (OBC) = W (OB) +W (BC)

= mg (OB) cos 180 + mg (BC) cos 90 = 4×10[-(1) (5) - 0] = -200 J

(c) W (OC) = mg (OC) cos 135 = 4×10 (5√2) (-1/√2) = -200 J

(d) Because F_g = mg is conservative

الشغل عبر اي مسار من O الى C = - 200 جول وهو لا يعتمد على المسار والشغل لاي مسار مغلق = صفرا

Prob. 7-63. An inclined plane of angle **θ=20.0°** has a spring of force constant ***k* = 500 N/m** fastened securely at the bottom so that the spring is parallel to the surface as shown in Figure P7.63.A block of mass ***m* = 2.50 kg** is placed on the plane at a

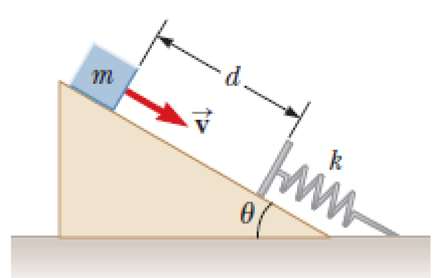


Figure P7.63
Problems 63 and 64.

distance ***d*** **= 0.300** m from the spring. From this position, the block is **projected downward toward the spring with speed *v* = 0.750 m/s**. By **what distance is the spring compressed** when the block momentarily **comes to rest**?

Spring will compressed a distance x.
The block moves a distance (d+x) ➔ h = (d+x) sin θ

$$W_s = -\Delta U_s = -\frac{1}{2}kx^2 = \frac{1}{2}(500)(x)^2 = 250x^2$$

$$W_g = -\Delta U_g = -mg(d+x)\sin\theta$$

$$K_f - K_i = W_s + W_g$$

$$\Rightarrow 0 - \frac{1}{2}mv_i^2 = 250x^2 + mg(d+x)\sin\theta$$

$$0 - \frac{1}{2}\times 2.5(0.75)^2 = 250x^2 + 250(0.3+x)\sin 20$$

Solve for x from quadratic equation

7.8 Relationship Between Conservative Forces and Potential Energy

$$W = \int_{x_i}^{x_f} F_x dx = -\Delta U$$

$$dU = -F_x dx \quad \text{Or} \quad F_x = -\frac{dU}{dx}$$

For a **deformed spring** $U_s = \frac{1}{2}k\,x^2$

$$F_x = -\frac{dU}{dx} \quad \Rightarrow F_x = -2\times\frac{1}{2}kx = -kx$$

The **gravitational potential energy** $U_g = mgy$

$$F_y = -\frac{dU}{dy} \quad \Rightarrow F_g = -mg$$

Generally for $U(x, y, z)$**, the force components is given by**

$$F_x = -\frac{\partial U}{\partial x}, \quad F_y = -\frac{\partial U}{\partial y}, \quad F_z = -\frac{\partial U}{\partial z}$$

[D] Prob.7-49 The potential energy of a system is of the form $U = 3x^3 y - 7x$, find the force F acting at point (1,1)

$$F_x = -\left.\frac{\partial U}{\partial x}\right|_{y=const.} = -(9x^2 y - 7) = -(9-7) = -2\ N$$

$$F_y = -\frac{\partial U}{\partial y} = -\ 3x^3 = -3, \quad \Rightarrow \vec{F} = -(3\hat{i} + 2\hat{j})\text{N}$$