Chapter 7 Energy of a System page 177

HWK: 5, 9, 11, 14, 29, 33, 43, 49, 63

Prob. 7-5. A block of mass $\boldsymbol{m}$**=2.50 kg** is pushed a distance **d = 2.20 m** along a **frictionless**, horizontal table by a constant applied force of magnitude $\boldsymbol{F}$ **=16.0** N directed at an angle $\theta = 25.0^\circ$ below the horizontal as shown in Figure P7.5. Determine the work done on the block by

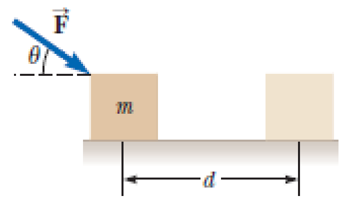

Figure P7.5

(a) the applied force,
(d) the net force on the block.
(b) the normal force exerted by the table,

(a) $W_F = F\,\Delta r\,\cos\theta = 2.5 \times 2.2 \times \cos 25 = 31.9\ J$

(b) $W_N = N\ \Delta r\ \cos 90 = \ 0\ J$

(c) $W_g = mg\Delta r\ \cos 90 = \ 0\ J$

(d) total work: $W_{tot} = 31.9 + 0 + 0 = 31.9 J = W_F$

Prob. 7-9 For $\vec{\mathbf{A}} = \ 3\hat{\mathbf{i}}\ + \hat{\mathbf{j}}\ - \hat{\mathbf{k}},\ \ \vec{\mathbf{B}} = \ -\hat{\mathbf{i}}\ + 2\hat{\mathbf{j}}\ + 5\hat{\mathbf{k}}$ and $\vec{\mathbf{C}} = 2\hat{\mathbf{j}}\ - 3\hat{\mathbf{k}}$ Find $\vec{\mathbf{C}}\cdot(\vec{\mathbf{A}} - \vec{\mathbf{B}})$

$$\vec{\mathbf{C}}\cdot(\vec{\mathbf{A}} - \vec{\mathbf{B}}) = (2\hat{\mathbf{j}}\ - 3\hat{\mathbf{k}}\ )\cdot[3\text{-}(\text{-}1)\hat{\mathbf{i}} + (1\text{-}2)\hat{\mathbf{j}} + (\text{-}\hat{\mathbf{k}}\text{-}5\hat{\mathbf{k}}\ )]$$

$$= (2\hat{\mathbf{j}}\ \text{-}3\hat{\mathbf{k}}\ )\cdot[4\hat{\mathbf{i}}\ \text{-}\hat{\mathbf{j}}\ \text{-}6\hat{\mathbf{k}}\ ] = 0 \times 4 + (2)(\text{-}1) + (\text{-}3)(\text{-}6) = 16$$

Prob.7-11 A force $\vec{\mathbf{F}} = (6\hat{\mathbf{i}}\text{-}2\hat{\mathbf{j}}\ )\ N$ acts on a particle that undergoes a displacement $\Delta\vec{\mathbf{r}} = (3\hat{\mathbf{i}} + 1\hat{\mathbf{j}}\ )\ m$. Find (a) the work done by the force on the particle and (b) the angle between $\vec{\mathbf{F}}$ and $\Delta\vec{\mathbf{r}}$.

$$\vec{F}\cdot\Delta\vec{r} = 6 \times 3\hat{\mathbf{i}}\cdot\hat{\mathbf{i}} + (\text{-}2)\hat{\mathbf{j}}\cdot 1\hat{\mathbf{j}} = 18 - 2 = 16\ \text{J}$$

$$\cos\theta = \frac{\vec{\mathbf{F}}\cdot\Delta\vec{\mathbf{r}}}{F\,\Delta r} = \frac{16}{\sqrt{40\times 10}} = \frac{16}{10\sqrt{2}}, \quad \theta \approx 37^0$$

Prob.7-14. The force acting on a particle varies as shown in Figure P7.14. Find the work done by the force on the particle as it moves (a) from x =0 to x = 8.00 m, (b) from x =8.00 m to x = 10.0 m, and (c) from x = 0 to x = 10.0 m.

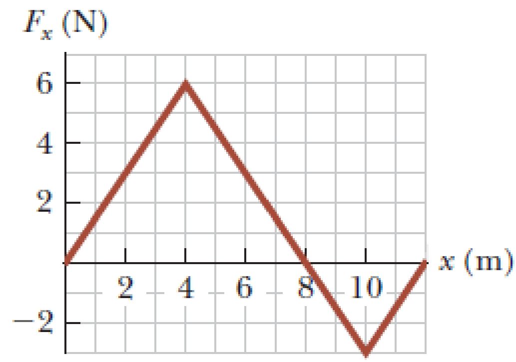

Figure P7.14

$$W = \int_{x_i}^{x_f} F_x dx = \text{area under the curve}$$

from x_i to x_f

(a) W1 = area of the triangle $\Delta = \frac{1}{2}(8)(6) = 24\ J$

(b) W2 = area of the triangle (from $\mathbf{x_i = 8.0}$ **to** $\mathbf{x_f = 10.0}$)

$$= \frac{1}{2}(2)(-3) = -3\ J$$

(b) W (x_i = 0.0 to x_f = 10.0) = W1 +W2 = 24 – 3 = 21.0 J

Prob.7-15. A particle is subject to a force F_x that varies with Position as shown in Figure P7.15. Find the work done by the force on the particle as it moves (a) from x = 0 to x = 5.00 m, (d) What is the total work done by the force over the distance x = 0 to x=15.0 m?

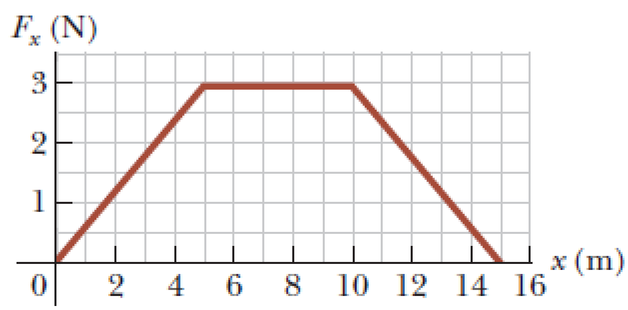

Figure P7.15 Problems 15 and 34.

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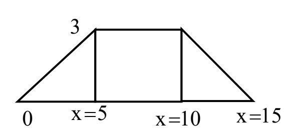
$$A = W = \frac{1}{2}[(15-0)+(10-5)]\times 3 = \frac{20}{2}\times 3 = 30J$$



Prob. 7-29. A force, where $\vec{\mathbf{F}} = (4x\,\hat{\mathbf{i}} - 3y\,\hat{\mathbf{j}})\ N$ is in newtons and x and y are in meters, acts on an object as the object moves **in the *x* direction** from the **origin** to ***x* =5.00 m**.

Find the work $W = \int \vec{F} \cdot d\vec{r}$ done by the force on the object.

$$d\vec{r} = dx\hat{i} + dy\hat{j}, \qquad \vec{F} = F_x\hat{i} + F_y\hat{j}$$

$$dW = \vec{F} \cdot d\vec{r} = F_x dx + F_y dy$$

$$W = \int_0^{x,y} \vec{F} \cdot d\vec{r} = \int_0^x F_x dx + \int_0^y F_y dy = \int_{x=0}^{x=5} 4x\,dx + \int_0^0 3y\,\underbrace{dy}_{0}$$

$$= \int_{x=0}^{x=5} 4x\,dx = 4\frac{x^2}{2}\Big|_0^5 + 0 = 2(25) = 50\,J$$

Prob. 7-31. A 3.00-kg object has a velocity $(6.00\,\hat{\mathbf{i}} + 2.00\,\hat{\mathbf{j}})$ m/s.

(a) What is its kinetic energy at this moment?

(b) What is the net work done on the object if its velocity changes to $(8.00\,\hat{\mathbf{i}} + 4.00\,\hat{\mathbf{j}})$ (m/s).

(*Note:* From the definition of the dot product, $v^2 = \mathbf{v} \cdot \mathbf{v}$.)

(a) $v^2 = \mathbf{v}.\mathbf{v} \Rightarrow \mathbf{v_i^2} = 6^2 + 4^2 = 40\ (m/s)^2$

or $v_i^2 = v_x^2 + v_y^2 = 6^2 + 4^2 = 40\ (m/s)^2$

$$K_i = \tfrac{1}{2}mv^2 = \frac{1}{2} \times 3 \times 40 = 60\,J$$

(b) $\mathbf{v_f^2} = 8^2 + 4^2 = 80\ (m/s)^2$

$$W = \Delta K = \frac{1}{2}m(v_f^2 - v_i^2) = \frac{1}{2} \times 3(80 - 60) = 30\,J$$

Prob. 7-33. A 0.300-kg particle has a speed of 2.00 m/s at point A and kinetic energy of 7.50 J at point B. What is (a) its kinetic energy at A? (b) its speed at B (c) the total work done on the particle as it moves from A to B?

(a) $K_A = \frac{1}{2}mv^2 = 0.3 \times 2^2 = 1.2\,J$

(b) $K_B = \frac{1}{2}mv_B^2 \quad \Rightarrow v_B = \sqrt{\frac{2K_B}{m}} = \sqrt{\frac{14}{0.3}} = 5\,m/s$

(c) $\sum W = K_f - K_i = 7.5 - 1.2 = 6.3\,J$

Prob. 7-34. A 4.00-kg particle is subject to a net force that varies with position as shown in Figure P7.15. The particle starts from rest at $x = 0$. What is its speed at (a) $x = 5.00$ m,

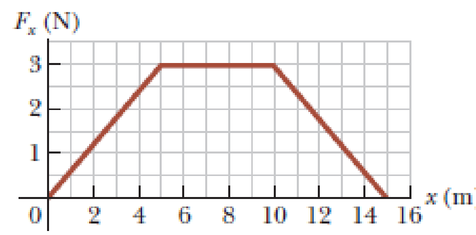
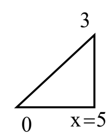


Figure P7.15 Problems 15 and 34.

W = area under the (F_x, x) curve

(a) $W = Area\ of\ triangle = \frac{1}{2}(5 \times 3) = 7.5J$



$$W = \Delta K = \frac{1}{2}m(v_f^2 - 0) \Rightarrow v_f = \sqrt{\frac{2\Delta K}{m}} = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2\times 7.5}{4}} = 1.94\ m/s$$

Prob. 7- 43. A 4.00-kg particle moves from the origin to Position ©, having coordinates $x = 5.00$ m and $y = 5.00$ m (Fig. P7.43). One force on the particle is the gravitational force acting in the negative y direction. Using Equation 7.3, calculate the work done by the gravitational force on the particle as it goes from O to along (a) the purple path, (b) the red path, and (c) the blue path. (d) Your results should all be identical. Why?

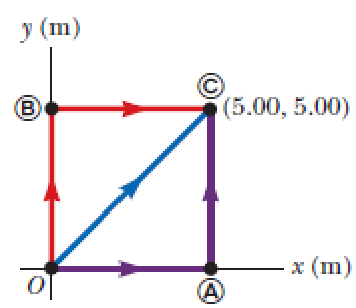


Figure P7.43
Problems 43 through 46.

(a) W (OAC) = W (OA) +W (AC)

= mg (OA) cos 90 +mg (AC) cos 180 = 4×10[0 – (5)1] = -200 J

(b) W (OBC) = W (OB) +W (BC)

= mg (OB) cos 180 + mg (BC) cos 90 = 4×10[-(1) (5) - 0] = -200 J

(c) W (OC) = mg (OC) cos 135 = 4×10 (5√2) (-1/√2) = -200 J

(d) Because $F_g = mg$ is conservative

الشغل عبر اي مسار من O الى C = - 200 جول وهو لا يعتمد على المسار والشغل لاي مسار مغلق = صفرا

Prob. 7-63. An inclined plane of angle **θ=20.0º** has a spring of force constant ***k*** **= 500 N/m** fastened securely at the bottom so that the spring is parallel to the surface as shown in Figure P7.63.A block of mass ***m*** **= 2.50 kg** is placed on the plane at a distance ***d*** **= 0.300** m from the spring. From this position, the block is **projected downward toward the spring with speed *v* = 0.750 m/s**.
By **what distance is the spring compressed** when the block momentarily **comes to rest**?

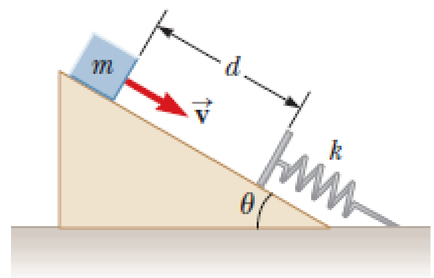


Figure P7.63
Problems 63 and 64.

Spring will compressed a distance x.
The block moves a distance (d+x) ➔ h = (d+x) sin θ

$$W_s = -\Delta U_s = -\frac{1}{2}kx^2 = \frac{1}{2}(500)(x)^2 = 250x^2$$

$$W_g = -\Delta U_g = -mg(d+x)\sin\theta$$

$$K_f - K_i = W_s + W_g$$

$$\Rightarrow 0 - \frac{1}{2}mv_i^2 = 250x^2 + mg(d+x)\sin\theta$$

$$0 - \frac{1}{2}\times 2.5(0.75)^2 = 250x^2 + 250(0.3+x)\sin 20$$

Solve for x from quadratic equation.

Prob.7-49 The potential energy of a system is of the form
$U = 3x^3 y - 7x$, find the force F acting at point (1,1)

$$F_x = -\left.\frac{\partial U}{\partial x}\right|_{y=const.} = -(9x^2 y - 7) = -(9-7) = -2\ N$$

$$F_y = -\frac{\partial U}{\partial y} = -\ 3x^3 = -3,$$

$$\Rightarrow \vec{F} = -(3\hat{i} + 2\hat{j})\quad N$$