Chapter 8
Conservation of Energy

ed.9	Ch.8 Conservation of Energy	**HWK 6,7,15, 23, 47,63**	Add. Prob.. 5, 10, 24, 16, 24,43
	Fall 2022	**Discussion ch8: 3, 5, 6, 7, 15, 23, 47, 63**	

8.1 Analysis Model: Nonisolated System (Energy)

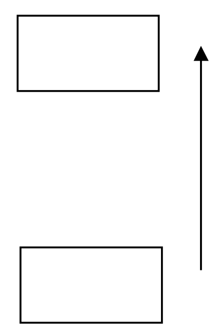
For nonisolated system energy changes due to the energy, that crosses the boundary of the system due to the interaction with the environment. This scenario is common in physics problems.

Work–kinetic energy theorem: For nonisolated systems. The system is the object and the work done by the external force changes its kinetic energy.

Energy transfer: heat, charge, matter, waves ………..

8.2 Analysis Model: Isolated System (Energy)

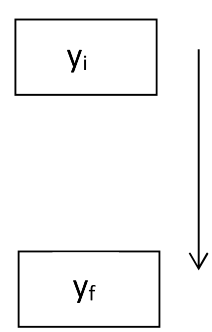
For isolated systems, no energy crosses the boundary the energy is conserved
Book–Earth system: a book lifted up



The work done on the system (object - Earth) by the external agent

$$W_F = \Delta U = (mg\,y_f - mgy_i)$$

As the book falls back to its original height from y_i to y_f , the work done by the gravitational force on the book is



$$W_g = -\Delta U_g \;=\; -\,[U_f \,-\, U_i] \;= -[mg\,y_f - mgy_i]$$

The book's kinetic energy changes such that

$$W_g = \Delta K = -\Delta U \qquad \Rightarrow \Delta K + \Delta U = 0$$

$$\Rightarrow \Delta(K+U) = 0$$

Or $\Delta E = 0 \qquad \Rightarrow E = K + U = \text{constant}$

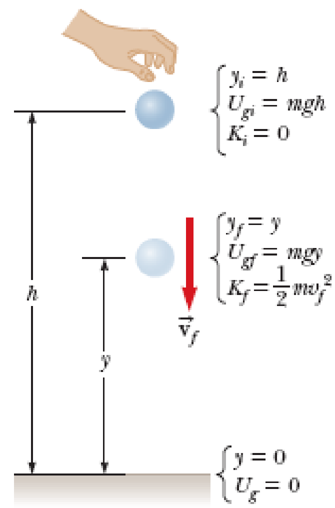
E is the total energy of the system, which is conserved (remains constant) for an isolated.

$$E_f = E_i \qquad \Rightarrow U_i + K_i = U_f + K_f$$

$$\frac{1}{2} m\, v_f^2 + m\, g\, y_f = \frac{1}{2} m\, v_i^2 + m\, g\, y_i$$

Example 8.1 Ball in Free Fall

A ball of mass *m* is dropped from a height *h* above the ground as shown in Active Figure 8.4.



(A) Neglecting air resistance, determine the speed of the ball when it is at a height *y* above the ground.

Solution

Ball-earth system (isolated)

$$E_f = E_i \;\Rightarrow\; U_i + K_i = U_f + K_f$$

$$mgh + 0 = mgy_f + \frac{1}{2} m v_f^2$$

$$gh = gy + \frac{1}{2} v_f^{\,2} \qquad v_f = \sqrt{2g(h-y)}$$

(B) Determine the speed of the ball at ***y*** if at the instant of release it already has an initial upward speed $\boldsymbol{v_i}$ at the initial altitude ***h***.

Solution

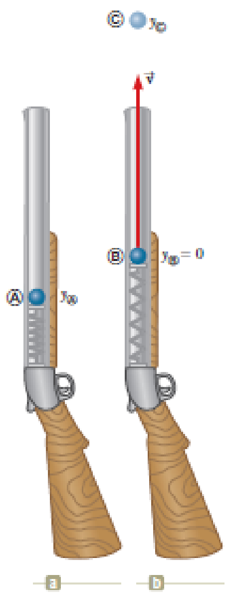
$$mgh + \frac{1}{2} m v_i^2 = mgy_f + \frac{1}{2} m v_f^2 \quad ,$$

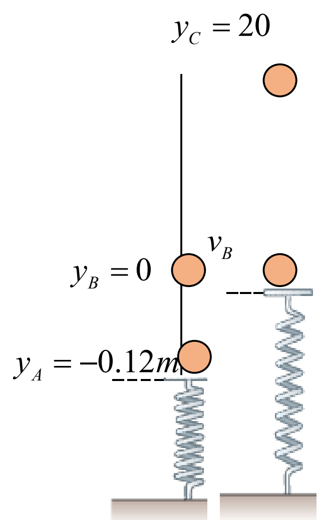
$$2(gh - gy) + v_i^2 = v_f^2 \qquad v_f = \sqrt{v_i^2 + 2g(h-y)}$$

Example 8.3

The Spring-Loaded Popgun

The launching mechanism of a popgun consists of a trigger-released spring (Fig. 8.6a). The spring is compressed to a position y_A, and the trigger is fired. The projectile of mass m rises to a position y_C above the position at which it leaves the spring, indicated in Active Figure 8.6b as position $y_B = 0$. Consider a firing of the gun for which m = 35.0 g, y_A = -0.120 m, and y_C = 20.0 m.





(A) Neglecting all resistive forces, determine the spring constant.

Solution

$$U_{si} + U_{gi} + K_i = U_{sf} + U_{gf} + K_f$$

$$\frac{1}{2}k(-0.12)^2 + mg(-0.12) + 0 = 0 + mg(20) + 0$$

$$k = \frac{2mg(20.12)}{0.0144} = \frac{2\times 0.35\times 10\times(20.12)}{0.0144} = 978\,N/m$$

(B) Find the speed of the projectile as it moves through the equilibrium position B (x=0) of the spring as shown in Active Figure 8.6b.

$$U_{sA} + U_{gA} + K_A = U_{sB} + U_{gB} + K_B$$

$$\frac{1}{2}k(-0.12)^2 + mg(-0.12) + 0 = 0 + 0 + \frac{1}{2}mv_B^2$$

$$v_B = \frac{k(-0.12)^2 - 2mg}{m} = 19.8 m/s$$

$$E_B = E_C$$

$$\frac{1}{2} m v_B^2 = mgy_C \Rightarrow v_B = \sqrt{2gy_C} \approx 20 m/s$$

8.3 Situations Involving Kinetic Friction

Assume a situation in which forces, including friction, are applied to the object, the total work done on the object by the net force $\sum \vec{F}$ when it moves $\Delta \vec{r}$ is

$$W = \int (\sum \vec{F}) \cdot d\vec{r} = \int m \vec{a} \cdot d\vec{r} = \int m \frac{d\vec{v}}{dt} \cdot d\vec{r} = \int m \vec{v} \cdot d\vec{v}$$

$$= \frac{1}{2} \int_{v_i}^{v_f} m \, d(\vec{v} \cdot \vec{v}) = \frac{1}{2} \int m \, d(v^2) = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

where v_i is the speed of the block when it is at $x=x_i$ and v_f is its speed at x_f.

$$\frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = \Delta K$$

$$W = \int (\sum \vec{F}) \cdot d\vec{r} = \int \vec{F}_{other} \cdot d\vec{r} + \int \vec{f}_k \cdot d\vec{r} = W_{otherF} - f_k \cdot d$$

$$\Delta K = W_{otherF} - f_k \cdot d$$

when a friction force acts on an object, the change in its kinetic energy is equal to the work done by all forces other than friction minus a term $f_k \cdot d$ associated with the friction force.

book–surface system with friction:

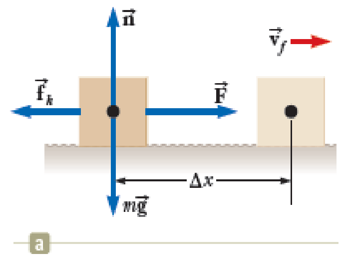
$$\Delta E = 0 \quad \Rightarrow \Delta K + \Delta E_{int} = 0$$

$\Delta K = -f_k \cdot d, \quad \Rightarrow \Delta E_{int} = f_k \cdot d$ (= increase in internal energy of the system)

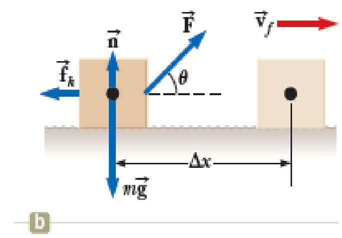
Friction transforms kinetic energy in a system to internal energy, and the increase in internal energy of the system is equal to its decrease in kinetic energy.

Example 8.4 A Block Pulled on a Rough Surface

A 6.0-kg block initially **at rest** is pulled to the right along a horizontal surface by a constant horizontal force of **12 N**.



(A) Find the speed of the block after it has moved 3.0 m if the surfaces in contact have a coefficient of kinetic friction of 0.15 (Fig. a)



Solution

$$(1)\ \Delta K = W_{otherF} - f_k \cdot d = F\Delta x - \mu_k n\ \Delta x$$

$$(2)\ K_f - K_i = W_{otherF} - f_k \cdot d$$
$$= F\Delta x - \mu_k n\ \Delta x$$

$$(3)\ K_f = K_i + W_{otherF} - f_k \cdot d = F\Delta x - \mu_k n\ \Delta x$$

The above three eqn. are the same

$$n = mg$$

$$\Delta K = F\ \Delta x - \mu_k mg\ \Delta x = 12\times 3 - 0.15\times 6\times 10\times 3 = 9\ J$$

$$\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}mv_f^2 - 0 = 9\ J$$

$$v_f = \sqrt{\frac{2\times 9}{m}} = \sqrt{\frac{2\times 9}{6}} = 1.73 m/s$$

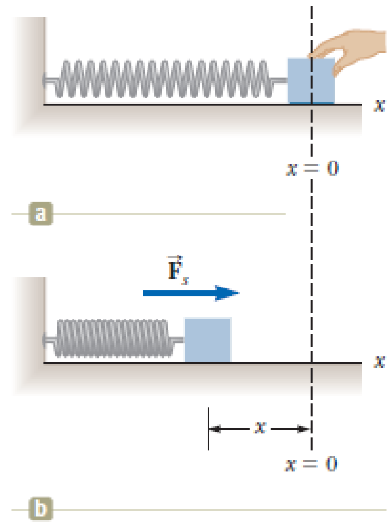
(B) Suppose the force **F** is applied at an angle θ as shown in active Figure 8.8b. At what angle should the force be applied to achieve the **largest possible speed** after the block has moved 3.0 m to the right?

$$f_k = \mu_k n = \mu_k (m_1 g - F \sin\theta)$$

$$\Delta K = W_{otherF} - f_k \cdot d = F \cos\theta \, \Delta x - \mu_k (mg - F \sin\theta) \, \Delta x$$

Derivative of K_f with respect to θ should be zero for v_f to be a maximum.

Example 8.6 A Block–Spring System



A block of mass 1.6 kg is attached to a horizontal spring that has a force constant of 1 000 N/m as shown in Figure 8.9. The spring is compressed 2.0 cm and is the released from rest.

(A) Calculate the speed of the block as it passes through the equilibrium position *x* = 0 if the surface is frictionless.

Solution

$$E_i(x) = E_f(0) \Rightarrow \frac{1}{2}kx^2 = \frac{1}{2}mv^2$$

$$\Rightarrow v^2 = \frac{1000 \times (0.02)^2}{1.6} = 0.25 \Rightarrow v = 0.5 \ m/s$$

(B) Calculate the speed of the block as it passes through the equilibrium position (x=0) if a constant friction force of 4.0 N retards its motion from the moment it is released.

Solution

$$W_s = -\Delta U_s = -(0 - \frac{1}{2}kx^2) = \frac{1}{2}(1000)(0.02)^2$$

$$K_f - K_i = W_{otherF} - f_k \cdot d = W_s - f_x . x$$

$$= \frac{1}{2}(1000)(0.02)^2 - 4 \times 0.02 = 0.12 \ J$$

$$\frac{1}{2}mv_f^2 - 0 = 0.12 \ J \qquad \Rightarrow v_f = \sqrt{\frac{2 \times 0.12}{1.6}} = 0.39 \ m/s$$

8.4 Changes in Mechanical Energy for Nonconservative Forces

When the object subjected to friction and system changes its energy. In this case, the mechanical energy of the system changes by ($-f_k\, d$) due to the force of kinetic friction, e.g. when the object moves on an incline that is not frictionless, there is a change in both the kinetic energy and the gravitational potential energy of the book–Earth system. Consequently,

$$\Delta E = E_f - E_i = -f_k \cdot d$$

If other forces also present:

$$\Delta E = \Delta K + \Delta U = W_{otherF} - f_k \cdot d$$

$$\Delta E = E_f - E_i = W_{otherF} - f_k \cdot d$$

$$E_f = \frac{1}{2}mv_f^2 + U_{gf} + U_{sf}$$

$$E_i = \frac{1}{2}mv_i^2 + U_{gi} + U_{sf}$$

Example 8.7 Crate Sliding Down a Ramp

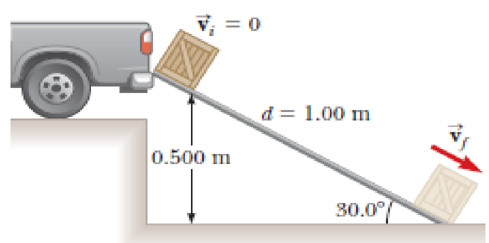


Figure 8.10 (Example 8.7) A crate slides down a ramp under the influence of gravity. The potential energy of the system decreases, whereas the kinetic energy increases.

A 3.00-kg crate slides down a ramp. The ramp is 1.00 m in length and inclined at an angle of 30.0° as shown in Figure 8.10. The crate starts from rest at the top, experiences a **constant friction force of magnitude 5.00 N**, and continues to move a short distance on the horizontal floor after it leaves the ramp.

(A) Use energy methods to determine the speed of the crate at the bottom of the ramp.

Solution

$$\Delta E = E_f - E_i = -f_k \cdot d$$

$$E_f = \frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}m\,v_f^2 - 0$$

$$E_i = \frac{1}{2}mv_i^2 + mgy_i = 0 + 3\times 10\times 5 = 150J\ ,$$

$$E_f = E_i - f_k\cdot d\,, \qquad \frac{1}{2}mv_f^2 = mgy_i - f_k d$$

$$v_f = \sqrt{2(mgy_i - f_k d\,)/\,m} = 2.58\ m/s$$

(B) How far does the crate slide on the horizontal floor if it continues to experience a friction force of magnitude 5.00 N?

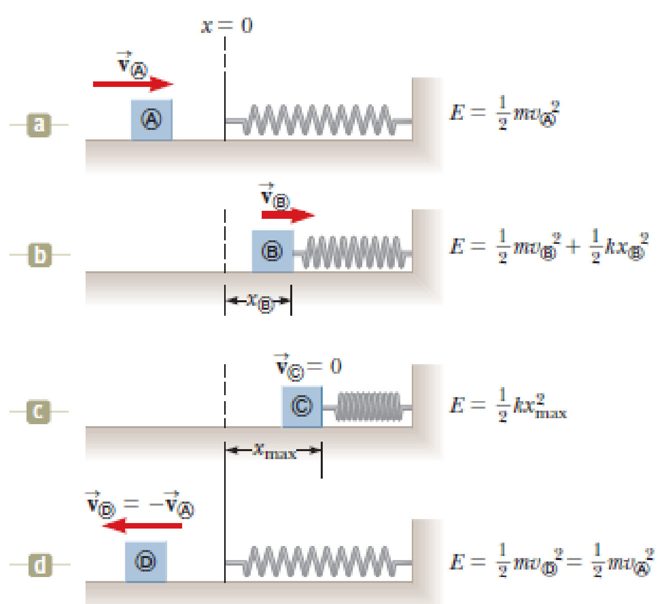
Max. d traveled with zero final speed

$$E_f = E_i - f_k\cdot d \qquad \Rightarrow 0 = \frac{1}{2}mv^2 - f_k\cdot d$$

$$\frac{mv^2}{2f_k} = d \qquad \Rightarrow d = \frac{3\times 2.54}{2\times 5} = 1.94m$$

Example 8.8 Block–Spring Collision

A block having a mass of **0.80 kg** is given an initial velocity $\mathbf{v_A = 1.2\ m/s}$ to the right and collides with a spring whose mass is negligible and whose force constant is ***k=50*** **N/m** as shown in Figure 8.11.



(A) Assuming the surface to be frictionless, calculate the maximum compression of the spring after the collision.

Solution

(a) $K_i + U_{si} = K_f + U_{sf}$

$$\frac{1}{2}mv_A^2 + 0 = 0 + \frac{1}{2}kx_C^2 \qquad x_C = \sqrt{\frac{k}{m}v_A^2} = 0.15m$$

(B) Suppose a constant force of kinetic friction acts between the block and the surface, with μ_k= 0.50. If the speed of the block at the moment it collides with the spring is v_B = 1.2 m/s, what is the maximum compression x_C in the spring?

Solution

$$\boldsymbol{E_f = E_i - f_k \cdot d}$$

$$\frac{1}{2}kx_C^2 = \frac{1}{2}mv_A^2 - f_k x_C$$

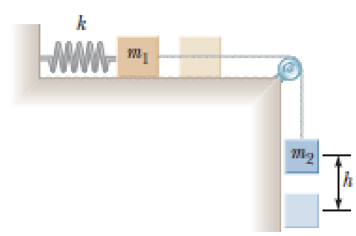
$$\frac{1}{2}\times 50 \times x_C^2 - \frac{1}{2}\times 0.8 \times 1.2^2 = -0.5\times 0.8\times 10\times x_C$$

$$\Rightarrow 25x_C^2 + 4.0x_C - 0.58 = 0$$

$$\boldsymbol{x_C = 0.094, \quad -25.0 \qquad \Rightarrow x_C = 0.094\ m}$$

Example 8.9 Connected Blocks in Motion

Two blocks are connected by a light string that passes over a frictionless pulley as shown in Figure 8.12. The block of mass ***m*₁** lies on a horizontal surface and is connected to a spring of force constant ***k***. The system is released from rest when the spring is unstretched. If the



hanging block of mass ***m*₂** falls a distance ***h*** before coming to rest, calculate the coefficient of kinetic friction between the block of mass ***m*₁** and the surface.

$$\Delta E_{mech} = E_f - E_i = U_{sf} - U_{gf} = -f_x \cdot h$$

$$\frac{1}{2}kh^2 + 0 - 0 - m_2gh = -\mu_k m_1 g \cdot h \qquad \mu_k = \frac{m_2 g - \frac{1}{2}kh^2}{m_1 g}$$

8.5 Power

The time rate of energy transfer is called the **instantaneous power** P

$$P = \frac{dW}{dt},$$

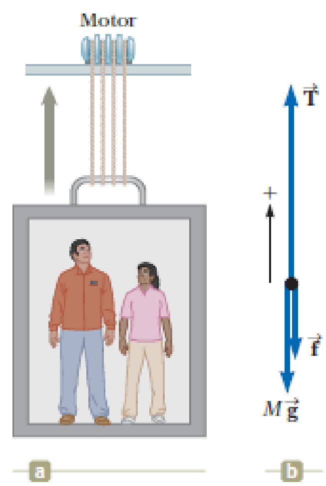
the **average power** during this interval is

$$\overline{P} = \frac{W}{\Delta t} \qquad \textbf{Since} \quad \mathbf{W = F \cdot \Delta r} \qquad \Rightarrow P = \frac{\vec{F} \cdot d\vec{r}}{dt} = \vec{F} \cdot \vec{v}$$

SI unit: J/s = watt= W, **1hp = 746 W** (horse-power),

kWh = 10^3 W. 60x60 sec (energy unit)

Example 8.11 Power Delivered by an Elevator Motor



An elevator car (Fig.a) has a mass of 1 600 kg and is carrying passengers having a combined mass of 200 kg. A constant friction force of 4 000 N retards its motion.

(A) How much power must a motor deliver to lift the elevator car and its passengers at a constant speed of 3.00 m/s?

a =0

T- f- mg = m(0) =0 T = f + mg = 4000 +18000

$$P = \vec{F} \cdot \vec{v} = Tv = 22000 \times 3 = 66000 \ W$$

[D] Problem 8-30 (28)[36 ch7 ed.6]

The electric motor of a model train accelerates the train from rest to 0.620 m/s in 21.0 ms. The total mass of the train is 875 g.

(a) Find the minimum power delivered to the train by electrical transmission
from the metal rails during the acceleration. (b) Why is it the minimum power?

$$P=\frac{W}{\Delta t}=\frac{\Delta K}{\Delta t}=\frac{K_f-0}{\Delta t}=\frac{mv^2}{2\Delta t}=\frac{0.875(0.620)^2}{2\times 21\times 10^{-3}}=8.01\ W$$

Selected Problems:

[D] Prob.8-6 A block of mass m = 5.00 kg is released from point A and slides on the frictionless track shown in Figure P8.6. Determine (a) the block's speed at points B and C and (b) the net work done by the gravitational force on the block as it moves from point A to point C.

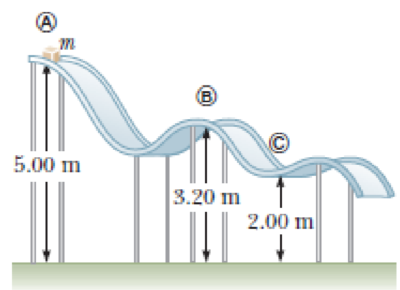


Figure P8.6

$$E_A=E_B \Rightarrow U_A+K_A=U_B+K_B$$

$$mgh_A+0=mgh_B+\frac{1}{2}mv_B^2$$

$$gh_A=gh_B+\frac{1}{2}v_f{}^2$$

$$v_B=\sqrt{2g(h_A-h_B)}=\sqrt{2\times 10(5-3.2)}=6m/s$$

$$v_C=\sqrt{2g(h_A-h_C)}=\sqrt{2\times 10(5-2)}=7.75m/s$$

$$W_g=-\Delta U=-mg(h_A-h_C)=-\mathbf{5\times 10(3)=-150J}$$

Same also: $W_g=\Delta K=(K_C-K_A)=K_C-0=\frac{1}{2}mv^2$

8-15. A block of mass m=2.00 kg is attached to a spring of force constant k=500 N/m as shown in Figure P8.15. The block is pulled to a position x_i = 5.00 cm to the right of equilibrium and released from rest. Find the speed the block has as it passes through equilibrium if (a) the horizontal surface is frictionless and (b) the coefficient of friction between block and surface is μ_k = 0.350.

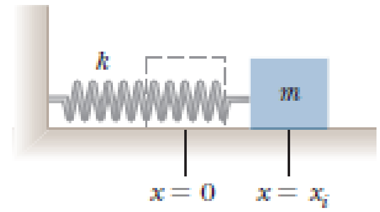


Figure P8.15

(a)By work Kinetic-Energy Theorem

$$W_s = -\Delta U_s = -(0 - \frac{1}{2}kx^2) = \frac{1}{2}(500)(0.05)^2 = 0.625J$$

$$K_f - K_i = W_s \qquad \Rightarrow \frac{1}{2}mv_f^2 - 0 = 0.625J$$

$$\Rightarrow v_f = \sqrt{\frac{2\times 0.625}{2.0}} = 0.791\, m/s$$

Block-spring as isolated system (alternative method)

$$K_f + U_{sf} = K_i + U_{si} \qquad \Rightarrow \frac{1}{2}mv_f^2 + 0 = 0 + \frac{1}{2}\times 500\times(0.05)^2$$

$$\Rightarrow v_f = \sqrt{\frac{2\times 0.625}{2.0}} = 0.791\, m/s$$

(b) $K_f = K_i + W_{others} \ - f_k d \qquad \Rightarrow \frac{1}{2}mv_f^2 = 0 + W_s - \mu_k mgd$

$$\frac{1}{2}mv_f^2 = 0 + 0.625 - (0.35)(2.00)(10)(0.05) = 0.275$$

$$\Rightarrow v_f = \sqrt{(2\times 0.275)/2} = 0.53\, m/s$$

8-23(19)(33)

A 5.00-kg block is set into motion up an inclined plane with an initial speed of v_i = 8.00 m/s (Fig.).The block comes to rest after

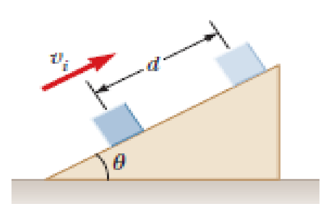
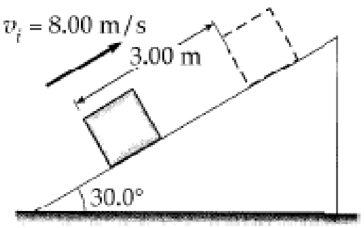


Figure P8.23

traveling $d = 3.00$ m along the plane, which is inclined at an angle of $\theta = 30^\circ$ to the horizontal. For this motion, determine

(a) the change in the block's kinetic energy,
(b) the change in the potential energy of the block–Earth system, and
(c) the friction force exerted on the block (assumed to be constant).
(d) What is the coefficient of kinetic friction?



(a) $\Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = 0 - \frac{1}{2}(5)(8^2) = -160\ J$

(b) $\Delta U = mg(y_f - y_i) = mgh = mg(3 \times \sin 30) = 73.5J$

(c) The mechanical energy converted due to friction is

$$\boldsymbol{E_f - E_i = \Delta E_{mech} = \Delta U + \Delta K = -f_k \cdot d = -160 + 73.5 = -86.5}$$

$$f_k = \frac{86.5}{3} = 28.8N$$

(d) $f_k = \mu_k n = \mu_k mg\cos\theta \qquad \Rightarrow \mu_k = \dfrac{28.8}{5 \times 10 \times \cos 30} \approx 0.68$

8-63. A **10.0-kg** block is released **from rest** at point **A** in Figure P8.63. the track is frictionless except for the portion between points **B** and **C**, which has a length of **6.00 m**. The block travels down the track, hits a spring of force constant **2 250 N/m**, and compresses the spring **0.300 m** from its equilibrium position before **coming to rest momentarily**.
Determine the coefficient of kinetic friction between the block and the rough surface between points **B** and **C**.

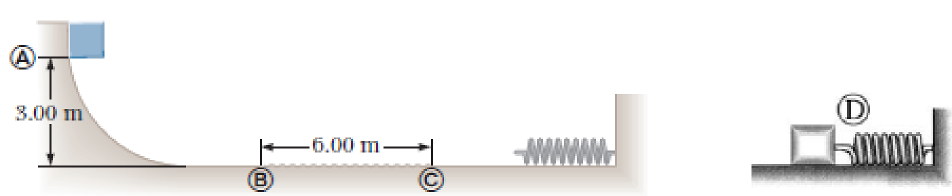


Figure P8.63

$$E_B = E_A \quad \Rightarrow mgh = \frac{1}{2}mv_B^2$$

$$\Rightarrow v_B^2 = \sqrt{2gh} = \sqrt{2\times 10\times 30} = \sqrt{60} = 7.75m/s$$

$$E_f = E_D = \frac{1}{2}kx_f{}^2 + \frac{1}{2}mv_f^2 + mgy_f$$

$$= \frac{1}{2}kx_f^2 + 0 + 0 = \frac{1}{2}\times 2250\times(0.3)^2 = 101.25J$$

$$E_i = E_A = mgy_A = 10\times 10\times 3 = 300J$$

$$E_D = E_A - f_k\cdot d \quad \Rightarrow \boxed{300 - 101.25 = f_k\cdot d}$$

$$f_k\cdot d = 198.8, \qquad f_k = \frac{198.8}{6} = 33.13N$$

$$f_k = \mu_k mg, \qquad \mu_k = \frac{33.13}{10\times 10} \approx 0.33$$

Find speed at C.

$$E_C = E_A - f_k d \qquad \Rightarrow \frac{1}{2}mv_C^2 = mgy_A - \mu_k mgd$$

Prob. 8-43. Review. A boy starts at rest and slides down a frictionless slide as in Figure P8.43. The bottom of the track is a height *h* above the ground. The boy then leaves the track horizontally, striking the ground at a distance *d* as shown. Using energy methods, determine the initial height *H* of the boy above the ground in terms of *h* and *d*.

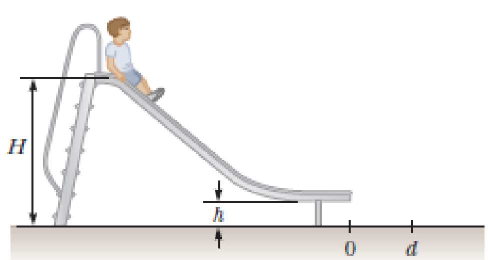
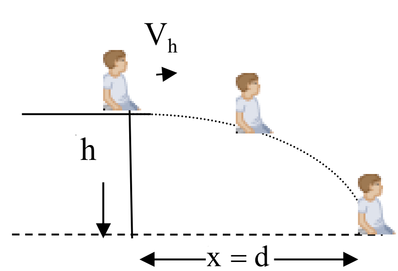


Figure P8.43



(a) $K_i + U_{gi} = K_f + U_{gf} \Rightarrow \frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f$

$$\frac{1}{2}mv_h^2 + mgh = mgH \qquad \Rightarrow v_h^2 = 2g(H-h)$$

bottom of the track. Determine the boy's speed as a projectile

$$y = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2h}{g}}$$

$$v_h = \frac{X}{t} = \frac{d}{\sqrt{2h/g}}$$

$$v_h^2 = 2g(H-h) = \frac{d^2}{2h/g} \Rightarrow H = (\frac{d^2}{2h} + 2h)/2$$