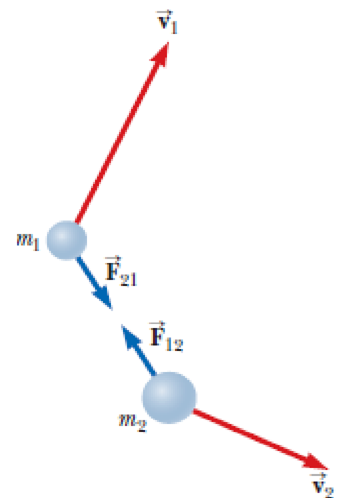
Chapter 9 p.247
Linear Momentum and Collisions

Ed. 9 5 lec	Linear Momentum and Collisions		Discussion: 4, 11, 14, 33, 44	Additional: 2, 9, 30,57,71
		Fall 2022	Discussion ch9: 3, 4, 11, 14, 18, 19, 33, 44, 71, 80,81	

9.1 Linear Momentum

Quantity of motion or linear momentum, related to the product of mass and velocity of the particle. When two isolated particle interact, From Newton's 3rd Law: the force **F12** exerted by object 1 on object 2 is equal in magnitude and opposite in direction to the force **F21** exerted by object 2 on object 1.



$\mathbf{F}_{21} = -\mathbf{F}_{12}$ (action force = - reaction force),

Or $\mathbf{F}_{21} + \mathbf{F}_{12} = 0$ $\qquad m_1\vec{a}_1 + m_2\vec{a}_2 = 0$

$$m_1\frac{d\vec{v}_1}{dt} + m_2\frac{d\vec{v}_2}{dt} = 0 \quad \Rightarrow \frac{d(m_1\vec{v}_1)}{dt} + \frac{d(m_2\vec{v}_2)}{dt} = 0$$

$$\frac{d}{dt}(m_1\vec{v}_1 + m_2\vec{v}_2) = 0 \quad \Rightarrow m_1\vec{v}_1 + m_2\vec{v}_2 = constant \;\; quantity$$

Linear Momentum: $\vec{\boldsymbol{p}} = \boldsymbol{m}\vec{\boldsymbol{v}}$, **SI-unit: kg.m/s**

$$m_1\vec{v}_1 + m_2\vec{v}_2 = \vec{p}_1 + \vec{p}_2 = \; constant \;\; quantity$$

The total momentum of isolated system of particles is conserved

$$\Delta\vec{p}_{tot} = 0$$

Force on <u>a particle</u> and its momentum:

$$\sum\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d\vec{p}}{dt}$$

Time rate change of linear momentum of a particle = net force acting on it

9.2 Analysis Model: Isolated System (Momentum)

$$\frac{d}{dt}(\vec{p}_1+\vec{p}_2)=0 \quad \Rightarrow \vec{p}_{tot}=\vec{p}_1+\vec{p}_2=constant$$

(total momentum of an isolated system of two particle remains constant)

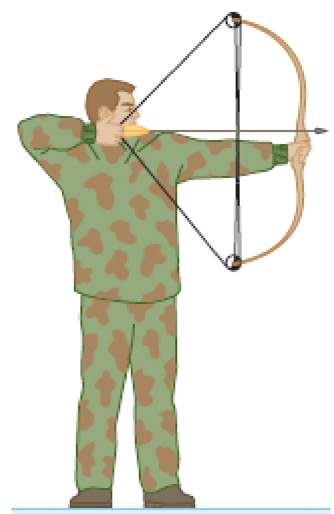
$$\vec{p}_{1i}+\vec{p}_{2i}=\vec{p}_{1f}+\vec{p}_{2f}$$

x-direction: $p_{1ix}+p_{2ix}=p_{1fx}+p_{2fx}$

y-direction: $p_{1iy}+p_{2iy}=p_{1fy}+p_{2fy}$

Momentum of an isolated *system* is conserved, the momentum of one particle within an isolated System is not necessarily conserved because other particles in the system may be interacting with it.

Example 9.1 The Archer



A **60-kg archer** stands at rest on **frictionless ice** and fires a **0.50-kg arrow** horizontally at **50 m/s** (Fig. 9.2). With what velocity does the archer move across the ice after firing the arrow?

Solution

(There is **no motion in the vertical direction**)

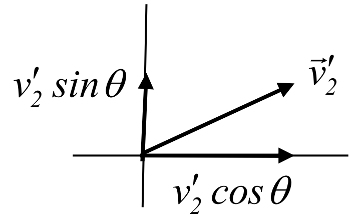
There are no external forces in the horizontal direction, and we can consider the system isolated in terms of momentum components in this direction.

Total horizontal momentum of the system before the arrow is fired is zero because nothing in the system is moving.

$$\vec{p}_{1i}+\vec{p}_{2i}=0=\vec{p}_{1f}+\vec{p}_{2f}$$

$$m_1\vec{v}_{1f}+m_2\vec{v}_{2f}=0 \qquad \vec{v}_{1f}=-\frac{m_2}{m_1}\vec{v}_{2f}=-\frac{0.5}{60}\times 50\,\hat{i}=-0.42\;\hat{i}\;\; m/s$$

What If? What if the arrow were fired in a direction that **makes an angle** θ with the horizontal? How will that change the recoil velocity of the archer? $\vec{v}_2$



In x- direction:

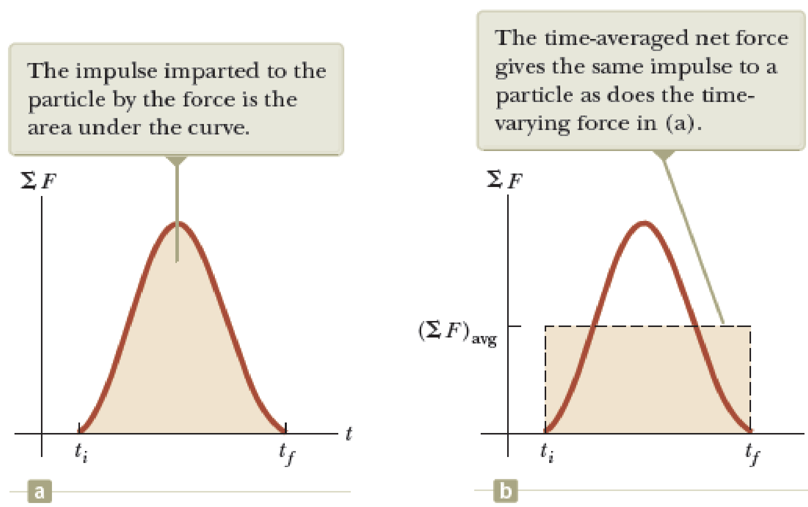
$$P_{fx} = m_1 v'_{1x} + m_2 v'_2 \cos\theta = 0 = P_{ix}$$

$$\Rightarrow v'_{1x} = -\frac{m_2}{m_1} v'_2 \cos\theta$$

$$m_1 v'_{1y} + m_2 v'_2 \sin\theta = 0 \qquad \Rightarrow v'_{1y} = -\frac{m_2}{m_1} v'_2 \sin\theta$$

9.3 Analysis Model: Nonisolated System (Momentum)

A system is **nonisolated** if a net force acts on the system for a time interval. Momentum being transferred to the system from the environment by means of the net force.



Net force on a particle:

$$\sum \vec{F} = \frac{d\vec{p}}{dt} \qquad or \qquad d\vec{p} = (\sum \vec{F})\, dt$$

Integrating: $$\Delta\vec{p} = \vec{p}_f - \vec{p}_i = \int_{t_i}^{t_f} \sum \vec{F}\, dt$$

Right side is the impulse of the net force;

$$\vec{I} = \int_{t_i}^{t_f} \sum \vec{F} \; dt$$ = area under the Force-time graph (Fig.a)

The **time-averaged net force** (Fig.b) gives the same impulse to a particle as does the time varying force in (a). $(\sum \vec{F})_{avg} = \overline{\vec{F}}$

$$\Rightarrow \vec{I} = \Delta \vec{p} = \int_{t_i}^{t_f} \vec{F}_{avg} \cdot dt$$

The impulse of net force on a particle = change in its momentum.

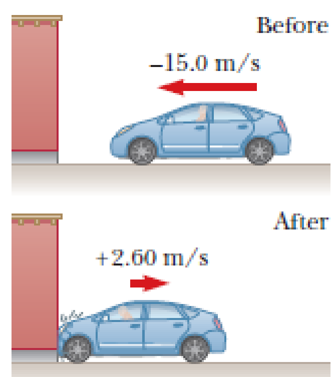
Since the **impulsive force** is of **unknown form** and **acts for a short time interval**, it is convenient to use the **time-averaged force** that has nearly the same effect and impulse

$$\boxed{\vec{I} = (\sum \vec{F})_{avg} \cdot \Delta t = \vec{F}_{avg} \cdot \Delta t = \Delta \vec{p}}$$

The impulse on a particle = average force **time** the time interval of action.

Example 9.3 How Good Are the Bumpers?

In a particular crash test, a car of **mass 1 500 kg** collides with a wall as shown in Figure 9.4. The initial and final velocities of the car are $\vec{v}_i = -15\,\hat{i}$ m/s and, $\vec{v}_f = +2.6\,\hat{i}$ m/s respectively. If **the collision lasts 0.150 s**, find the impulse caused by the collision and the average force exerted on the car.



Solution

$$\vec{I} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i = m(\vec{v}_f - \vec{v}_i)$$

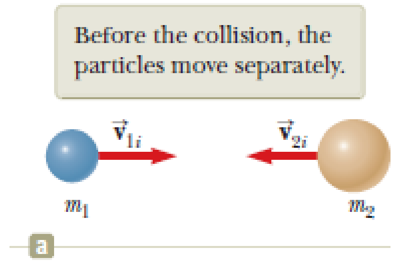
$$= 1500(2.6 - (-15))\hat{i} = 2.64 \times 10^4 \hat{i} \;\; kg.m/s$$

$$\vec{I} = \Delta\vec{p} = \vec{F}_{avg} \cdot \Delta t$$

$$\Rightarrow \vec{F}_{avg} = \frac{\vec{I}}{\Delta t} = \frac{2.64 \times 10^4}{0.15}\hat{i} = 1.76 \times 10^5 \;\hat{i} \;\; N$$

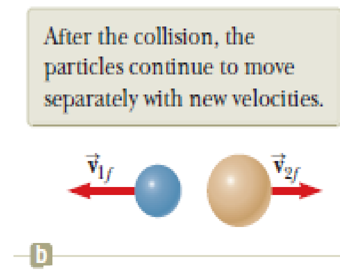
9.4 Collisions in One Dimension

When two particles of masses m_1 and m_2 collide, the impulsive forces is internal to the system of the two particles. Therefore, the two particles form an isolated system and the momentum of the system must be conserved in *any* collision.



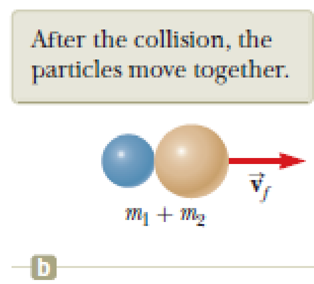
Elastic collision: total kinetic energy and total momentum of the system are conserved the same before and after the collision.

Inelastic collision: is one in which the total kinetic energy of the system is not the same before and after the collision (even though the momentum of the system is conserved).



Perfectly inelastic: collisions: **the objects stick together** after they collide.

When the colliding objects do not stick together but some kinetic energy is transformed or transferred, the collision is called **inelastic.**



Elastic collision in one-dimension:

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$
$$\Rightarrow m_1 (v_{1i} - v_{1f}) = -m_2 (v_{2i} - v_{2f})$$

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

$$\Rightarrow m_1 (v_{1i}^2 - v_{1f}^2) = m_2 (v_{2f}^2 - v_{2i}^2)$$
$$\Rightarrow m_1 (v_{1i} - v_{1f})(v_{1i} + v_{1f}) = -m_2 (v_{2i} - v_{2f})(v_{2f} + v_{2i})$$

Divide by first eq.

$$(v_{1i} + v_{1f}) = (v_{2i} + v_{2f}) \quad \Rightarrow (v_{1i} - v_{2i}) = -(v_{1f} - v_{2f})$$

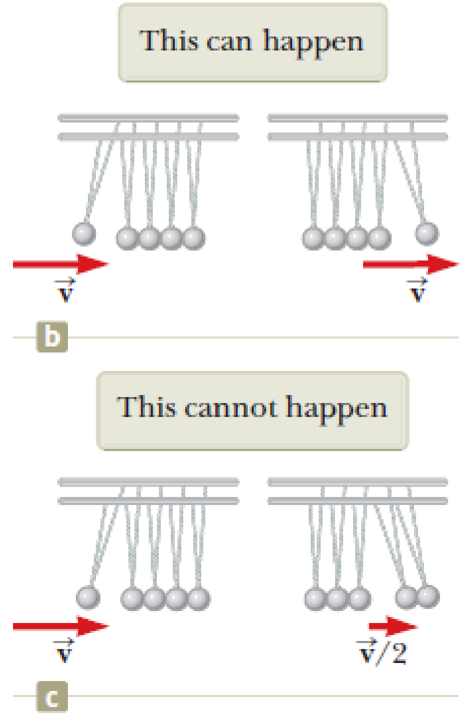
[Only for Elastic Collisions]

Relative velocity before collision = negative relative velocity after collision

Example 9.4:

The Executive Stress Reliever

Illustrates conservation of momentum and kinetic energy. Five identical hard balls supported by strings of equal lengths. Ball 1 pulled out and released collides elastically with 2, **1 stops and ball 5 moves** out as shown in Figure 9.8b. If balls 1 and 2 are pulled out and released, they stop after the collision and balls 4 and 5 swing out, and so forth. Is it ever possible that when ball 1 is released, it stops after the collision and balls 4 and 5 will swing out on the opposite side and travel with half the speed of ball 1 as in Figure 9.8c?



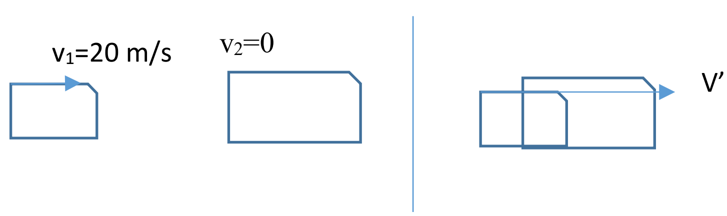
$$\left.\begin{aligned} m v_i &= m v_f \\ \frac{1}{2} m v^2 &= \frac{1}{2} m\ v^2 \end{aligned}\right\} \textit{case b (collision is elastic)}$$

$$\left.\begin{aligned} mv &= (m+m)\frac{v}{2} = mv \\ \frac{1}{2} m v^2 &\neq \frac{1}{2}(2m)\ \frac{v^2}{4} \end{aligned}\right\} \textit{case c (collision is elastic)}$$

Case c Never occur if ball 1 stops

Example 9.5 Carry Collision Insurance!

A **1 800-kg** car stopped at a traffic light **is struck from the rear** by a **900-kg car**. The two cars become entangled,



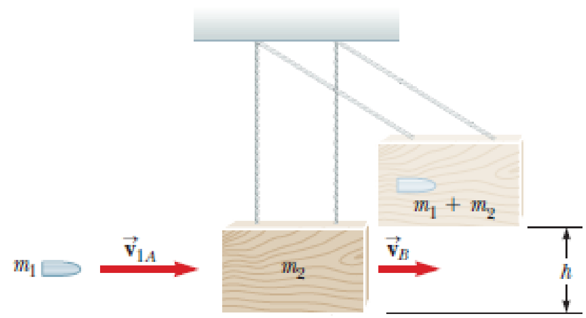
moving along the same path as that of the originally moving car. If the **smaller car** were moving at **20.0 m/s** before the collision, what is the velocity of the **entangled cars** after the collision?

Solution

$$m_1 v_{1i} = (m_1 + m_2) V_f$$

$$\Rightarrow V_f = \frac{900 \times 20}{2700} = 6.67\ m/s$$

Example 9.6 The Ballistic Pendulum



The ballistic pendulum (Fig. 9.9) is an apparatus used to measure the speed of a fast-moving projectile such as a bullet. A projectile of mass m_1 is fired into a large block of wood of mass m_2 suspended from some light wires. The projectile embeds in the block, and the entire system swings through a height h. How can we **determine the speed** of the projectile from a measurement of h?

Solution

P conservation: $m_1 v_{1A} = (m_1 + m_2) v_B$

$$\Rightarrow v_{1A} = \frac{(m_1 + m_2) v_B}{m_1}$$

E conservation: $\frac{1}{2}(m_1 + m_2)\, v_B^2 = (m_1 + m_2) gh$

$$\Rightarrow v_B = \sqrt{2gh}$$

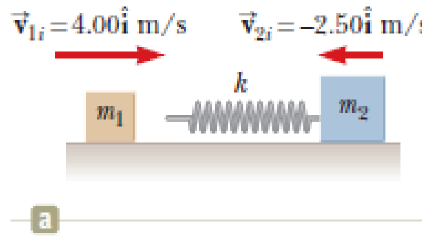
$$\Rightarrow v_{1A} = \frac{(m_1 + m_2)}{m_1}\sqrt{2gh}$$

Example 9.7 A Two-Body Collision with a Spring

A block of mass m_1 **= 1.60 kg** initially moving **to the right with a speed** of **4.00 m/s** on a frictionless, horizontal track collides with a light spring attached to a second block of mass m_2 **= 2.10 kg** initially moving to the **left with a speed of 2.50 m/s** as shown in Figure 9.10a. The spring constant is **600 N/m.**

(A) Find the velocities of the two blocks after the collision.

Solution



$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

$$1.6(4) + 2.1(-2.5) = 1.6v_1' + 2.1v_2'$$

$$6.4 - 5.25 = 1.6v_1' + 2.1v_2'$$

$$1.15 = 1.6v_1' + 2.1v_2'$$

$$(v_1 - v_2) = (v_2' - v_1') \qquad 4 + 2.5 = 6.5 = v_2' - v_1' \qquad \Rightarrow v_2' = 6.5 + v_1'$$

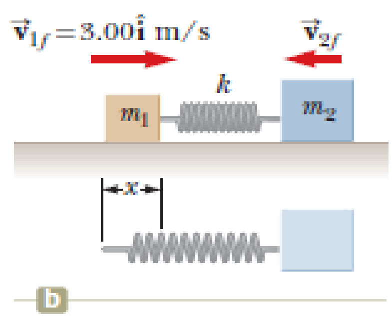
$$1.15 = 1.6v_1' + 2.1(6.5 + v_1')$$

$$-12.5 = 3.7v_1' \qquad \Rightarrow v_1' = \frac{-12.5}{3.7} = -3.38\ m/s$$

$$v_2' = 6.5 + v_1' \qquad \Rightarrow v_2' = 6.5 - 3.38 = 3.12\,m/s$$

(B) Determine the **velocity of block 2** during the collision, at the **instant block 1 is moving to the right** with a **velocity of 3.00 m/s** as in Figure 9.10b.

Solution



$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

$$1.6(4) + 2.1(-2.5) = 1.6(3) + 2.1v_2'$$

$$1.15 = 4.8 + 2.1v_2'$$

$$v_2' = \frac{-3.65}{2.1} = -1.74\,m/s$$

9.5 Collisions in Two Dimensions

Momentum of a system of two particles is conserved when the system is isolated. The momentum in each of the directions *x, y,* and *z* is conserved. For such two-dimensional collisions:

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$$

x-direction: $p_{1ix} + p_{2ix} = p_{1fx} + p_{2fx}$

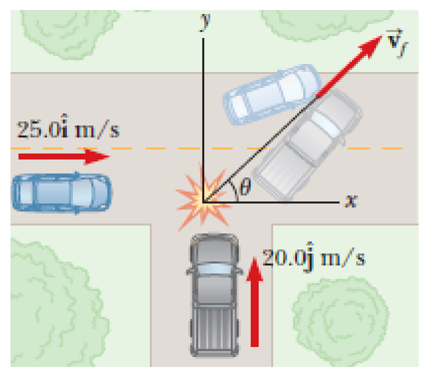
$$\Rightarrow m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx}$$

y-direction: $p_{1iy} + p_{2iy} = p_{1fy} + p_{2fy}$

$$\Rightarrow m_1 v_{1iy} + m_2 v_{2iy} = m_1 v_{1fy} + m_2 v_{2fy}$$

Example 9.8 Collision at an Intersection

A **1 500-kg car** traveling east with a speed of **25.0 m/s** collides at an intersection with a **2 500-kg truck** traveling **north at a speed of 20.0 m/s** as shown in Figure 9.12. Find the direction and magnitude of **the velocity of the wreckage after the collision**, assuming the vehicles stick together after the collision.



Solution

$$P_{xi} = P_{xf} \qquad m_1 v_{1ix} = (m_1 + m_2)\, v_f \cos\theta$$

$$P_{yi} = P_{yf} \qquad m_2 v_{2iy} = (m_1 + m_2)\, v_f \sin\theta$$

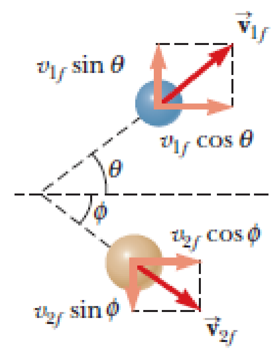
$$\tan\theta = \frac{m_2 v_{2iy}}{m_1 v_{1ix}} \Rightarrow \theta = \tan^{-1}(15/20) = 53.1^0$$

$$v_f = \frac{m_1 v_{1ix}}{(m_1 + m_2)\cos\theta} = \frac{15 \times 25}{40 \times \cos 35.1} = 15.6\ m/s$$

Example 9.9 Proton–Proton Collision

A proton collides elastically with **another proton** that **is initially at rest**. The incoming proton has an **initial speed of 3.50** $\times$ **10^5 m/s** and makes a glancing collision with the second proton as in active **Figure 9.11**. (At close separations, the protons exert a repulsive electrostatic force on each other.) After the collision, **one proton moves off at an angle of** 37^o to the original direction of motion and **the second deflects at an angle** φ **off to the same axis**. Find the **final speeds** of the two protons **and the angle** φ

Solution



x: $m_1 v_{1i} = m_1 v_{1f}\cos\theta + m_2 v_{2f}\cos\varphi$

y; $0 = m_1 v_{1f}\sin\theta - m_2 v_{2f}\sin\varphi$

m$_1$=m$_2$

$$v_{1i} = v_{1f}\cos\theta + v_{2f}\cos\varphi$$

$$0 = v_{1f}\sin\theta + v_{2f}\sin\varphi$$

$$v_{1i} - v_{1f}\cos\theta = v_{2f}\cos\varphi$$
$$\Rightarrow v_{1i}^2 - 2v_{1i}v_{1f}\cos\theta + v_{1f}^2\cos^2\theta = v_{2f}^2\cos^2\varphi$$

$$v_{1f}\sin\theta = v_{2f}\sin\varphi \qquad \Rightarrow v_{1f}^2\sin^2\theta = v_{2f}^2\sin^2\varphi$$

adding:

$$v_{1i}^2 - 2v_{1i}v_{1f}\cos\theta + v_{1f}^2(\cos^2\theta + \sin^2\theta) = v_{2f}^2(\cos^2\varphi + \sin^2\varphi)$$

using K.E conservation: $v_{1i}^2 = v_{1f}^2 + v_{2f}^2$ and substituting

$$v_{1f}^2 - v_{1i}v_{1f}\cos\theta = 0 \qquad \Rightarrow v_{1f} = 0, \qquad or$$

$$v_{1f} = v_{1i}\cos\theta = 3.5\times10^5\cos 37 = 2.8\times10^5$$

$v_{2f}{}^{2} = \sqrt{\left(3.5^2 - 2.8^2\right) \times 10^{10}} = 2.11 \times 10^5\, m/s$, and

$$\varphi = \sin^{-1}\left(\frac{v_{1f}\ \sin\theta}{v_{2f}}\right) = 53^o$$

9.6 The Center of Mass

The system may be collection of particles, or extended large object. The center of mass of the system locates a point such that if all the mass of the system were concentrated at that point. The system moves as if the net external force were applied to a single particle located at the center of mass.

The position vector of the center of mass of many particle system:

$$\vec{R}_{cm} = X_{cm}\,\hat{i} + Y_{cm}\,\hat{j} + Z_{cm}\,\hat{j}$$

where: $X_{cm} = \dfrac{\sum_i m_i x_i}{\sum_i m_i} = \dfrac{1}{M}\sum_i m_i x_i = \dfrac{m_1 x_1 + m_2 x_2 + \cdots}{m_1 + m_2 + \cdots}$

$$Y_{cm} = \frac{\sum m_i y_i}{M}; \qquad Z_{cm} = \frac{\sum m_i z_i}{M}, \qquad M = \sum_i m_i = m_1 + m_2 + \cdots$$

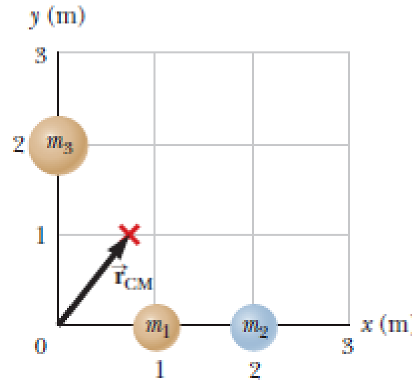
In vector notation:

$\vec{R}_{cm} = \dfrac{1}{M}\sum_i m_i \vec{r}_i$ where: $\vec{r}_i = x\,\hat{i} + y\,\hat{j} + z\,\hat{j}$

For extended objects:

$$\vec{r}_{cm} \approx \frac{1}{M}\sum_i \Delta m_i \vec{r}_i\,; \quad \vec{r}_{cm} = \lim_{\Delta m_i \to 0} \frac{1}{M}\sum_i \Delta m_i \quad \Rightarrow \vec{r}_{cm} = \frac{1}{M}\int \vec{r}\,dm$$

Example 9.10



The Center of Mass of Three Particles

A system consists of three particles located as shown in Figure 9.18. Find the center of mass of the system. The masses of the particles are

$m_1 = m_2 = 1.0$ kg and $m_3 = 2.0$ kg.

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

$$x_{cm} = \frac{(1 \times 1) + (1 \times 2) + (2 \times 0)}{1 + 1 + 2} = \frac{3kg.m}{4kg} = 0.75m$$

$$y_{cm} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3}$$

$$y_{cm} = \frac{(1 \times 0) + (1 \times 0) + (2 \times 2)}{1 + 1 + 2} = \frac{4kg.m}{4kg} = 1.0m$$

$$\vec{r}_{cm} = x_{cm}\hat{i} + y_{cm}\hat{j} = 0.75\hat{i} + 1.0\hat{j} \;\; m$$

Problem Solution

Problem9-3.

At one instant, a **17.5-kg** sled is moving over a horizontal surface of snow at **3.50 m/s**. After **8.75 s** has elapsed, the sled stops.
Use a momentum approach to find the average friction force acting on the sled while it was moving.

$$\overline{\vec{F}}\Delta t = \Delta \vec{p} = m(\vec{v}_f - \vec{v}_i)$$

$$= 17.5(0 - 3.5)\hat{i} = 61.25\,\hat{i}\ \ kg.m/s$$

$$\overline{\vec{F}} = \frac{-61.25}{8.75} = -7.0\ N$$

Problem 9-4. A 3.00-kg particle has a velocity of ($3.0\,\hat{i}$ - $4.0\,\hat{j}$) m/s. (a) Find its *x* and *y* components of momentum. (b) Find the magnitude and direction of its momentum.

$$P_x = mv_x = 3\times 3 = 9\,kg.m/s$$

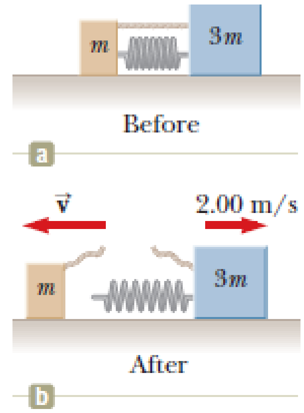
$$P_y = mv_y = 3\times(-4) = -12\,kg.m/s$$

$$p = \sqrt{p_x^2 + p_y^2} = \sqrt{9^2 + (-12)^2} = 15\,kg.m/s$$

$$tan\ \theta\ = \frac{p_y}{p_x}\ \ \Rightarrow \theta\ =\ tan^{-1}(-12/9) = -53.1^0, 307^0$$

Problem 9-11.

Two blocks of masses ***m*** and **3*m*** are placed on a frictionless, horizontal surface. A light spring is attached to the more massive block, and the blocks are pushed together with the **spring between them** (Fig. P9.9). A **cord initially holding the blocks together is burned**; after that happens, the block of mass **3*m* moves to the right** with a speed of **2.00 m/s**.



(a) What is the velocity of the block of mass *m*?

(b) Find the system's original elastic potential energy, taking

m= 0.350 kg.

(c) Is the original energy in the spring or in the cord?

(d) Explain your answer to part (c).

(e) Is the momentum of the system conserved in the bursting-apart process?

Explain how that is possible considering

(f) If there are large forces acting and

(g) there is no motion beforehand and plenty of motion afterward?

(a) $p_{1ix} + p_{2ix} = p_{1fx} + p_{2fx}$

$$\Rightarrow m(0) + 3m(0) = mv_{1fx} + 3m(2) = 0 \qquad \Rightarrow v_{1f} = -6.0\ m/s$$

(b) $E_i = E_f$

$$\Rightarrow U_s = \frac{1}{2}kx^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}(3m)v_2^2 = \frac{1}{2}(0.35)[6^2 + 3\times 2^2] = 8.4\ J$$

(c) It is in the spring.

(d) cord stores no energy, it is non-stretchable

(e) yes.

(f) Large forces acting in short time, but impulses produces equal but opposite change in momenta.

(g) Both momentum changes from zero (no motion) when cord exists to the maximum possible values

Problem 9-13. An estimated force–time curve for a baseball struck by a bat is shown in Figure P9.13. From this curve, determine (a) the **magnitude** of **the impulse delivered** to the ball and (b) the **average force** exerted on the ball.

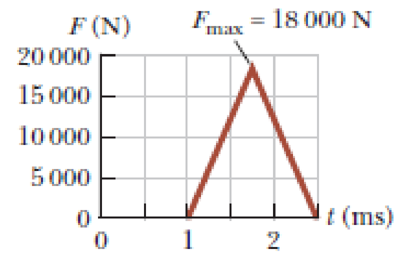


Figure P9.13

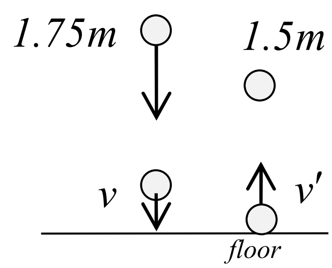
(a) Impulse:

$$I=\int F_x dt = Area\ \ of\ the\ triangle\ =\frac{1}{2}(1.5\times10^{-3})(18000)=13.50\,N.s$$

(b) $$\bar{F}=\frac{\Delta p}{\Delta t}=\frac{I}{\Delta t}=\frac{13.5}{1.5\times10^{-3}}=9000\ N$$

Problem 9-14. Review.

After a 0.300-kg rubber ball is dropped from a height of 1.75 m, it bounces off a concrete floor and rebounds to a height of 1.50 m. (a) Determine the magnitude and direction of **the impulse** delivered to the ball by the floor. (b) **Estimate the time the ball is in contact** with the floor **and use this** estimate to calculate the **average force** the floor exerts on the ball.



$$mgh=\frac{1}{2}mv^2 \qquad v=\sqrt{2gh}=\sqrt{2\times10\times1.75}=5.92\ m/s$$

$$v'=\sqrt{2gh'}=\sqrt{2\times10\times1.5}=5.48\ m/s$$

$$h=\frac{1}{2}gt^2 \qquad \Rightarrow t_1=\sqrt{\frac{2(1.75)}{g}}=0.592s$$

$$\Rightarrow t'=\sqrt{\frac{2(1.5)}{g}}=0.548s, \qquad \Delta t=0.044s$$

$$\boxed{I=\vec{\bar{F}}\Delta t=\Delta\vec{p}=m(\vec{v}_f-\vec{v}_i)}$$

$$\vec{I} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i = m(\vec{v}_f - \vec{v}_i)$$

$$= 0.3[5.48 - (-5.92)]\hat{j} = 0.3 \times 11.40\hat{j} \ kg.m/s$$

$$\vec{I} = \Delta\vec{p} = \vec{F}_{avg} \cdot \Delta t$$

$$\Rightarrow \vec{F}_{avg} = \frac{\Delta\vec{p}}{\Delta t} = \frac{3.42}{0.05}\hat{j} = 68.4\ \hat{j}\ N$$

Problem 9-18

A tennis player receives a shot with the ball (**0.060 0 kg**) traveling horizontally at **50.0 m/s** and returns the shot with the ball traveling horizontally at **40.0 m/s** in the opposite direction. (a) What is the impulse delivered to the ball by the tennis racquet? (b) What work does the racquet do on the ball?

(a) Impulse: $\vec{I} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i = m(\vec{v}_f - \vec{v}_i)$ (let v_i in the –x direction)

$$= 0.060\ (40 - (-50))\hat{i} = 5.40\ \hat{i} \quad N.s$$

(b) $W = \Delta K = \frac{1}{2} \times 0.060\left(40^2 - 50^2\right) = \ -27.0\ J$

Problem 9-19

The magnitude of the net force exerted in the x direction on a **2.50-kg** particle varies in time as *shown in Figure P9.19*. Find **(a)** the impulse of the force over the **5.00-s** time interval, **(b)** The final velocity the particle attains if it is originally at rest.

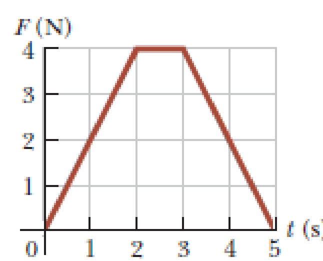


Figure P9.19

(c) Its final velocity if its original velocity is **2.00** $\hat{i}$ **m/s**, and

(d) The average force exerted on the particle for the time interval between **0** and **5.00 s.**

(a) $\vec{I} = \int_{t_i}^{t_f} (\sum \vec{F})_{avg} dt$ = *area under the force-time graph*

$$= \tfrac{1}{2}[5+1][4] = 12 \text{ N.s}$$

(b) $\vec{I} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i = m(\vec{v}' - 0) = 2.5v'$

$$v' = \frac{I}{2.5} = \frac{12\hat{i}}{2.5} = 4.8\hat{i} \ m/s$$

(c) $\vec{I} = 12\hat{i} = 2.5(\vec{v}' - (-2\hat{i}))$ $\quad\quad \vec{v}' = \frac{\vec{I}}{2.5} = \frac{7\hat{i}}{2.5} = 2.8\hat{i} \ m/s$

(d) $\vec{I} = \vec{\overline{F}}\,\Delta t \quad \Rightarrow \overline{F} = \frac{12\hat{i}}{5} = 2.4\,\hat{i}\,N$

Problem 9-30.

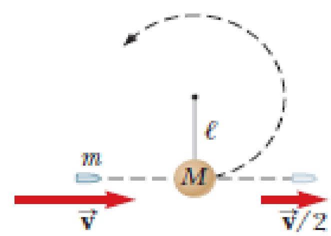


Figure P9.30

As shown in Figure P9.26, a bullet of mass *m* and speed *v* passes completely through a pendulum bob of mass *M*. The bullet emerges with a speed of *v*/2. The pendulum bob is suspended by a stiff rod (*not* a string) of length *l*, and negligible mass. What is the minimum value of *v* such that the pendulum bob will barely swing through a complete vertical circle?

Momentum : $mv = m\frac{v}{2} + MV \quad\quad \Rightarrow v = \frac{2MV}{m}$

Energy: $\frac{1}{2}MV^2 = Mg(2l) \quad \Rightarrow V = 2\sqrt{gl} \quad\quad \Rightarrow v = \frac{4M\sqrt{gl}}{m}$

Problem 9-33. Two blocks are free to slide along the frictionless, wooden track shown in Figure P9.33. The block of mass $\boldsymbol{m_1}$ **= 5.00 kg** is released from the position shown, at height $\boldsymbol{h}$**= 5.00 m** above the flat part of the track.

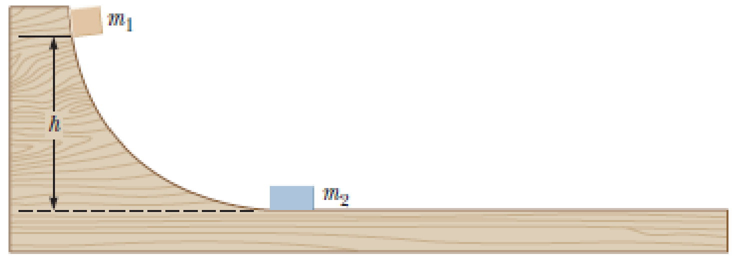


Figure P9.33

Protruding from its front end is the north pole of a strong magnet, which repeles the north pole of an identical magnet embedded in the back end of the block of mass $\boldsymbol{m_2}$ **= 10.0 kg**, **initially at rest**. The two blocks never touch. Calculate the maximum height to which m_1 rises after the elastic collision.

$$mgh = \frac{1}{2}mv_1^2 \qquad v_1=\sqrt{2gh_1}=\sqrt{2\times10\times5}=10\ m/s$$

$$m_1v_1+m_2v_2=m_1v_1'+m_2v_2'$$

$$\Rightarrow\ 5\times10+m_2(0)=5v_1'+10v_2' \quad \Rightarrow 50=5v_1'+10v_2'$$

$$(v_1 - v_2)=(v_2' - v_1') \qquad \Rightarrow v_2'=10+v_1'$$

$$5v_1'=50-10(10+v_1')=50-100-10v_1'$$
$$15v_1'=-50 \qquad \Rightarrow v_1'=-3.33\ m/s$$

$$mgh=\frac{1}{2}mv_1'^2 \qquad h_2=\frac{3.33^2}{2\times10}=0.554\ m$$

Problem 9-44

The mass of the blue puck in Figure P9.44 is 20.0% greater than the mass of the green puck. Before colliding, the pucks approach each other **with momenta of equal magnitudes and opposite directions**, and the green puck **has an initial speed of 10.0 m/s**.

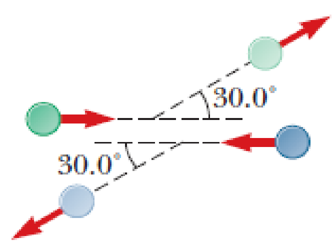


Figure P9.44

Find the speeds the pucks have **after the collision if half the kinetic energy of the system becomes internal energy** during the collision.

$$m_B = m_G + .2m_G = 1.2m_G$$

$$\vec{P}_i = \vec{p}_{1i} + \vec{p}_{2i} = 0, \qquad \Rightarrow m_G v_{Gi} - 1.2m_G v_{Bi} = 0$$

$$\Rightarrow v_{Bi} = \frac{m_G v_{Gi}}{1.2m_G} = \frac{10}{1.2} = 8.33\,m/s$$

But since $\vec{P}_i = \vec{P}_f = \vec{p}_{1f} + \vec{p}_{2f} = 0$

$$mv_{Gf} = 1.2mv_{Bf}$$

$$K_i = \frac{1}{2}m_G(10)^2 + \frac{1}{2}(1.2m_B)(8.33)^2 = \frac{1}{2}m \cdot 183$$

$$K_f = \frac{1}{2}K_i \quad \Rightarrow [\frac{1}{2}m_G v_{Gf}{}^2 + \frac{1}{2}(1.2m_G)(v_{Bf})^2] = \frac{1}{2}(\frac{1}{2}m_G \cdot 183)$$

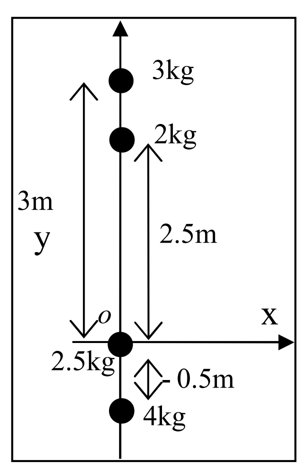
$$v_{Gf}^2 + 1.2v_{Bf}^2 = 91.7, \qquad mv_{Gf} = 1.2mv_{Bf}$$

$$1.2^2 v_{Bf}^2 + 1.2v_{Bf}^2 = 91.7$$

$$2.64v_{Bf}^2 = 91.7 \qquad \Rightarrow v_{Bf} = \sqrt{\frac{91.7}{2.64}} = 5.89m/s^2$$

Problem 9-45.

Four objects are situated along the ***y* axis** as follows: a **2.00-kg** object **is at +3.00 m**, a **3.00-kg** object **is at +2.50 m**, a **2.50-kg** object **is at the origin**, and a **4.00-kg** object **is at - 0.500 m**. Where is the center of mass of these objects?



$$y_{cm} = \frac{2\times3+3\times2.5+2.5\times0+4\times(-0.5)}{2+3+2.5+4} = \frac{11.5}{11.5} = 1.0m$$

$$x_{cm} = \frac{2\times0+3\times0+2.5\times0+4\times(0)}{2+3+2.5+4} = \frac{0}{11.5} = 0m$$

$$\vec{r}_{cm} = x_{cm}\hat{i} + y_{cm}\hat{j} = 0\hat{i} + 1\hat{j}\ \ m$$

Problem 9-71

A **1.25-kg** wooden block rests on a table over a large hole as in Figure P9.71. A **5.00-g** bullet with an **initial velocity** $\boldsymbol{v_i}$ is fired upward into the bottom of the block and remains in the block after the collision. The block and bullet rise to a maximum height of **22.0 cm**. (a) Describe how you would find the initial velocity of the bullet using ideas you have learned in this chapter. (b) Calculate the initial velocity of the bullet from the information provided.

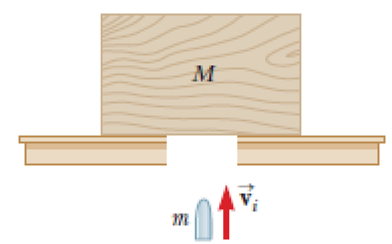


Figure P9.71
Problems 71 and 72.

(a) P conservation: $m_1 v_{1i} = (m_1+m_2)V_f \Rightarrow v_{1i} = \dfrac{(m_1+m_2)V_f}{m_1}$

E conservation: $\dfrac{1}{2}(m_1+m_2)\,V_f^2 = (m_1+m_2)gh \quad \Rightarrow V_f = \sqrt{2gh}$

(b) $v_{1i} = \dfrac{(m_1+m_2)}{m_1}\sqrt{2gh} = \dfrac{1.255}{0.005}\times\sqrt{(20\times0.22)} = 526.5\ m/s$

Problem 9-81.

A bullet of mass m is fired into a block of mass M initially at rest at the edge of a frictionless table of height h (Fig. P9.81).The bullet remains in the block, and after impact the block lands a distance d from the bottom of the table.

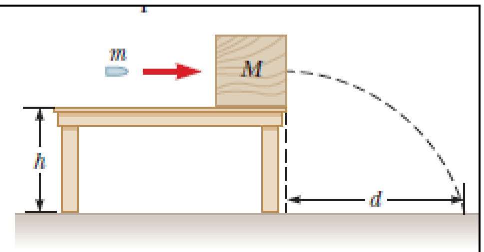
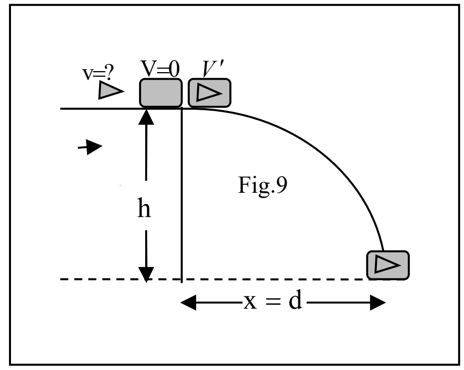


Figure P9.81 Problems 81 and 82.

Determine the initial speed of the bullet.



$$y=\frac{1}{2}gt^2 \Rightarrow t=\sqrt{\frac{2y}{g}}$$

$$V'=\frac{X}{t}=\frac{d}{t}$$

$$mv+0=(M+m)V' \quad \Rightarrow v = \frac{M+m}{m}\times\frac{d}{t}$$

Problem 9-80. A **5.00-g** bullet moving with an initial speed of $v_i = 400\,m/s$ is fired into and passes through a **1.00-kg** block as shown in Figure P 9.73. The block, initially at rest on a frictionless, horizontal surface, is connected to a spring with force constant **900 N/m**. The block moves ***d* = 5.00** cm to the right after impact before being brought to rest by the spring. Find **(a)** the speed at which the bullet emerges from the block and **(b)** the amount of initial kinetic energy of the bullet that is converted into internal energy in the bullet–block system during the collision.

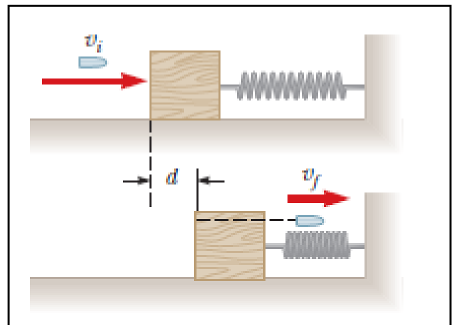
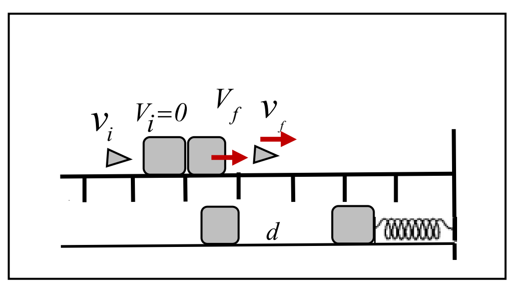


Figure P9.89



(a) $V_i = 0$ الكتلة الكبيرة ساكنة $v_i = 400\,m/s$ الرصاصة تضرب بسرعة

$$mv_i + MV_i = mv_f + MV_f$$

$$(0.005)\times 400 + 0 = (0.005)v_f + 1\times V_f$$

مطلوب حساب v_f ولذلك يجب حساب V_f وهي سرعة *M* بعد خروج الرصاصة مباشرة

M تضغط النابض أقصى مسافة d عندها سكنت فتتحول طاقتها الحركية الى وضع مختزنة في النابض ، بالنسبة الى M والنابض فقط يكون

$$(K_{B1}+U_{s1})=(K_{B2}+U_{s2})$$

$$\frac{1}{2}MV_{Bf}^2=\frac{1}{2}kx^2 \qquad \Rightarrow V_{Bf}=\sqrt{\frac{900\times(0.05)^2}{1}}=30\times 0.05=1.5m/s$$

$$0.005\times 400+0=0.005v_f+1\times V_f$$

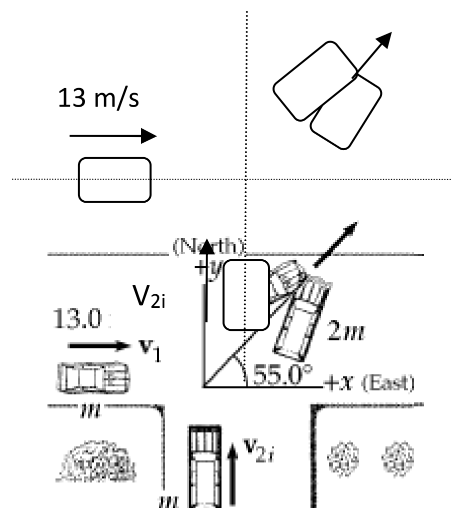
$$v_f=\frac{2-1.5}{0.005}=100m/s$$

(b) $K_{bullet})_f-K_{bullet})_{initial}=(K_{B2}+U_{s2})$

$$\frac{1}{2}(0.005)(100^2-400^2)=0.0025\times(1-16)\times 10^4=375J$$

Problem 9-36

Two automobiles of equal mass approach an intersection. One vehicle is traveling with speed **13.0 m/s** toward **the east**, and the other is traveling **north with speed v_{2i}**. Neither driver sees the other. The vehicles **collide in the intersection and stick together**, leaving parallel skid marks at an angle of **55.0° north of east**. The speed limit for both roads is **35 mi/h**, and the driver of the northward-moving vehicle **claims he was within the speed limit** when the collision occurred. **Is he telling the truth**? Explain your reasoning.



$$P_{xi}=P_{xf}: \quad M\times(13)=2M\cdot v_f\cos 55$$

$$P_{yi} = P_{yf} : \quad M \times v_{2i} = 2M \cdot v_f \ \sin 55$$

$$v_{2i} = 13 \tan (55) = 18.6 \ m/h$$

$$V_f = 18.6 \times \frac{3600}{1609} = 41.6 \ mi/h$$

No , he lies $v_{2i} = 41.6\ mi/h > 35\ mi/h$

Problem 9-51

A **2.00-kg** particle has a velocity ($2.0\,\hat{i} - 3.0\,\hat{j}$) m/s, and a **3.00-kg** particle has a velocity ($1.0\,\hat{i} + 6.0\,\hat{j}$) m/s. Find

(a) the velocity of the center of mass and

(b) the total momentum of the system.

(a) $$\vec{v}_{cm} = \frac{1}{M}\sum_i m_i \vec{v}_i = \frac{1}{5}\left[2(2.0\,\hat{i} - 3.0\,\hat{j}) + 3(1.0\,\hat{i} + 6.0\,\hat{j})\right]$$

$$= 1.4\,\hat{i} + 2.4\,\hat{j} \ m/s$$

(b) $$\vec{P} = M\vec{V}_{cm} = 5\left(1.4\,\hat{i} + 2.4\,\hat{j}\right) = 7.0\,\hat{i} + 12.0\,\hat{j} \quad kg.m/s$$

Problem 9-55

A ball of mass **0.200 kg** with a velocity of **1.50** $\hat{i}$ **m/s** meets a ball of mass **0.300 kg** with a velocity of **- 0.400** $\hat{i}$ **m/s** in a head-on, elastic collision.

(a) Find their velocities after the collision.

(b) Find the velocity of their center of mass before and after the collision.

(a) 1D-collision in x-direction:

$0.2\times1.5+0.3(-0.4)=0.2v_{1f}+0.3v_f$ $\qquad$ $0.18=0.2v_{1f}+0.3v_f$	
Relative velocity eq.: $v_{1i}-v_{2i}=-(v_{1f}-v_{2f})$	

$$1.5-(-0.4)=1.9=v_{2f}-v_{1f}$$

$$0.18=0.2v_{1f}+0.3(1.9+v_{1f}) \quad \Rightarrow 0.5v_{1f}=-0.39$$

$$\Rightarrow v_{1f}=-0.78\ m/s, \qquad v_{2f}=1.12\ m/s$$

(b) before:

$\vec{V}_{cm}=\dfrac{1}{0.5}\left[0.2\times(1.5)\hat{i}+0.3\times(-0.4\,\hat{i})\right]=0.36\,\hat{i}\ \ m/s$ = $\vec{V}_{cm}$ (after)