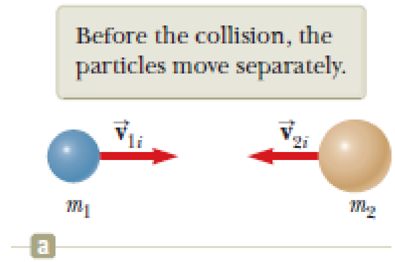
Chapter 9 *p.247*
Linear Momentum and Collisions

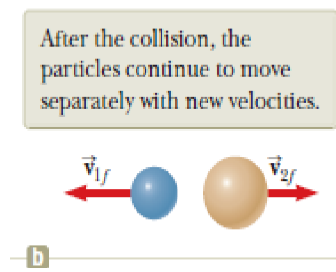
Ed. 9	Linear Momentum and Collisions	5 lecs.	Discussion: 4, 11, 14, 33, 44	Additional: 2, 9, 30,57,71

9.4 Collisions in One Dimension



When two particles of masses m_1 and m_2 collide, the impulsive forces is internal to the system of the two particles. Therefore, the two particles form an isolated system and the momentum of the system must be conserved in *any* collision.

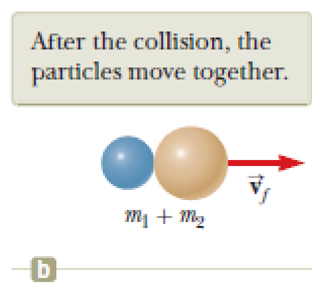
Elastic collision: total kinetic energy and total momentum of the system are conserved the same before and after the collision.



Inelastic collision: is one in which the total kinetic energy of the system is not the same before and after the collision (even though the momentum of the system is conserved).

Perfectly inelastic: collisions: **the objects stick together** after they collide.

When the colliding objects do not stick together but some kinetic energy is transformed or transferred, the collision is called **inelastic.**



Elastic collision in one-dimension:

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$
$$\Rightarrow m_1 (v_{1i} - v_{1f}) = -m_2 (v_{2i} - v_{2f})$$

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

$$\Rightarrow m_1(v_{1i}^2 - v_{1f}^2) = m_2(v_{2f}^2 - v_{2i}^2)$$
$$\Rightarrow m_1(v_{1i} - v_{1f})(v_{1i} + v_{1f}) = -m_2(v_{2i} - v_{2f})(v_{2f} + v_{2i})$$

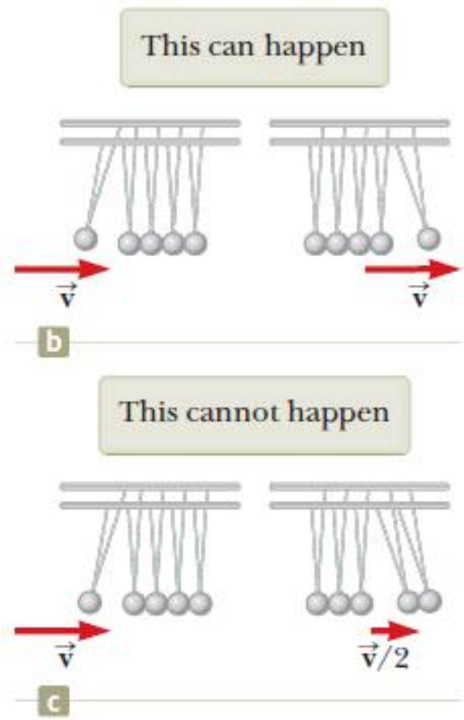
Divide by first eq.

$$(v_{1i} + v_{1f}) = (v_{2i} + v_{2f}) \quad \Rightarrow (v_{1i} - v_{2i}) = -(v_{1f} - v_{2f})$$

[Only for Elastic Collisions]

Relative velocity before collision = negative relative velocity after collision

Example 9.4 The Executive Stress Reliever



Illustrates conservation of momentum and kinetic energy. Five identical hard balls supported by strings of equal lengths. Ball 1 pulled out and released collides elastically with 2, **1 stops and ball 5 moves** out as shown in Figure 9.8b. If balls 1 and 2 are pulled out and released, they stop after the collision and balls 4 and 5 swing out, and so forth. Is it ever possible that when ball 1 is released, it stops after the collision and balls 4 and 5 will swing out on the opposite side and travel with half the speed of ball 1 as in Figure 9.8c?

$$\left.\begin{aligned} mv_i &= mv_f \\ \frac{1}{2}mv^2 &= \frac{1}{2}m\,v^2 \end{aligned}\right\} \textit{case b (collision is elastic)}$$

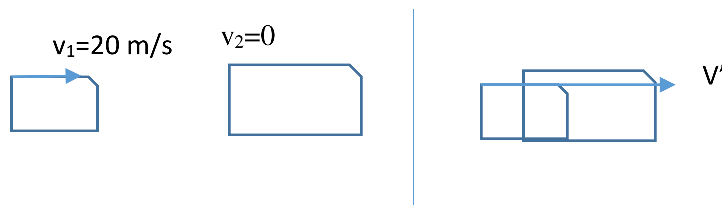
$$\left.\begin{aligned} mv &= (m+m)\frac{v}{2} = mv \\ \frac{1}{2}mv^2 &\neq \frac{1}{2}(2m)\,\frac{v^2}{4} \end{aligned}\right\} \textit{case c (collision is elastic)}$$

Case c Never occur if ball 1 stops

Example 9.5 Carry Collision Insurance!

A **1 800-kg** car stopped at a traffic light **is struck from the rear** by a **900-kg car**. The two cars become entangled, moving along the same path as that of the originally moving car. If the **smaller car** were moving at **20.0 m/s** before the collision, what is the velocity of the **entangled cars** after the collision?

Solution



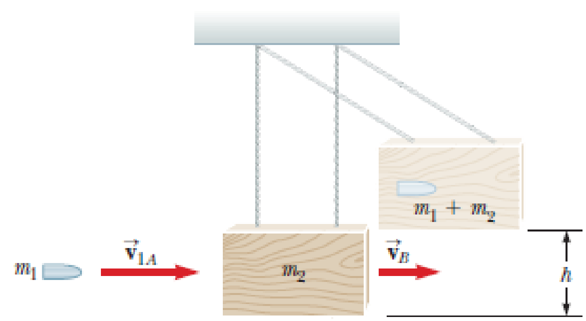
$$m_1 v_{1i} = (m_1 + m_2) V_f$$

$$\Rightarrow V_f = \frac{900 \times 20}{2700} = 6.67 \ m/s$$

Example 9.6 The Ballistic Pendulum

The ballistic pendulum (Fig. 9.9) is an apparatus used to measure the speed of a fast-moving projectile such as a bullet.

A projectile of mass m_1 is fired into a large block of wood of mass m_2 suspended from some light wires. The projectile embeds in the block, and the entire system swings through a height h. How can we **determine the speed** of the projectile from a measurement of h?



Solution

P conservation: $m_1 v_{1A} = (m_1 + m_2) v_B \qquad \Rightarrow v_{1A} = \dfrac{(m_1 + m_2) v_B}{m_1}$

E conservation: $\dfrac{1}{2}(m_1 + m_2)\, v_B^2 = (m_1 + m_2) gh \qquad \Rightarrow v_B = \sqrt{2gh}$

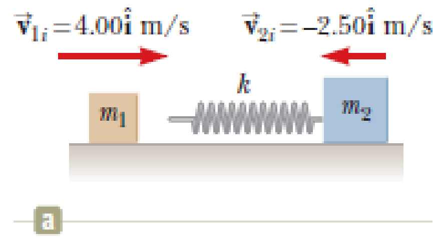
$$\Rightarrow v_{1A} = \frac{(m_1 + m_2)}{m_1}\sqrt{2gh}$$

Example 9.7 A Two-Body Collision with a Spring

A block of mass m_1 **= 1.60 kg** initially moving **to the right with a speed** of **4.00 m/s** on a frictionless, horizontal track collides with a light spring attached to a second block of mass m_2 **= 2.10 kg** initially moving to the **left with a speed of 2.50 m/s** as shown in Figure 9.10a. The spring constant is **600 N/m.**

(A) Find the velocities of the two blocks after the collision.

Solution



$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

$$1.6(4) + 2.1(-2.5) = 1.6 v_1' + 2.1 v_2'$$

$$6.4 - 5.25 = 1.6 v_1' + 2.1 v_2'$$

$$1.15 = 1.6 v_1' + 2.1 v_2'$$

$$(v_1 - v_2) = (v_2' - v_1') \qquad 4 + 2.5 = 6.5 = v_2' - v_1' \qquad \Rightarrow v_2' = 6.5 + v_1'$$

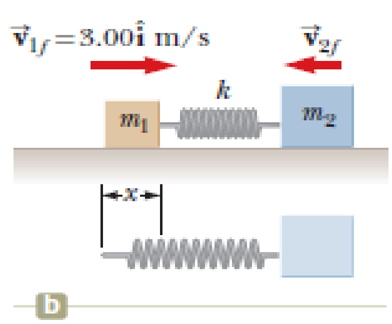
$$1.15 = 1.6 v_1' + 2.1(6.5 + v_1')$$

$$-12.5 = 3.7 v_1' \qquad \Rightarrow v_1' = \frac{-12.5}{3.7} = -3.38 \; m/s$$

$$v_2' = 6.5 + v_1' \qquad \Rightarrow v_2' = 6.5 - 3.38 = 3.12 \, m/s$$

(B) Determine the **velocity of block 2** during the collision, at the **instant block 1 is moving to the right** with a **velocity of 3.00 m/s** as in Figure 9.10b.

Solution



$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

$$1.6(4) + 2.1(-2.5) = 1.6(3) + 2.1 v_2'$$

$$1.15 = 4.8 + 2.1 v_2'$$

$$v_2' = \frac{-3.65}{2.1} = -1.74 \, m/s$$