# *Chapter 9 p.247*
# *Linear Momentum and Collisions*

| Ed.9<br>5 lecs. | Linear Momentum<br>and Collisions | Fall 2022 | Discussion ch9: 3, 4, 11, 14, 18, 19, 33, 44, 71, 80,81 |
|---|---|---|---|

## *Problem Solution*

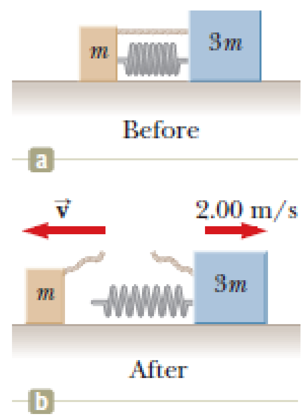
### Problem9-3.

At one instant, a **17.5-kg** sled is moving over a horizontal surface of snow at **3.50 m/s**. After **8.75 s** has elapsed, the sled stops.
Use a momentum approach to find the average friction force acting on the sled while it was moving.

$$\overline{\vec{F}}\Delta t = \Delta\vec{p} = m(\vec{v}_f - \vec{v}_i)$$

$$= 17.5(0 - 3.5)\hat{i} = 61.25\,\hat{i}\ \ kg.m/s$$

$$\overline{\vec{F}} = \frac{-61.25}{8.75} = -7.0\ N$$




**Problem 9-4.** A 3.00-kg particle has a velocity of ($3.0\,\hat{i}$ - $4.0\,\hat{j}$) m/s.
(a) Find its *x* and *y* components of momentum.
(b) Find the magnitude and direction of its momentum.

$$P_x = mv_x = 3\times 3 = 9\,kg.m/s$$

$$P_y = mv_y = 3\times(-4) = -12\,kg.m/s$$

$$p = \sqrt{p_x^2 + p_y^2} = \sqrt{9^2 + (-12)^2} = 15\,kg.m/s$$

$$tan\ \theta = \frac{p_y}{p_x} \Rightarrow \theta = tan^{-1}(-12/9) = -53.1^0, 307^0$$

## Problem 9-11.

Two blocks of masses ***m*** and **3*m*** are placed on a frictionless, horizontal surface. A light spring is attached to the more massive block, and the blocks are pushed together with the **spring between them** (Fig. P9.9). A **cord initially holding the blocks together is burned**; after that happens, the block of mass **3*m* moves to the right** with a speed of **2.00 m/s**.

(a) What is the velocity of the block of mass *m*?
(b) Find the system's original elastic potential energy, taking
*m*= 0.350 kg.
(c) Is the original energy in the spring or in the cord?
(d) Explain your answer to part (c).
(e) Is the momentum of the system conserved in the bursting-apart process?
Explain how that is possible considering
(f) If there are large forces acting and
(g) there is no motion beforehand and plenty of motion afterward?

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(a) $p_{1ix} + p_{2ix} = p_{1fx} + p_{2fx}$

$$\Rightarrow m(0) + 3m(0) = mv_{1fx} + 3m(2) = 0 \qquad \Rightarrow v_{1f} = -6.0\ m/s$$

(b) $E_i = E_f$

$$\Rightarrow U_s = \frac{1}{2}kx^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}(3m)v_2^2 = \frac{1}{2}(0.35)[6^2 + 3\times 2^2] = 8.4\ J$$

(c) It is in the spring.
(d) cord stores no energy, it is non-stretchable
(e) yes.
(f) Large forces acting in short time, but impulses produces equal but opposite change in momenta.

(g) Both momentum changes from zero (no motion) when cord exists to the maximum possible values

**Problem 9-13.** An estimated force–time curve for a baseball struck by a bat is shown in Figure P9.13. From this curve, determine (a) the **magnitude** of **the impulse delivered** to the ball and (b) the **average force** exerted on the ball.

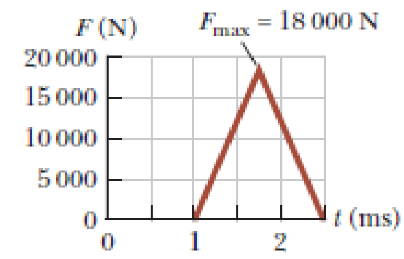


Figure P9.13

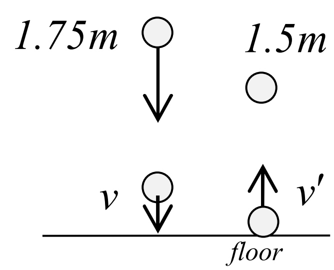
(a) Impulse:

$$I = \int F_x dt = Area\ \ of\ the\ triangle\ = \frac{1}{2}(1.5 \times 10^{-3})(18000) = 13.50\, N.s$$

(b) $$\overline{F} = \frac{\Delta p}{\Delta t} = \frac{I}{\Delta t} = \frac{13.5}{1.5 \times 10^{-3}} = 9000\ N$$

**Problem 9-14. Review.**

After a 0.300-kg rubber ball is dropped from a height of 1.75 m, it bounces off a concrete floor and rebounds to a height of 1.50 m. (a) Determine the magnitude and direction of **the impulse** delivered to the ball by the floor. (b) **Estimate the time the ball is in contact** with the floor **and use this** estimate to calculate the **average force** the floor exerts on the ball.




$$mgh = \frac{1}{2}mv^2 \qquad v = \sqrt{2gh} = \sqrt{2 \times 10 \times 1.75} = 5.92\ m/s$$

$$v' = \sqrt{2gh'} = \sqrt{2 \times 10 \times 1.5} = 5.48\ m/s$$

$$h = \frac{1}{2}gt^2 \qquad \Rightarrow t_1 = \sqrt{\frac{2(1.75)}{g}} = 0.592s$$

$$\Rightarrow t' = \sqrt{\frac{2(1.5)}{g}} = 0.548s, \qquad \Delta t = 0.044s$$

$$\boxed{I = \overline{\vec{F}}\Delta t = \Delta \vec{p} = m(\vec{v}_f - \vec{v}_i)}$$

$$\vec{I} = \Delta \vec{p} = \vec{p}_f - \vec{p}_i = m(\vec{v}_f - \vec{v}_i)$$

$$= 0.3[5.48 - (-5.92)]\hat{j} = 0.3 \times 11.40\hat{j} \ kg.m/s$$

$$\vec{I} = \Delta \vec{p} = \vec{F}_{avg} \cdot \Delta t$$

$$\Rightarrow \vec{F}_{avg} = \frac{\Delta \vec{p}}{\Delta t} = \frac{3.42}{0.05}\hat{j} = 68.4\ \hat{j}\ N$$

## Problem 9-18

A tennis player receives a shot with the ball (**0.060 0 kg**) traveling horizontally at **50.0 m/s** and returns the shot with the ball traveling horizontally at **40.0 m/s** in the opposite direction. (a) What is the impulse delivered to the ball by the tennis racquet? (b) What work does the racquet do on the ball?

(a) Impulse: $\vec{I} = \Delta \vec{p} = \vec{p}_f - \vec{p}_i = m(\vec{v}_f - \vec{v}_i)$ (let $v_i$ in the –x direction)

$$= 0.060\ (40 - (-50))\hat{i} = 5.40\ \hat{i} \quad N.s$$

(b) $W = \Delta K = \frac{1}{2} \times 0.060\left(40^2 - 50^2\right) = \ -27.0\ J$

## Problem 9-19

The magnitude of the net force exerted in the *x* direction on a **2.50-kg** particle varies in time as *shown in Figure P9.19*. Find **(a)** the impulse of the force over the **5.00-s** time interval, **(b)** The final

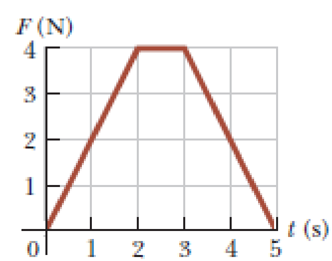


Figure P9.19

velocity the particle attains if it is originally at rest. **(c)** Its final velocity if its original velocity is **2.00** $\hat{i}$ **m/s**, and

**(d)** The average force exerted on the particle for the time interval between **0** and **5.00 s.**

(a) $\vec{I} = \int_{t_i}^{t_f} (\sum \vec{F})_{avg} dt$ = *area under the force-time graph*

**= ½[5+1][4]=12 N.s**

(b) $\vec{I} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i = m(\vec{v}' - 0) = 2.5v'$

$$v' = \frac{I}{2.5} = \frac{12\hat{i}}{2.5} = 4.8\hat{i} \; m/s$$

(c) $\vec{I} = 12\hat{i} = 2.5(\vec{v}' - (-2\hat{i}))$ $\quad \vec{v}' = \frac{\vec{I}}{2.5} = \frac{7\hat{i}}{2.5} = 2.8\hat{i} \; m/s$

(d) $\vec{I} = \vec{\overline{F}} \Delta t \quad \Rightarrow \overline{F} = \frac{12\hat{i}}{5} = 2.4 \, \hat{i} \, N$

**Problem 9-30.**

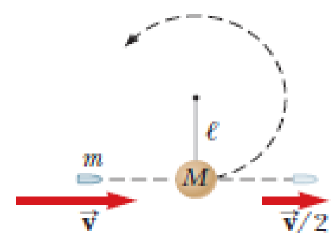


Figure P9.30

As shown in Figure P9.26, a bullet of mass *m* and speed *v* passes completely through a pendulum bob of mass *M*. The bullet emerges with a speed of *v*/2. The pendulum bob is suspended by a stiff rod (*not* a string) of length *l*, and negligible mass. What is the minimum value of *v* such that the pendulum bob will barely swing through a complete vertical circle?

**Momentum** : $mv = m\frac{v}{2} + MV \qquad \Rightarrow v = \frac{2MV}{m}$

**Energy:** $\frac{1}{2}MV^2 = Mg(2l) \quad \Rightarrow V = 2\sqrt{gl} \qquad \Rightarrow v = \frac{4M\sqrt{gl}}{m}$

**Problem 9-33.** Two blocks are free to slide along the frictionless, wooden track shown in Figure P9.33. The block of mass $\boldsymbol{m_1}$ **= 5.00 kg** is released from the position shown, at height $\boldsymbol{h}$**= 5.00 m** above the flat part of the track.

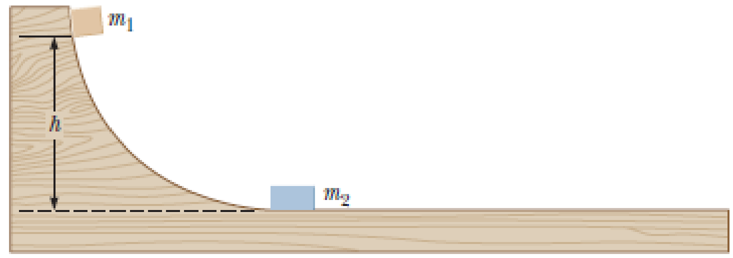



Figure P9.33

Protruding from its front end is the north pole of a strong magnet, which repels the north pole of an identical magnet embedded in the back end of the block of mass $\boldsymbol{m_2}$ **= 10.0 kg**, **initially at rest**. The two blocks never touch. Calculate the maximum height to which $m_1$ rises after the elastic collision.

$$mgh = \frac{1}{2}mv_1^2 \qquad v_1=\sqrt{2gh_1}=\sqrt{2\times10\times5}=10\ m/s$$

$$m_1v_1+m_2v_2=m_1v_1'+m_2v_2'$$

$$\Rightarrow\ 5\times10+m_2(0)=5v_1'+10v_2' \qquad \Rightarrow 50=5v_1'+10v_2'$$

$$(v_1-v_2)=(v_2'-v_1') \qquad \Rightarrow v_2'=10+v_1'$$

$$5v_1'=50-10(10+v_1')=50-100-10v_1'$$

$$15v_1'=-50 \qquad \Rightarrow v_1'=-3.33\ m/s$$

$$mgh=\frac{1}{2}mv_1'^2 \qquad h_2=\frac{3.33^2}{2\times10}=0.554\ m$$

**Problem 9-44**

The mass of the blue puck in Figure P9.44 is 20.0% greater than the mass of the green puck. Before colliding, the pucks approach each other **with momenta of equal magnitudes and opposite directions**, and the green puck **has an initial speed of 10.0 m/s**.

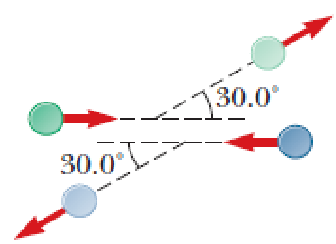



Figure P9.44

Find the speeds the pucks have **after the collision if half the kinetic energy of the system becomes internal energy** during the collision.

$$m_B = m_G + .2m_G = 1.2m_G$$

$$\vec{P}_i = \vec{p}_{1i} + \vec{p}_{2i} = 0, \qquad \Rightarrow m_G v_{Gi} - 1.2m_G v_{Bi} = 0$$

$$\Rightarrow v_{Bi} = \frac{m_G v_{Gi}}{1.2m_G} = \frac{10}{1.2} = 8.33\, m/s$$

But since $\vec{P}_i = \vec{P}_f = \vec{p}_{1f} + \vec{p}_{2f} = 0$

$$mv_{Gf} = 1.2mv_{Bf}$$

$$K_i = \frac{1}{2}m_G(10)^2 + \frac{1}{2}(1.2m_B)(8.33)^2 = \frac{1}{2}m \cdot 183$$

$$K_f = \frac{1}{2}K_i \quad \Rightarrow [\frac{1}{2}m_G v_{Gf}{}^2 + \frac{1}{2}(1.2m_G)(v_{Bf})^2] = \frac{1}{2}(\frac{1}{2}m_G \cdot 183)$$

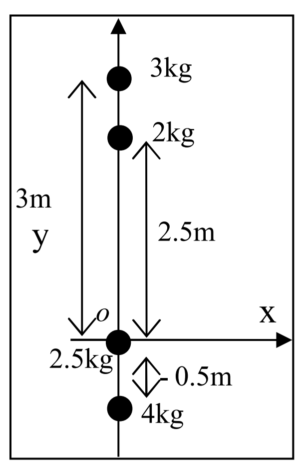
$$v_{Gf}^2 + 1.2v_{Bf}^2 = 91.7, \qquad mv_{Gf} = 1.2mv_{Bf}$$

$$1.2^2 v_{Bf}^2 + 1.2v_{Bf}^2 = 91.7$$

$$2.64v_{Bf}^2 = 91.7 \qquad \Rightarrow v_{Bf} = \sqrt{\frac{91.7}{2.64}} = 5.89m/s^2$$

**Problem 9-45.**

Four objects are situated along the ***y* axis** as follows: a **2.00-kg** object **is at +3.00 m**, a **3.00-kg** object **is at +2.50 m**, a **2.50-kg** object **is at the origin**, and a **4.00-kg** object **is at - 0.500 m**. Where is the center of mass of these objects?



$$y_{cm} = \frac{2 \times 3 + 3 \times 2.5 + 2.5 \times 0 + 4 \times (-0.5)}{2 + 3 + 2.5 + 4} = \frac{11.5}{11.5} = 1.0m$$

$$x_{cm} = \frac{2 \times 0 + 3 \times 0 + 2.5 \times 0 + 4 \times (0)}{2 + 3 + 2.5 + 4} = \frac{0}{11.5} = 0m$$

$$\vec{r}_{cm} = x_{cm}\hat{i} + y_{cm}\hat{j} = 0\hat{i} + 1\hat{j} \ \ m$$

## Problem 9-71

A **1.25-kg** wooden block rests on a table over a large hole as in Figure P9.71. A **5.00-g** bullet with an **initial velocity $v_i$** is fired upward into the bottom of the block and remains in the block after the collision. The block and bullet rise to a maximum height of **22.0 cm**. (a) Describe how you would find the initial velocity of the bullet using ideas you have learned in this chapter. (b) Calculate the initial velocity of the bullet from the information provided.

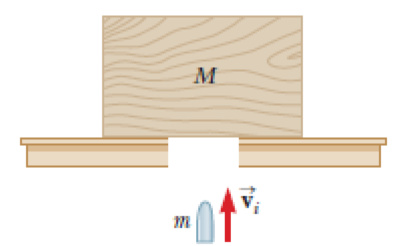


Figure P9.71
Problems 71 and 72.

(a) P conservation: $m_1 v_{1i} = (m_1 + m_2)V_f \Rightarrow v_{1i} = \dfrac{(m_1 + m_2)V_f}{m_1}$

E conservation: $\dfrac{1}{2}(m_1 + m_2)\, V_f^2 = (m_1 + m_2)gh \quad \Rightarrow V_f = \sqrt{2gh}$

(b) $v_{1i} = \dfrac{(m_1 + m_2)}{m_1}\sqrt{2gh} = \dfrac{1.255}{0.005} \times \sqrt{(20 \times 0.22)} = 526.5 \ m/s$

## Problem 9-81.

A bullet of mass *m* is fired into a block of mass *M* initially at rest at the edge of a frictionless table of height *h* (Fig. P9.81).The bullet remains in the block, and after impact the block lands a distance *d* from the bottom of the table.

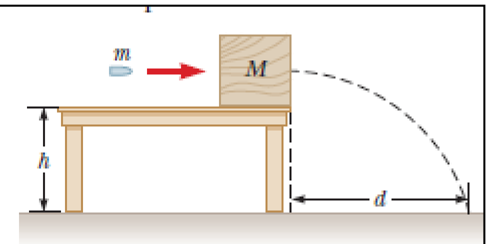
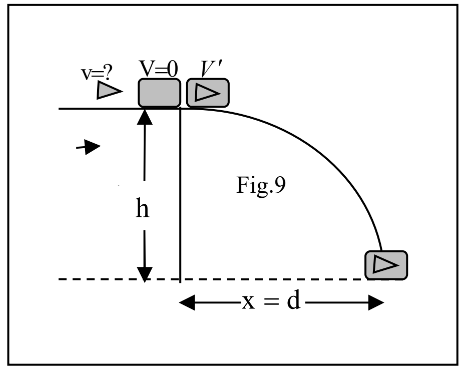


Figure P9.81 Problems 81 and 82.

Determine the initial speed of the bullet.




$$y=\frac{1}{2}gt^2 \Rightarrow t=\sqrt{\frac{2y}{g}}$$

$$V'=\frac{X}{t}=\frac{d}{t}$$

$$mv+0=(M+m)V' \quad \Rightarrow v=\frac{M+m}{m}\times\frac{d}{t}$$

**Problem 9-80.** A **5.00-g** bullet moving with an initial speed of $v_i = 400\, m/s$ is fired into and passes through a **1.00-kg** block as shown in Figure P 9.73. The block, initially at rest on a frictionless, horizontal surface, is connected to a spring with force constant **900 N/m**. The block moves ***d*** **= 5.00** cm to the right after impact before being brought to rest by the spring. Find **(a)** the speed at which the bullet emerges from the block and **(b)** the amount of initial kinetic energy of the bullet that is converted into internal energy in the bullet–block system during the collision.

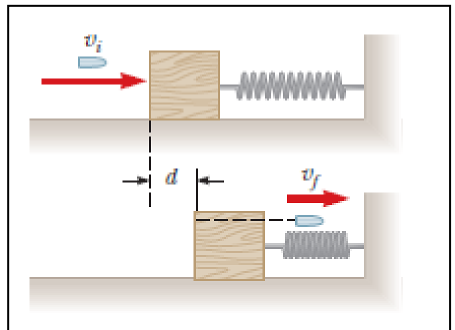
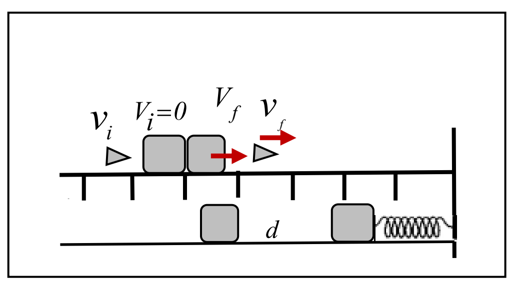


Figure P9.89




**(a)** $V_i = 0$ **الكتلة الكبيرة ساكنة** $v_i = 400\, m/s$ **الرصاصة تضرب بسرعة**

$$mv_i + MV_i = mv_f + MV_f$$

$$(0.005)\times 400 + 0 = (0.005)v_f + 1\times V_f$$

**مطلوب حساب** $v_f$ **ولذلك يجب حساب** $V_f$ **وهي سرعة *M* بعد خروج الرصاصة مباشرة**

M تضغط النابض أقصى مسافة d عندها سكنت فتتحول طاقتها الحركية الى وضع مختزنة في النابض ، بالنسبة الى M والنابض فقط يكون

$$(K_{B1}+U_{s1})=(K_{B2}+U_{s2})$$

$$\frac{1}{2}MV_{Bf}^2=\frac{1}{2}kx^2 \qquad \Rightarrow V_{Bf}=\sqrt{\frac{900\times(0.05)^2}{1}}=30\times0.05=1.5m/s$$

$$0.005\times400+0=0.005v_f+1\times V_f$$

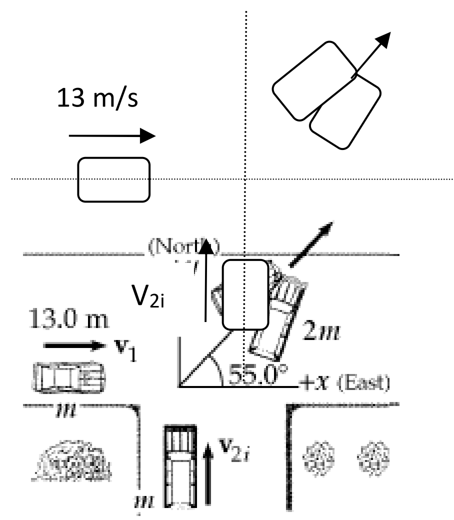
$$v_f=\frac{2-1.5}{0.005}=100m/s$$

**(b)** $K_{bullet})_f-K_{bullet})_{initial}=(K_{B2}+U_{s2})$

$$\frac{1}{2}(0.005)(100^2-400^2)=0.0025\times(1-16)\times10^4=375J$$

## Problem 9-36

Two automobiles of equal mass approach an intersection. One vehicle is traveling with speed **13.0 m/s** toward **the east**, and the other is traveling **north with speed $v_{2i}$.** Neither driver sees the other. The vehicles **collide in the intersection and stick together**, leaving parallel skid marks at an angle of **55.0° north of east**. The speed limit for both roads is **35 mi/h**, and the driver of the northward-moving vehicle **claims he was within the speed limit** when the collision occurred. **Is he telling the truth**? Explain your reasoning.




$$P_{xi}=P_{xf}: \quad M\times(13)=2M\cdot v_f\cos55$$

$$P_{yi} = P_{yf} : \quad M \times v_{2i} = 2M \cdot v_f \ \sin 55$$

$$v_{2i} = 13 \tan(55) = 18.6 \ m/h$$

$$V_f = 18.6 \times \frac{3600}{1609} = 41.6 \ mi/h$$

*No, he lies* $v_{2i} = 41.6 \ mi/h > 35 \ mi/h$

## Problem 9-51

A **2.00-kg** particle has a velocity ($2.0\,\hat{i} - 3.0\,\hat{j}$) m/s, and a **3.00-kg** particle has a velocity ($1.0\,\hat{i} + 6.0\,\hat{j}$) m/s. Find

**(a)** the velocity of the center of mass and

**(b)** the total momentum of the system.

(a) $$\vec{v}_{cm} = \frac{1}{M}\sum_i m_i\vec{v}_i = \frac{1}{5}\left[2(2.0\,\hat{i} - 3.0\,\hat{j}) + 3(1.0\,\hat{i} + 6.0\,\hat{j})\right]$$

$$= 1.4\,\hat{i} + 2.4\,\hat{j} \ m/s$$

(b) $$\vec{P} = M\vec{V}_{cm} = 5\left(1.4\,\hat{i} + 2.4\,\hat{j}\right) = 7.0\,\hat{i} + 12.0\,\hat{j} \quad kg.m/s$$

## Problem 9-55

A ball of mass **0.200 kg** with a velocity of **1.50** $\hat{i}$ **m/s** meets a ball of mass **0.300 kg** with a velocity of **- 0.400** $\hat{i}$ **m/s** in a head-on, elastic collision.

(a) Find their velocities after the collision.

(b) Find the velocity of their center of mass before and after the collision.

**(a) 1D-collision in x-direction:**

$$0.2 \times 1.5 + 0.3(-0.4) = 0.2v_{1f} + 0.3v_f \qquad 0.18 = 0.2v_{1f} + 0.3v_f$$

**Relative velocity eq.:** $v_{1i} - v_{2i} = -(v_{1f} - v_{2f})$

$$1.5 - (-0.4) = 1.9 = v_{2f} - v_{1f}$$

$$0.18 = 0.2v_{1f} + 0.3(1.9 + v_{1f}) \quad \Rightarrow 0.5v_{1f} = -0.39$$

$$\Rightarrow v_{1f} = -0.78 \ m/s, \qquad v_{2f} = 1.12 \ m/s$$

(b) before:

$$\vec{v}_{cm} = \frac{1}{0.5}\left[0.2 \times (1.5)\hat{i} + 0.3 \times (-0.4\,\hat{i})\right] = 0.36\,\hat{i} \ \ m/s \ = \vec{v}_{cm} \text{ (after)}$$