4.7 Simplification of a Force and Couple System

Sometimes it is convenient to reduce a system of forces and couple moments acting on a body to a simpler form by replacing it with an *equivalent system*, consisting of a single resultant force acting at a specific point and a resultant couple moment. A system is equivalent if the *external effects* it produces on a body are the same as those caused by the original force and couple moment system. In this context, the external effects of a system refer to the *translating and rotating motion* of the body if the body is free to move, or it refers to the *reactive forces* at the supports if the body is held fixed.

For example, consider holding the stick in Fig. 4–34*a*, which is subjected to the force **F** at point *A*. If we attach a pair of equal but opposite forces **F** and **−F** at point *B*, which is *on the line of action* of **F**, Fig. 4–34*b*, we observe that **−F** at *B* and **F** at *A* will cancel each other, leaving only **F** at *B*, Fig. 4–34*c*. Force **F** has now been moved from *A* to *B* without modifying its *external effects* on the stick; i.e., the reaction at the grip remains the same. This demonstrates the *principle of transmissibility*, which states that a force acting on a body (stick) is a *sliding vector* since it can be applied at any point along its line of action.

We can also use the above procedure to move a force to a point that is *not* on the line of action of the force. If **F** is applied perpendicular to the stick, as in Fig. 4–35*a*, then we can attach a pair of equal but opposite forces **F** and **−F** to *B*, Fig. 4–35*b*. Force **F** is now applied at *B*, and the other two forces, **F** at *A* and **−F** at *B*, form a couple that produces the couple moment $M = Fd$, Fig. 4–35*c*. Therefore, the force **F** can be moved from *A* to *B* provided a couple moment **M** is added to maintain an equivalent system. This couple moment is determined by taking the moment of **F** about *B*. Since **M** is actually a *free vector*, it can act at any point on the stick. In both cases the systems are equivalent, which causes a downward force **F** and clockwise couple moment $M = Fd$ to be felt at the grip.

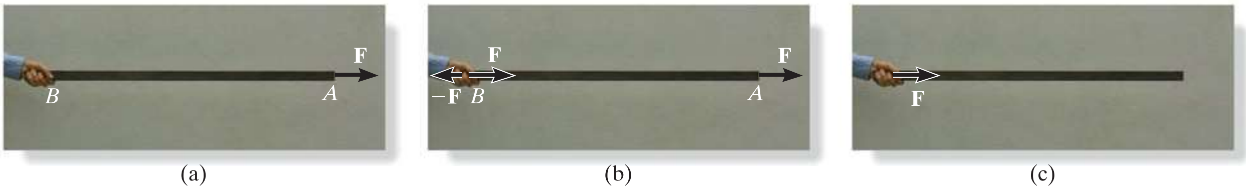


Fig. 4–34

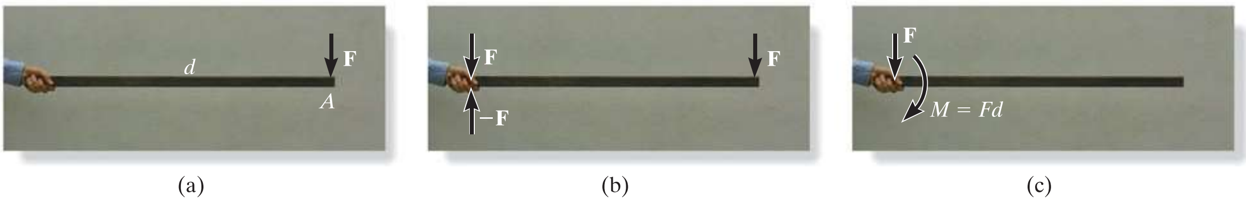


Fig. 4–35

System of Forces and Couple Moments. Using the above method, a system of several forces and couple moments acting on a body can be reduced to an equivalent single resultant force acting at a point O and a resultant couple moment. For example, in Fig. 4–36*a*, O is not on the line of action of $\mathbf{F}_1$, and so this force can be moved to point O provided a couple moment $(\mathbf{M}_O)_1 = \mathbf{r}_1 \times \mathbf{F}$ is added to the body. Similarly, the couple moment $(\mathbf{M}_O)_2 = \mathbf{r}_2 \times \mathbf{F}_2$ should be added to the body when we move $\mathbf{F}_2$ to point O. Finally, since the couple moment $\mathbf{M}$ is a free vector, it can just be moved to point O. By doing this, we obtain the equivalent system shown in Fig. 4–36*b*, which produces the same external effects (support reactions) on the body as that of the force and couple system shown in Fig. 4–36*a*. If we sum the forces and couple moments, we obtain the resultant force $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$ and the resultant couple moment $(\mathbf{M}_R)_O = \mathbf{M} + (\mathbf{M}_O)_1 + (\mathbf{M}_O)_2$, Fig. 4–36*c*.

Notice that $\mathbf{F}_R$ is independent of the location of point O since it is simply a summation of the forces. However, $(\mathbf{M}_R)_O$ depends upon this location since the moments $\mathbf{M}_1$ and $\mathbf{M}_2$ are determined using the position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$, which extend from O to each force. Also note that $(\mathbf{M}_R)_O$ is a free vector and can act at *any point* on the body, although point O is generally chosen as its point of application.

We can generalize the above method of reducing a force and couple system to an equivalent resultant force $\mathbf{F}_R$ acting at point O and a resultant couple moment $(\mathbf{M}_R)_O$ by using the following two equations.

$$\mathbf{F}_R = \Sigma \mathbf{F}$$
$$(\mathbf{M}_R)_O = \Sigma \mathbf{M}_O + \Sigma \mathbf{M} \qquad (4\text{–}17)$$

The first equation states that the resultant force of the system is equivalent to the sum of all the forces; and the second equation states that the resultant couple moment of the system is equivalent to the sum of all the couple moments $\Sigma \mathbf{M}$ plus the moments of all the forces $\Sigma \mathbf{M}_O$ about point O. If the force system lies in the x–y plane and any couple moments are perpendicular to this plane, then the above equations reduce to the following three scalar equations.

$$(F_R)_x = \Sigma F_x$$
$$(F_R)_y = \Sigma F_y$$
$$(M_R)_O = \Sigma M_O + \Sigma M \qquad (4\text{–}18)$$

Here the resultant force is determined from the vector sum of its two components $(F_R)_x$ and $(F_R)_y$.

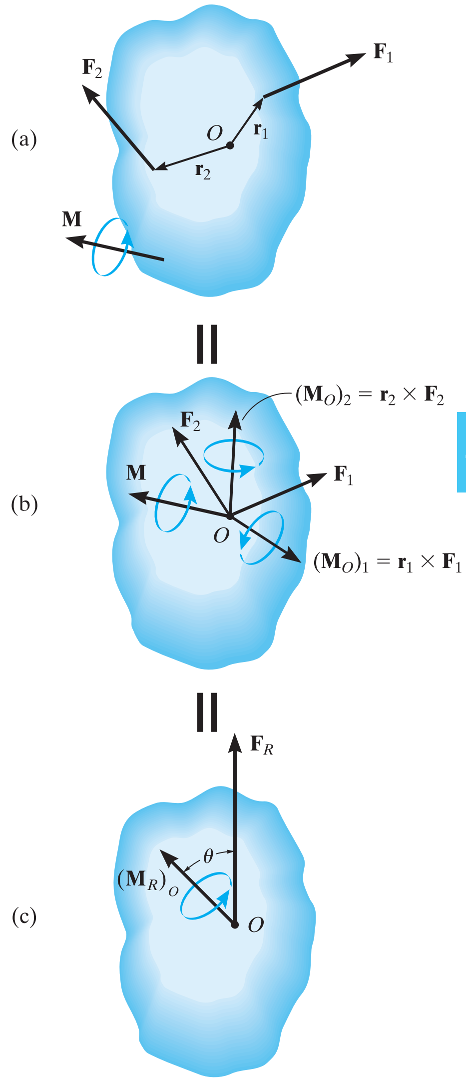
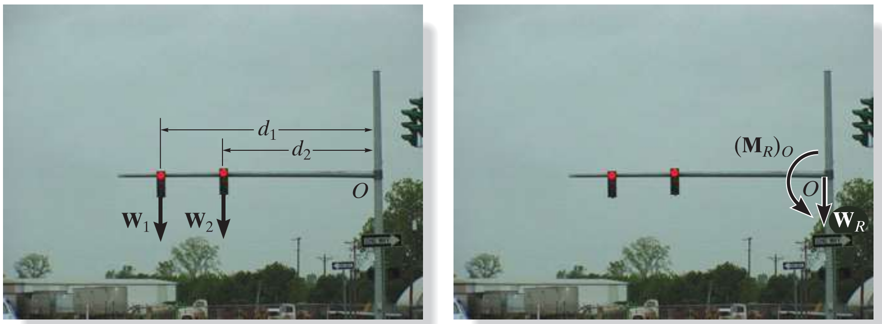


Fig. 4–36

4



The weights of these traffic lights can be replaced by their equivalent resultant force $W_R = W_1 + W_2$ and a couple moment $(M_R)_O = W_1 d_1 + W_2 d_2$ at the support, O. In both cases the support must provide the same resistance to translation and rotation in order to keep the member in the horizontal position.

Procedure for Analysis

The following points should be kept in mind when simplifying a force and couple moment system to an equivalent resultant force and couple system.

- Establish the coordinate axes with the origin located at point O and the axes having a selected orientation.

Force Summation.

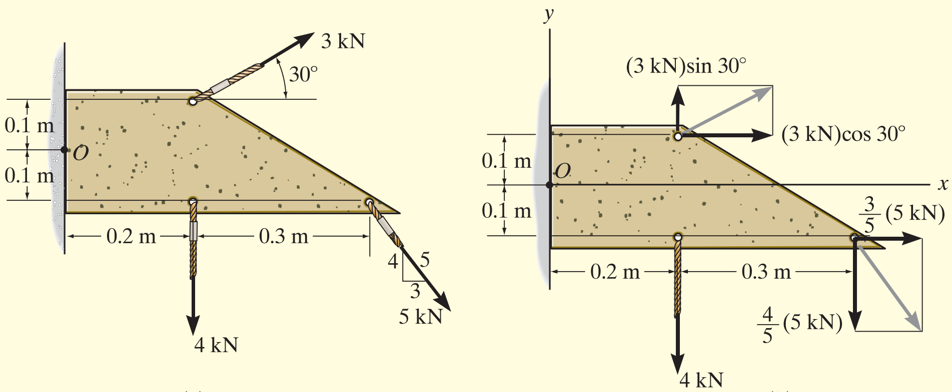
- If the force system is *coplanar*, resolve each force into its x and y components. If a component is directed along the positive x or y axis, it represents a positive scalar; whereas if it is directed along the negative x or y axis, it is a negative scalar.
- In three dimensions, represent each force as a Cartesian vector before summing the forces.

Moment Summation.

- When determining the moments of a *coplanar* force system about point O, it is generally advantageous to use the principle of moments, i.e., determine the moments of the components of each force, rather than the moment of the force itself.
- In three dimensions use the vector cross product to determine the moment of each force about point O. Here the position vectors extend from O to any point on the line of action of each force.

EXAMPLE 4.14

Replace the force and couple system shown in Fig. 4–37*a* by an equivalent resultant force and couple moment acting at point *O*.



(a) (b)

SOLUTION

Force Summation. The 3 kN and 5 kN forces are resolved into their x and y components as shown in Fig. 4–37*b*. We have

$$\overset{+}{\rightarrow} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = (3 \text{ kN})\cos 30^\circ + \left(\tfrac{3}{5}\right)(5 \text{ kN}) = 5.598 \text{ kN} \rightarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = (3 \text{ kN})\sin 30^\circ - \left(\tfrac{4}{5}\right)(5 \text{ kN}) - 4 \text{ kN} = -6.50 \text{ kN} = 6.50 \text{ kN}\downarrow$$

Using the Pythagorean theorem, Fig. 4–37*c*, the magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^{\,2} + (F_R)_y^{\,2}} = \sqrt{(5.598 \text{ kN})^2 + (6.50 \text{ kN})^2} = 8.58 \text{ kN} \qquad \textit{Ans.}$$

Its direction θ is

$$\theta = \tan^{-1}\left(\frac{(F_R)_y}{(F_R)_x}\right) = \tan^{-1}\left(\frac{6.50 \text{ kN}}{5.598 \text{ kN}}\right) = 49.3^\circ \qquad \textit{Ans.}$$

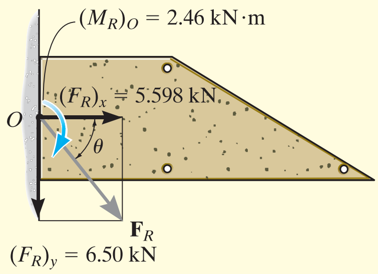
Moment Summation. The moments of 3 kN and 5 kN about point *O* will be determined using their x and y components. Referring to Fig. 4–37*b*, we have

$$\circlearrowleft + (M_R)_O = \Sigma M_O;$$

$$(M_R)_O = (3 \text{ kN})\sin 30^\circ(0.2 \text{ m}) - (3 \text{ kN})\cos 30^\circ(0.1 \text{ m}) + \left(\tfrac{3}{5}\right)(5 \text{ kN})\,(0.1 \text{ m}) - \left(\tfrac{4}{5}\right)(5 \text{ kN})\,(0.5 \text{ m}) - (4 \text{ kN})(0.2 \text{ m})$$

$$= -2.46 \text{ kN}\cdot\text{m} = 2.46 \text{ kN}\cdot\text{m} \circlearrowright \qquad \textit{Ans.}$$

This clockwise moment is shown in Fig. 4–37*c*.



(c)

Fig. 4–37

NOTE: Realize that the resultant force and couple moment in Fig. 4–37*c* will produce the same external effects or reactions at the supports as those produced by the force system, Fig. 4–37*a*.

4

EXAMPLE 4.15

4

Replace the force and couple system acting on the member in Fig. 4–38*a* by an equivalent resultant force and couple moment acting at point *O*.

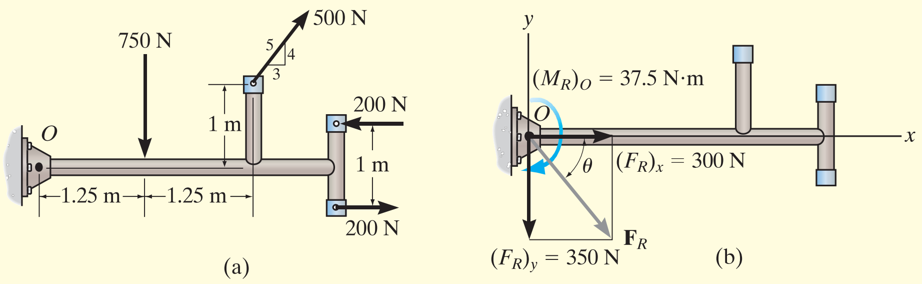


Fig. 4–38

SOLUTION

Force Summation. Since the couple forces of 200 N are equal but opposite, they produce a zero resultant force, and so it is not necessary to consider them in the force summation. The 500-N force is resolved into its *x* and *y* components, thus,

$$\overset{+}{\rightarrow} (F_R)_x = \Sigma F_x; \quad (F_R)_x = \left(\tfrac{3}{5}\right)(500\text{ N}) = 300\text{ N} \rightarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \quad (F_R)_y = (500\text{ N})\left(\tfrac{4}{5}\right) - 750\text{ N} = -350\text{ N} = 350\text{ N}\downarrow$$

From Fig. 4–15*b*, the magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2}$$

$$= \sqrt{(300\text{ N})^2 + (350\text{ N})^2} = 461\text{ N} \qquad \textit{Ans.}$$

And the angle θ is

$$\theta = \tan^{-1}\left(\frac{(F_R)_y}{(F_R)_x}\right) = \tan^{-1}\left(\frac{350\text{ N}}{300\text{ N}}\right) = 49.4^\circ \qquad \textit{Ans.}$$

Moment Summation. Since the couple moment is a free vector, it can act at any point on the member. Referring to Fig. 4–38*a*, we have

$$\circlearrowleft + (M_R)_O = \Sigma M_O + \Sigma M$$

$$(M_R)_O = (500\text{ N})\left(\tfrac{4}{5}\right)(2.5\text{ m}) - (500\text{ N})\left(\tfrac{3}{5}\right)(1\text{ m})$$

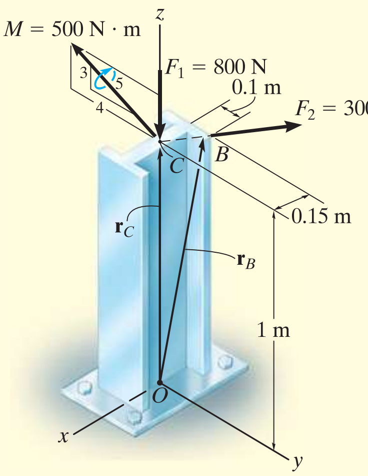
$$- (750\text{ N})(1.25\text{ m}) + 200\text{ N}\cdot\text{m}$$

$$= -37.5\text{ N}\cdot\text{m} = 37.5\text{ N}\cdot\text{m} \circlearrowright \qquad \textit{Ans.}$$

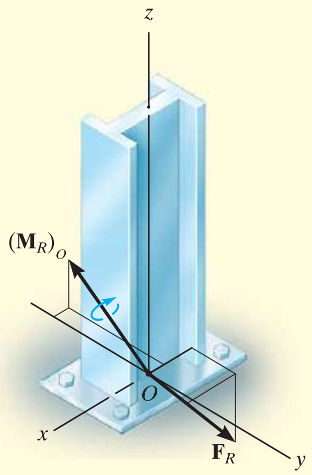
This clockwise moment is shown in Fig. 4–38*b*.

EXAMPLE 4.16

The structural member is subjected to a couple moment **M** and forces $\mathbf{F}_1$ and $\mathbf{F}_2$ in Fig. 4–39*a*. Replace this system by an equivalent resultant force and couple moment acting at its base, point *O*.



(a)



(b)

Fig. 4–39

SOLUTION *(VECTOR ANALYSIS)*

The three-dimensional aspects of the problem can be simplified by using a Cartesian vector analysis. Expressing the forces and couple moment as Cartesian vectors, we have

$$\mathbf{F}_1 = \{-800\mathbf{k}\}\ \text{N}$$

$$\mathbf{F}_2 = (300\ \text{N})\mathbf{u}_{CB}$$

$$= (300\ \text{N})\left(\frac{\mathbf{r}_{CB}}{r_{CB}}\right)$$

$$= 300\ \text{N}\left[\frac{\{-0.15\mathbf{i} + 0.1\mathbf{j}\}\ \text{m}}{\sqrt{(-0.15\ \text{m})^2 + (0.1\ \text{m})^2}}\right] = \{-249.6\mathbf{i} + 166.4\mathbf{j}\}\ \text{N}$$

$$\mathbf{M} = -500\left(\tfrac{4}{5}\right)\mathbf{j} + 500\left(\tfrac{3}{5}\right)\mathbf{k} = \{-400\mathbf{j} + 300\mathbf{k}\}\ \text{N}\cdot\text{m}$$

Force Summation.

$$\mathbf{F}_R = \Sigma\mathbf{F}; \qquad \mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 = -800\mathbf{k} - 249.6\mathbf{i} + 166.4\mathbf{j}$$

$$= \{-250\mathbf{i} + 166\mathbf{j} - 800\mathbf{k}\}\ \text{N} \qquad \textit{Ans.}$$

Moment Summation.

$$(\mathbf{M}_R)_o = \Sigma\mathbf{M} + \Sigma\mathbf{M}_O$$

$$(\mathbf{M}_R)_o = \mathbf{M} + \mathbf{r}_C \times \mathbf{F}_1 + \mathbf{r}_B \times \mathbf{F}_2$$

$$(\mathbf{M}_R)_o = (-400\mathbf{j} + 300\mathbf{k}) + (1\mathbf{k}) \times (-800\mathbf{k}) + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.15 & 0.1 & 1 \\ -249.6 & 166.4 & 0 \end{vmatrix}$$

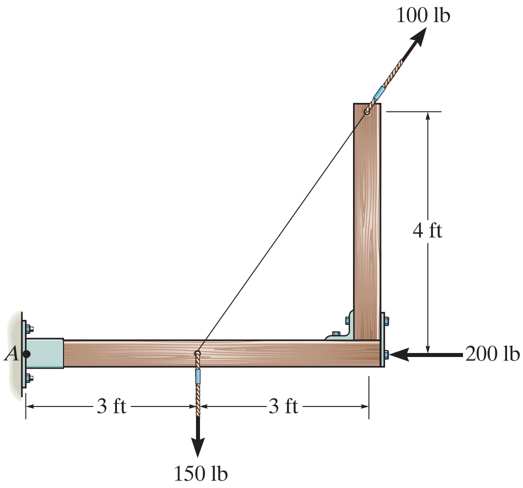
$$= (-400\mathbf{j} + 300\mathbf{k}) + (\mathbf{0}) + (-166.4\mathbf{i} - 249.6\mathbf{j})$$

$$= \{-166\mathbf{i} - 650\mathbf{j} + 300\mathbf{k}\}\ \text{N}\cdot\text{m} \qquad \textit{Ans.}$$

The results are shown in Fig. 4–39*b*.

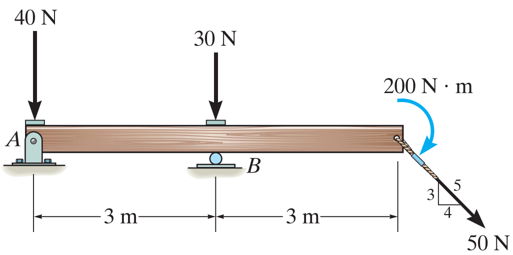
FUNDAMENTAL PROBLEMS

F4–25. Replace the loading system by an equivalent resultant force and couple moment acting at point A.



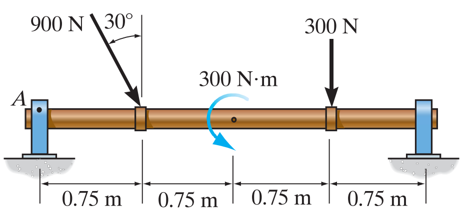
F4–25

F4–26. Replace the loading system by an equivalent resultant force and couple moment acting at point A.



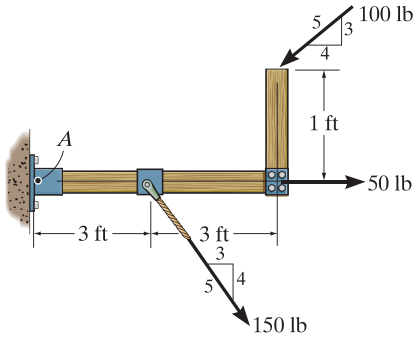
F4–26

F4–27. Replace the loading system by an equivalent resultant force and couple moment acting at point A.



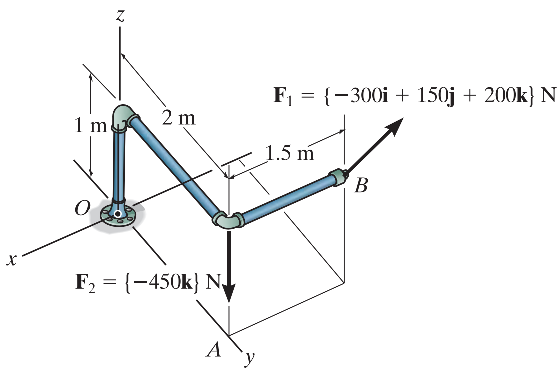
F4–27

F4–28. Replace the loading system by an equivalent resultant force and couple moment acting at point A.



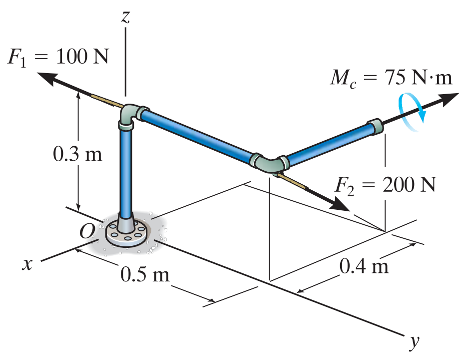
F4–28

F4–29. Replace the loading system by an equivalent resultant force and couple moment acting at point O.



F4–29

F4–30. Replace the loading system by an equivalent resultant force and couple moment acting at point O.

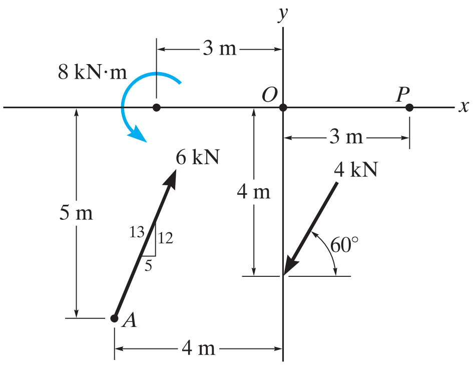


F4–30

PROBLEMS

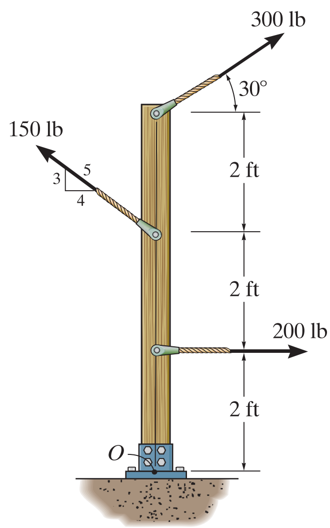
4–97. Replace the force and couple system by an equivalent force and couple moment at point O.

4–98. Replace the force and couple system by an equivalent force and couple moment at point P.



Probs. 4–97/98

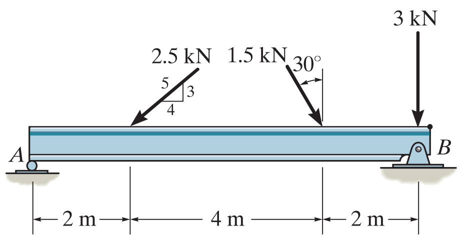
4–101. Replace the force system acting on the post by a resultant force and couple moment at point O.



Prob. 4–101

4–99. Replace the force system acting on the beam by an equivalent force and couple moment at point A.

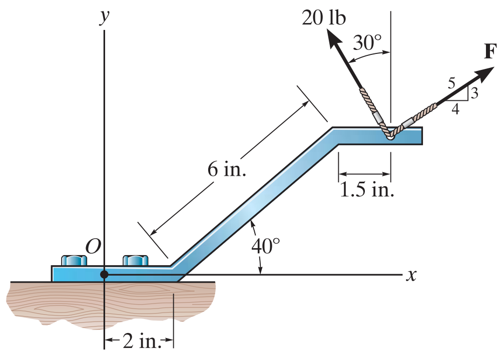
***4–100.** Replace the force system acting on the beam by an equivalent force and couple moment at point B.



Probs. 4–99/100

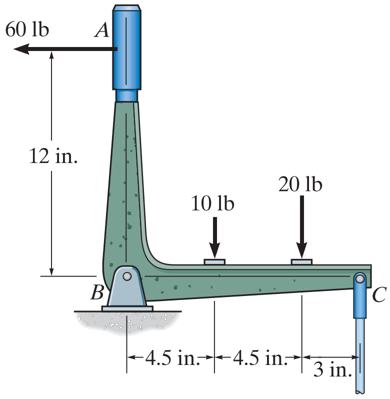
4–102. Replace the two forces by an equivalent resultant force and couple moment at point O. Set $F = 20$ lb.

4–103. Replace the two forces by an equivalent resultant force and couple moment at point O. Set $F = 15$ lb.



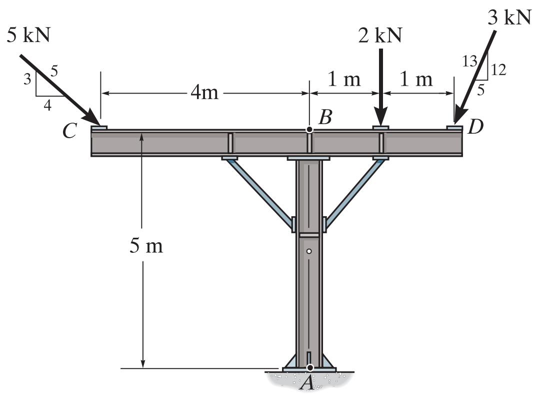
Probs. 4–102/103

*4–104. Replace the force system acting on the crank by a resultant force, and specify where its line of action intersects BA measured from the pin at B.



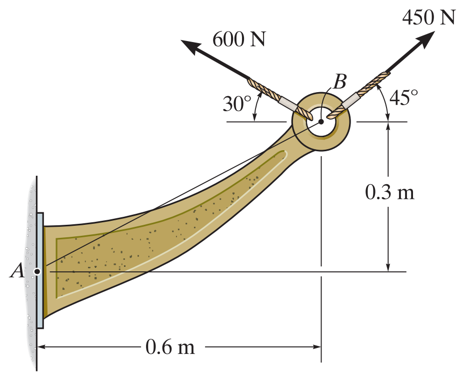
Prob. 4–104

4–105. Replace the force system acting on the frame by a resultant force and couple moment at point A.



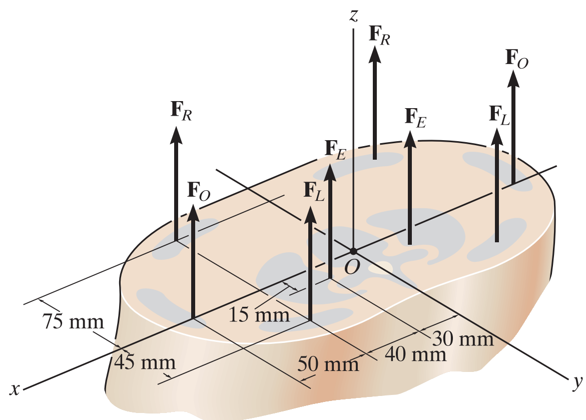
Prob. 4–105

4–106. Replace the force system acting on the bracket by a resultant force and couple moment at point A.



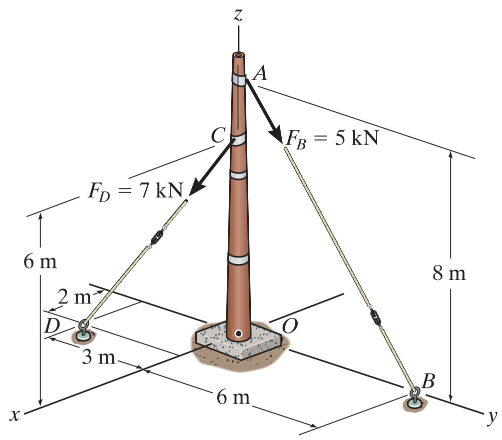
Prob. 4–106

4–107. A biomechanical model of the lumbar region of the human trunk is shown. The forces acting in the four muscle groups consist of $F_R = 35$ N for the rectus, $F_O = 45$ N for the oblique, $F_L = 23$ N for the lumbar latissimus dorsi, and $F_E = 32$ N for the erector spinae. These loadings are symmetric with respect to the y–z plane. Replace this system of parallel forces by an equivalent force and couple moment acting at the spine, point O. Express the results in Cartesian vector form.



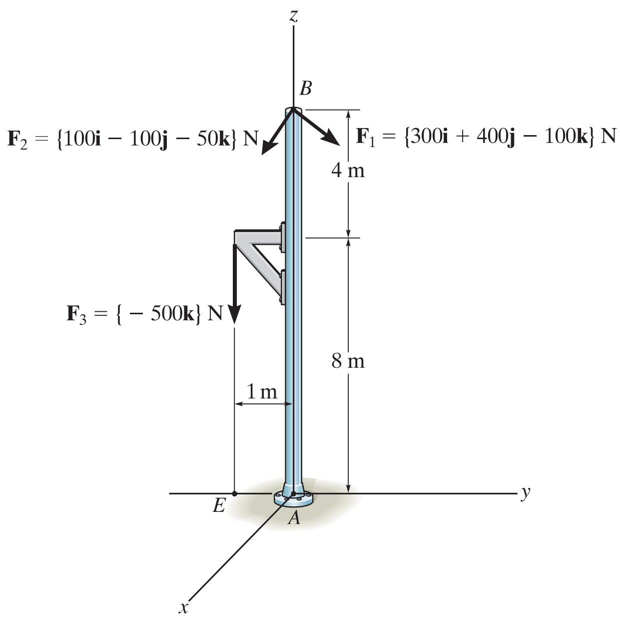
Prob. 4–107

*4–108. Replace the two forces acting on the post by a resultant force and couple moment at point O. Express the results in Cartesian vector form.



Prob. 4–108

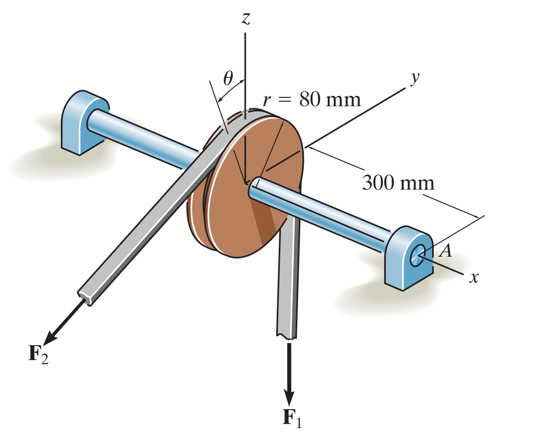
4–109. Replace the force system by an equivalent force and couple moment at point A.



Prob. 4–109

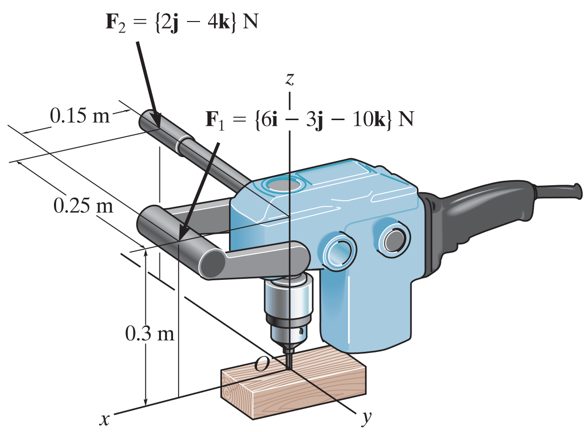
4–110. The belt passing over the pulley is subjected to forces $\mathbf{F}_1$ and $\mathbf{F}_2$, each having a magnitude of 40 N. $\mathbf{F}_1$ acts in the $-\mathbf{k}$ direction. Replace these forces by an equivalent force and couple moment at point A. Express the result in Cartesian vector form. Set $\theta = 0°$ so that $\mathbf{F}_2$ acts in the $-\mathbf{j}$ direction.

4–111. The belt passing over the pulley is subjected to two forces $\mathbf{F}_1$ and $\mathbf{F}_2$, each having a magnitude of 40 N. $\mathbf{F}_1$ acts in the $-\mathbf{k}$ direction. Replace these forces by an equivalent force and couple moment at point A. Express the result in Cartesian vector form. Take $\theta = 45°$.



Probs. 4–110/111

*4–112. Handle forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are applied to the electric drill. Replace this force system by an equivalent resultant force and couple moment acting at point O. Express the results in Cartesian vector from.



Prob. 4–112

4.8 Further Simplification of a Force and Couple System

In the preceding section, we developed a way to reduce a force and couple moment system acting on a rigid body into an equivalent resultant force $\mathbf{F}_R$ acting at a specific point O and a resultant couple moment $(\mathbf{M}_R)_O$. The force system can be further reduced to an equivalent single resultant force provided the lines of action of $\mathbf{F}_R$ and $(\mathbf{M}_R)_O$ are *perpendicular* to each other. Because of this condition, only concurrent, coplanar, and parallel force systems can be further simplified.

Concurrent Force System. Since a *concurrent force system* is one in which the lines of action of all the forces intersect at a common point O, Fig. 4–40*a*, then the force system produces no moment about this point. As a result, the equivalent system can be represented by a single resultant force $\mathbf{F}_R = \Sigma\mathbf{F}$ acting at O, Fig. 4–40*b*.

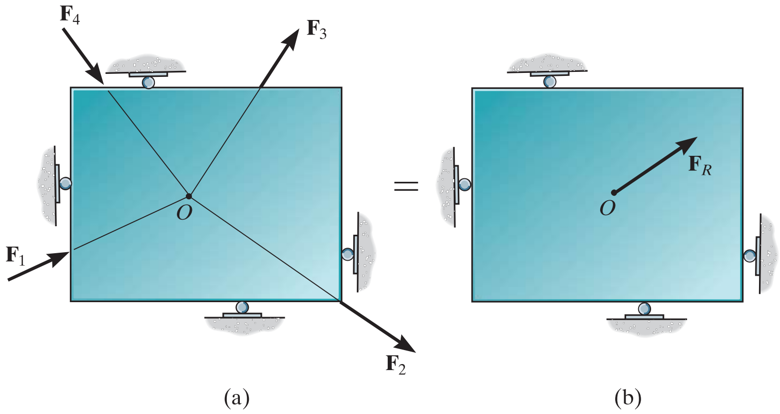


Fig. 4–40

Coplanar Force System. In the case of a *coplanar force system*, the lines of action of all the forces lie in the same plane, Fig. 4–41*a*, and so the resultant force $\mathbf{F}_R = \Sigma\mathbf{F}$ of this system also lies in this plane. Furthermore, the moment of each of the forces about any point O is directed perpendicular to this plane. Thus, the resultant moment $(\mathbf{M}_R)_O$ and resultant force $\mathbf{F}_R$ will be *mutually perpendicular*, Fig. 4–41*b*. The resultant moment can be replaced by moving the resultant force $\mathbf{F}_R$ a perpendicular or moment arm distance d away from point O such that $\mathbf{F}_R$ produces the *same moment* $(\mathbf{M}_R)_O$ about point O, Fig. 4–41*c*. This distance d can be determined from the scalar equation $(M_R)_O = F_R d = \Sigma M_O$ or $d = (M_R)_O/F_R$.

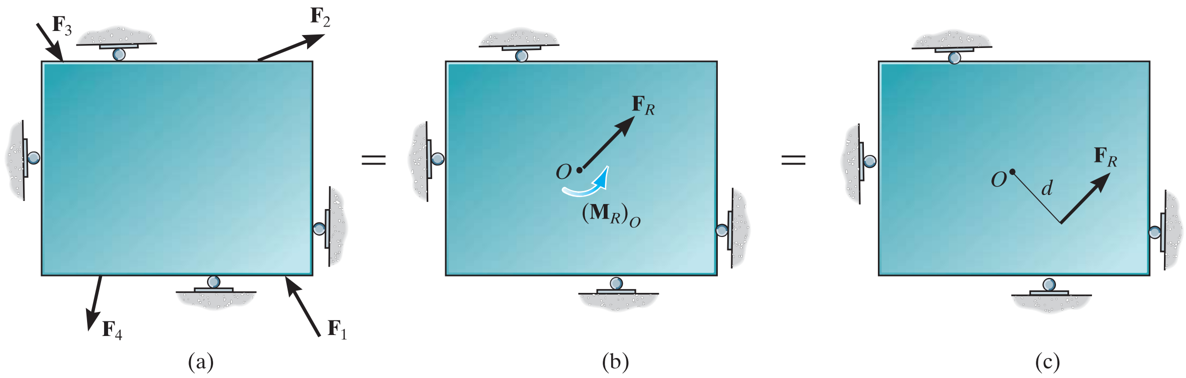


Fig. 4–41

Parallel Force System. The *parallel force system* shown in Fig. 4–42*a* consists of forces that are all parallel to the z axis. Thus, the resultant force $\mathbf{F}_R = \Sigma \mathbf{F}$ at point O must also be parallel to this axis, Fig. 4–42*b*. The moment produced by each force lies in the plane of the plate, and so the resultant couple moment, $(\mathbf{M}_R)_O$, will also lie in this plane, along the moment axis a since $\mathbf{F}_R$ and $(\mathbf{M}_R)_O$ are mutually perpendicular. As a result, the force system can be further reduced to an equivalent single resultant force $\mathbf{F}_R$, acting through point P located on the perpendicular b axis, Fig. 4–42*c*. The distance d along this axis from point O requires $(M_R)_O = F_R d = \Sigma M_O$ or $d = \Sigma M_O / F_R$.

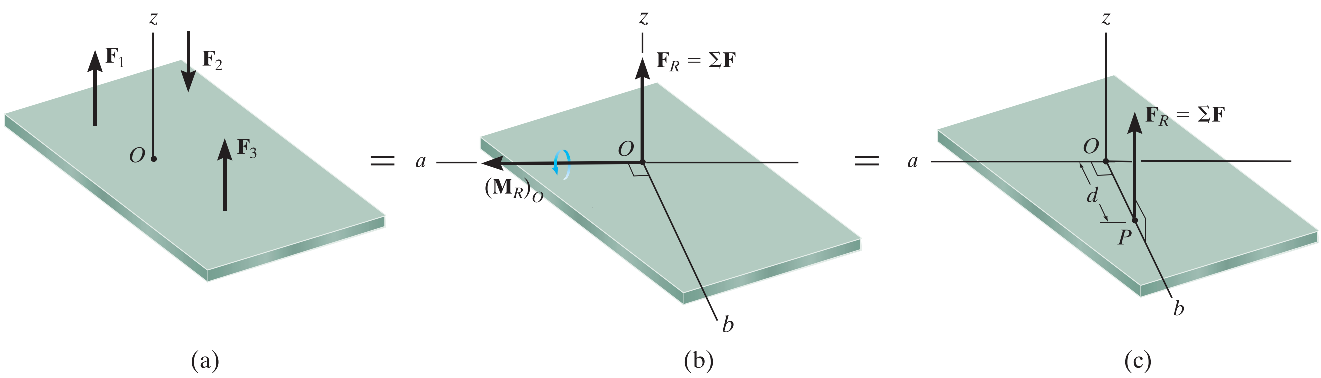


Fig. 4–42

The four cable forces are all concurrent at point O on this bridge tower. Consequently they produce no resultant moment there, only a resultant force $\mathbf{F}_R$. Note that the designers have positioned the cables so that $\mathbf{F}_R$ is directed *along* the bridge tower directly to the support, so that it does not cause any bending of the tower.

4

Procedure for Analysis

The technique used to reduce a coplanar or parallel force system to a single resultant force follows a similar procedure outlined in the previous section.

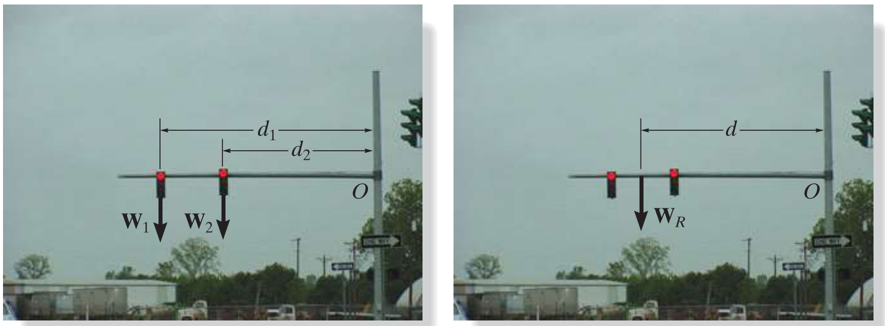
- Establish the x, y, z, axes and locate the resultant force $\mathbf{F}_R$ an arbitrary distance away from the origin of the coordinates.

Force Summation.

- The resultant force is equal to the sum of all the forces in the system.
- For a coplanar force system, resolve each force into its x and y components. Positive components are directed along the positive x and y axes, and negative components are directed along the negative x and y axes.

Moment Summation.

- The moment of the resultant force about point O is equal to the sum of all the couple moments in the system plus the moments of all the forces in the system about O.
- This moment condition is used to find the location of the resultant force from point O.



Here the weights of the traffic lights are replaced by their resultant force $W_R = W_1 + W_2$ which acts at a distance $d = (W_1 d_1 + W_2 d_2)/\,W_R$ from O. Both systems are equivalent.

Reduction to a Wrench. In general, a three-dimensional force and couple moment system will have an equivalent resultant force $\mathbf{F}_R$ acting at point O and a resultant couple moment $(\mathbf{M}_R)_O$ that are *not perpendicular* to one another, as shown in Fig. 4–43*a*. Although a force system such as this cannot be further reduced to an equivalent single resultant force, the resultant couple moment $(\mathbf{M}_R)_O$ can be resolved into components parallel and perpendicular to the line of action of $\mathbf{F}_R$, Fig. 4–43*a*. The perpendicular component $\mathbf{M}_\perp$ can be replaced if we move $\mathbf{F}_R$ to point P, a distance d from point O along the b axis, Fig. 4–43*b*. As seen, this axis is perpendicular to both the a axis and the line of action of $\mathbf{F}_R$. The location of P can be determined from $d = M_\perp / F_R$. Finally, because $\mathbf{M}_\|$ is a free vector, it can be moved to point P, Fig. 4–43*c*. This combination of a resultant force $\mathbf{F}_R$ and collinear couple moment $\mathbf{M}_\|$ will tend to translate and rotate the body about its axis and is referred to as a *wrench* or *screw*. A wrench is the simplest system that can represent any general force and couple moment system acting on a body.

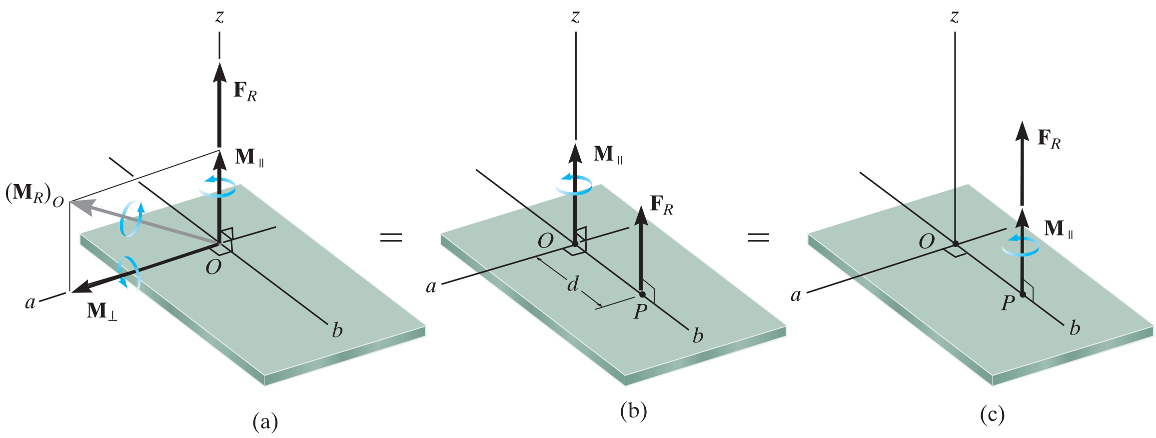


Fig. 4–43

EXAMPLE 4.17

Replace the force and couple moment system acting on the beam in Fig. 4–44*a* by an equivalent resultant force, and find where its line of action intersects the beam, measured from point O.

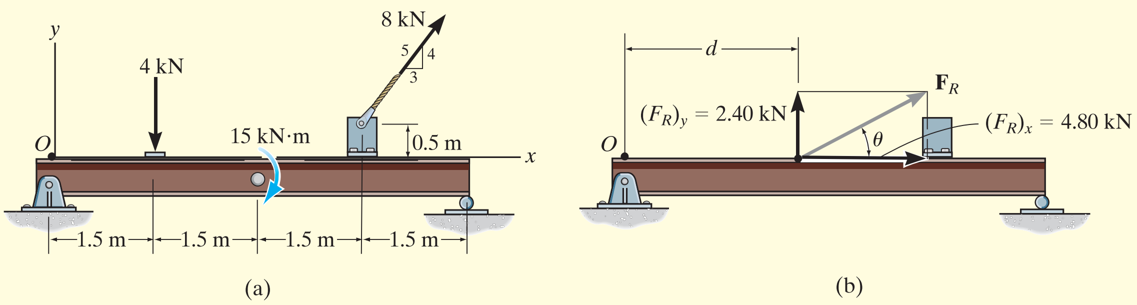


Fig. 4–44

SOLUTION

Force Summation. Summing the force components,

$$\xrightarrow{+} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = 8\text{ kN}\left(\tfrac{3}{5}\right) = 4.80\text{ kN} \rightarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = -4\text{ kN} + 8\text{ kN}\left(\tfrac{4}{5}\right) = 2.40\text{ kN}\uparrow$$

From Fig. 4–44*b*, the magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(4.80\text{ kN})^2 + (2.40\text{ kN})^2} = 5.37\text{ kN} \qquad \textit{Ans.}$$

The angle θ is

$$\theta = \tan^{-1}\left(\frac{2.40\text{ kN}}{4.80\text{ kN}}\right) = 26.6^\circ \qquad \textit{Ans.}$$

Moment Summation. We must equate the moment of $\mathbf{F}_R$ about point O in Fig. 4–44*b* to the sum of the moments of the force and couple moment system about point O in Fig. 4–44*a*. Since the line of action of $(\mathbf{F}_R)_x$ acts through point O, *only* $(\mathbf{F}_R)_y$ *produces a moment* about this point. Thus,

$$\circlearrowleft + (M_R)_O = \Sigma M_O; \qquad 2.40\text{ kN}(d) = -(4\text{ kN})(1.5\text{ m}) - 15\text{ kN}\cdot\text{m}$$

$$-\left[8\text{ kN}\left(\tfrac{3}{5}\right)\right](0.5\text{ m}) + \left[8\text{ kN}\left(\tfrac{4}{5}\right)\right](4.5\text{ m})$$

$$d = 2.25\text{ m} \qquad \textit{Ans.}$$

EXAMPLE 4.18

The jib crane shown in Fig. 4–45*a* is subjected to three coplanar forces. Replace this loading by an equivalent resultant force and specify where the resultant's line of action intersects the column AB and boom BC.

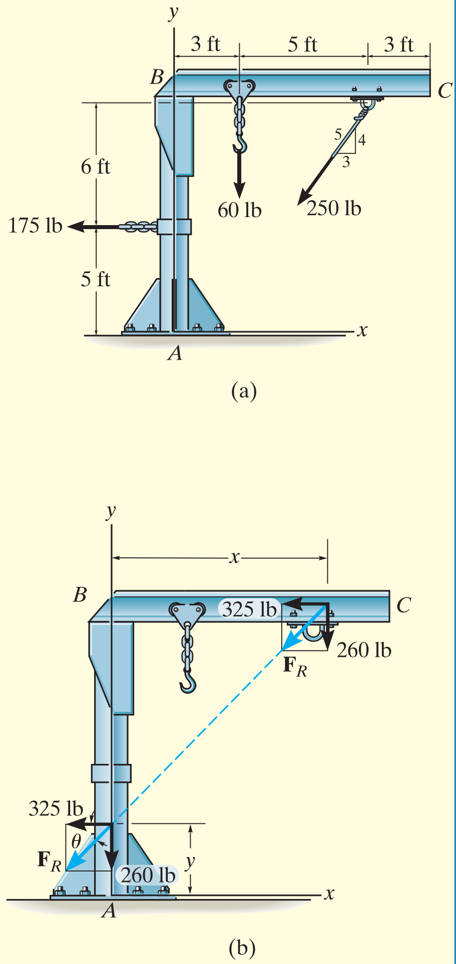


Fig. 4–45

SOLUTION

Force Summation. Resolving the 250-lb force into x and y components and summing the force components yields

$$\overset{+}{\rightarrow}(F_R)_x = \Sigma F_x; \quad (F_R)_x = -250\text{ lb}\left(\tfrac{3}{5}\right) - 175\text{ lb} = -325\text{ lb} = 325\text{ lb} \leftarrow$$

$$+\uparrow(F_R)_y = \Sigma F_y; \quad (F_R)_y = -250\text{ lb}\left(\tfrac{4}{5}\right) - 60\text{ lb} = -260\text{ lb} = 260\text{ lb}\downarrow$$

As shown by the vector addition in Fig. 4–45*b*,

$$F_R = \sqrt{(325\text{ lb})^2 + (260\text{ lb})^2} = 416\text{ lb} \qquad \textit{Ans.}$$

$$\theta = \tan^{-1}\left(\frac{260\text{ lb}}{325\text{ lb}}\right) = 38.7^\circ \ \measuredangle \qquad \textit{Ans.}$$

Moment Summation. Moments will be summed about point A. Assuming the line of action of $\mathbf{F}_R$ *intersects* AB at a distance y from A, Fig. 4–45*b*, we have

$$\circlearrowleft + (M_R)_A = \Sigma M_A; \qquad 325\text{ lb }(y) + 260\text{ lb }(0)$$

$$= 175\text{ lb }(5\text{ ft}) - 60\text{ lb }(3\text{ ft}) + 250\text{ lb}\left(\tfrac{3}{5}\right)(11\text{ ft}) - 250\text{ lb}\left(\tfrac{4}{5}\right)(8\text{ ft})$$

$$y = 2.29\text{ ft} \qquad \textit{Ans.}$$

By the principle of transmissibility, $\mathbf{F}_R$ can be placed at a distance x where it intersects BC, Fig. 4–45*b*. In this case we have

$$\circlearrowleft + (M_R)_A = \Sigma M_A; \qquad 325\text{ lb }(11\text{ ft}) - 260\text{ lb }(x)$$

$$= 175\text{ lb }(5\text{ ft}) - 60\text{ lb }(3\text{ ft}) + 250\text{ lb}\left(\tfrac{3}{5}\right)(11\text{ ft}) - 250\text{ lb}\left(\tfrac{4}{5}\right)(8\text{ ft})$$

$$x = 10.9\text{ ft} \qquad \textit{Ans.}$$

4

EXAMPLE 4.19

The slab in Fig. 4–46*a* is subjected to four parallel forces. Determine the magnitude and direction of a resultant force equivalent to the given force system, and locate its point of application on the slab.

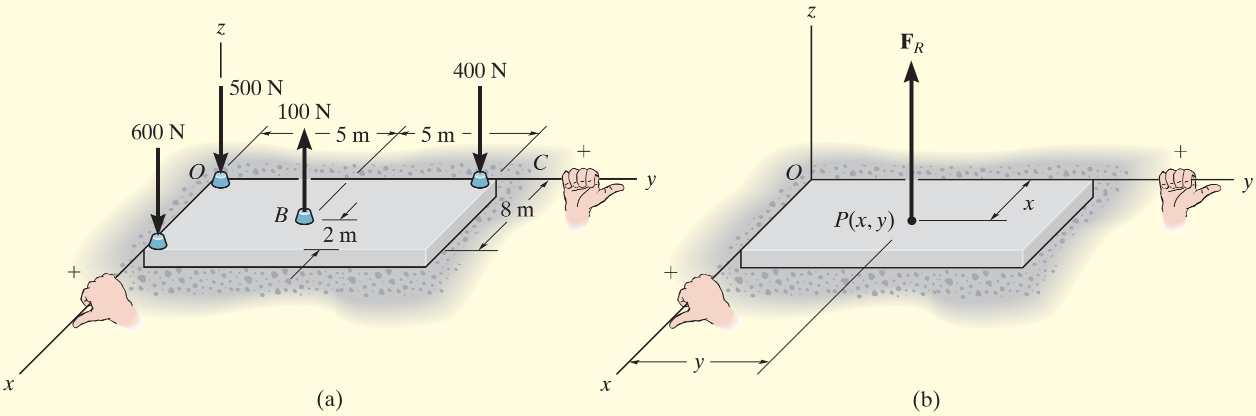


Fig. 4–46

SOLUTION *(SCALAR ANALYSIS)*

Force Summation. From Fig. 4–46*a*, the resultant force is

$+\uparrow F_R = \Sigma F;$ $\qquad F_R = -600\text{ N} + 100\text{ N} - 400\text{ N} - 500\text{ N}$

$$= -1400\text{ N} = 1400\text{ N}\downarrow \qquad \textit{Ans.}$$

Moment Summation. We require the moment about the *x* axis of the resultant force, Fig. 4–46*b*, to be equal to the sum of the moments about the *x* axis of all the forces in the system, Fig. 4–46*a*. The moment arms are determined from the *y* coordinates, since these coordinates represent the *perpendicular distances* from the *x* axis to the lines of action of the forces. Using the right-hand rule, we have

$(M_R)_x = \Sigma M_x;$

$$-(1400\text{ N})y = 600\text{ N}(0) + 100\text{ N}(5\text{ m}) - 400\text{ N}(10\text{ m}) + 500\text{ N}(0)$$

$$-1400y = -3500 \qquad y = 2.50\text{ m} \qquad \textit{Ans.}$$

In a similar manner, a moment equation can be written about the *y* axis using moment arms defined by the *x* coordinates of each force.

$(M_R)_y = \Sigma M_y;$

$$(1400\text{ N})x = 600\text{ N}(8\text{ m}) - 100\text{ N}(6\text{ m}) + 400\text{ N}(0) + 500\text{ N}(0)$$

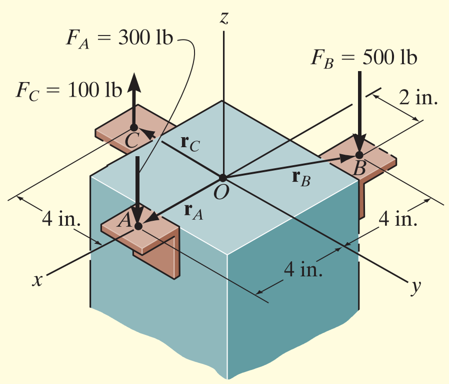
$$1400x = 4200$$

$$x = 3\text{ m} \qquad \textit{Ans.}$$

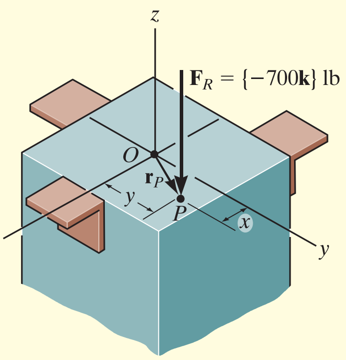
NOTE: A force of $F_R = 1400$ N placed at point $P(3.00\text{ m}, 2.50\text{ m})$ on the slab, Fig. 4–46*b*, is therefore equivalent to the parallel force system acting on the slab in Fig. 4–46*a*.

EXAMPLE 4.20

Replace the force system in Fig. 4–47*a* by an equivalent resultant force and specify its point of application on the pedestal.



(a)



(b)

Fig. 4–47

SOLUTION

Force Summation. Here we will demonstrate a vector analysis. Summing forces,

$$\mathbf{F}_R = \Sigma\mathbf{F};\ \mathbf{F}_R = \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C$$

$$= \{-300\mathbf{k}\}\text{ lb} + \{-500\mathbf{k}\}\text{ lb} + \{100\mathbf{k}\}\text{ lb}$$

$$= \{-700\mathbf{k}\}\text{ lb} \qquad \textit{Ans.}$$

Location. Moments will be summed about point O. The resultant force $\mathbf{F}_R$ is assumed to act through point P $(x, y, 0)$, Fig. 4–47*b*. Thus

$$(\mathbf{M}_R)_O = \Sigma\mathbf{M}_O;$$

$$\mathbf{r}_P \times \mathbf{F}_R = (\mathbf{r}_A \times \mathbf{F}_A) + (\mathbf{r}_B \times \mathbf{F}_B) + (\mathbf{r}_C \times \mathbf{F}_C)$$

$$(x\mathbf{i} + y\mathbf{j}) \times (-700\mathbf{k}) = [(4\mathbf{i}) \times (-300\mathbf{k})]$$

$$+ [(-4\mathbf{i} + 2\mathbf{j}) \times (-500\mathbf{k})] + [(-4\mathbf{j}) \times (100\mathbf{k})]$$

$$-700x(\mathbf{i} \times \mathbf{k}) - 700y(\mathbf{j} \times \mathbf{k}) = -1200(\mathbf{i} \times \mathbf{k}) + 2000(\mathbf{i} \times \mathbf{k})$$

$$- 1000(\mathbf{j} \times \mathbf{k}) - 400(\mathbf{j} \times \mathbf{k})$$

$$700x\mathbf{j} - 700y\mathbf{i} = 1200\mathbf{j} - 2000\mathbf{j} - 1000\mathbf{i} - 400\mathbf{i}$$

Equating the **i** and **j** components,

$$-700y = -1400 \qquad (1)$$

$$y = 2\text{ in.} \qquad \textit{Ans.}$$

$$700x = -800 \qquad (2)$$

$$x = -1.14\text{ in.} \qquad \textit{Ans.}$$

The negative sign indicates that the x coordinate of point P is negative.

NOTE: It is also possible to establish Eq. 1 and 2 directly by summing moments about the x and y axes. Using the right-hand rule, we have

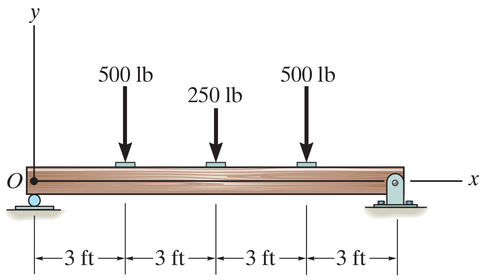
$$(M_R)_x = \Sigma M_x; \qquad -700y = -100\text{ lb}(4\text{ in.}) - 500\text{ lb}(2\text{ in.})$$

$$(M_R)_y = \Sigma M_y; \qquad 700x = 300\text{ lb}(4\text{ in.}) - 500\text{ lb}(4\text{ in.})$$

4

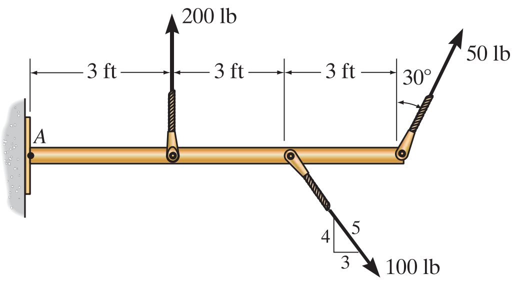
FUNDAMENTAL PROBLEMS

F4–31. Replace the loading system by an equivalent resultant force and specify where the resultant's line of action intersects the beam measured from *O*.



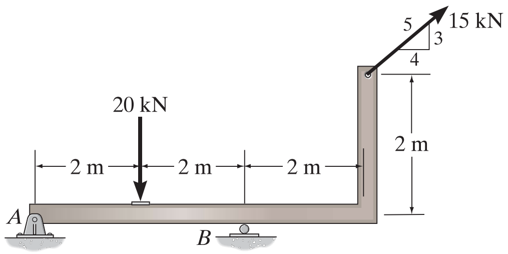
F4–31

F4–32. Replace the loading system by an equivalent resultant force and specify where the resultant's line of action intersects the member measured from *A*.



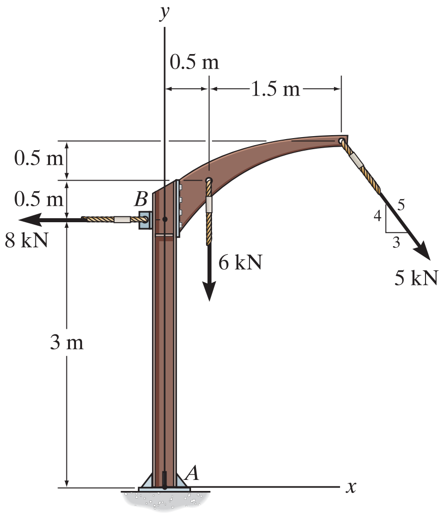
F4–32

F4–33. Replace the loading system by an equivalent resultant force and specify where the resultant's line of action intersects the horizontal segment of the member measured from *A*.



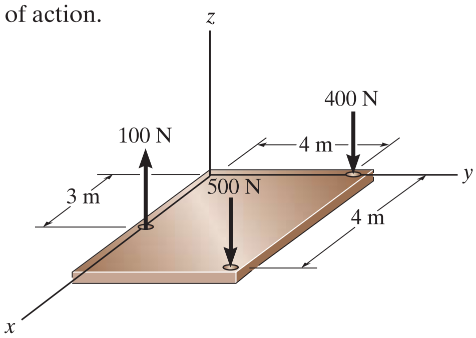
F4–33

F4–34. Replace the loading system by an equivalent resultant force and specify where the resultant's line of action intersects the member *AB* measured from *A*.



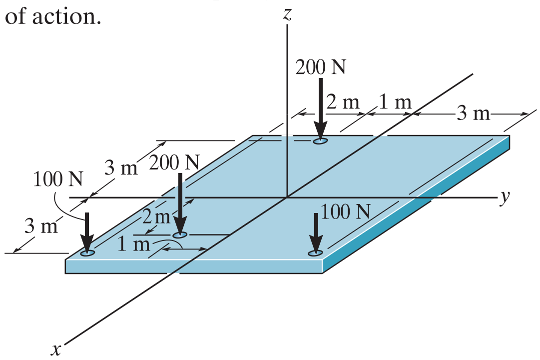
F4–34

F4–35. Replace the loading shown by an equivalent single resultant force and specify the *x* and *y* coordinates of its line of action.



F4–35

F4–36. Replace the loading shown by an equivalent single resultant force and specify the *x* and *y* coordinates of its line of action.

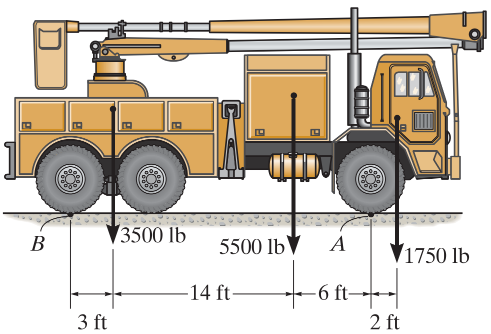


F4–36

PROBLEMS

4–113. The weights of the various components of the truck are shown. Replace this system of forces by an equivalent resultant force and specify its location measured from *B*.

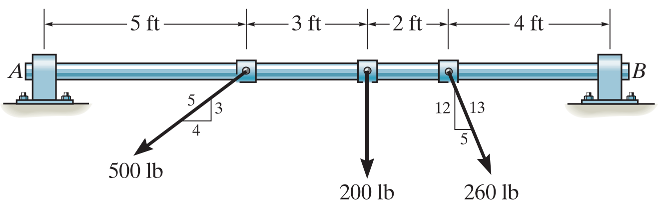
4–114. The weights of the various components of the truck are shown. Replace this system of forces by an equivalent resultant force and specify its location measured from point *A*.



Probs. 4–113/114

4–115. Replace the three forces acting on the shaft by a single resultant force. Specify where the force acts, measured from end *A*.

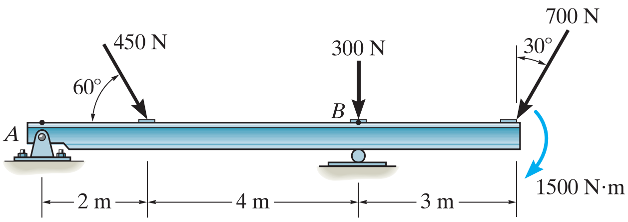
***4–116.** Replace the three forces acting on the shaft by a single resultant force. Specify where the force acts, measured from end *B*.



Probs. 4–115/116

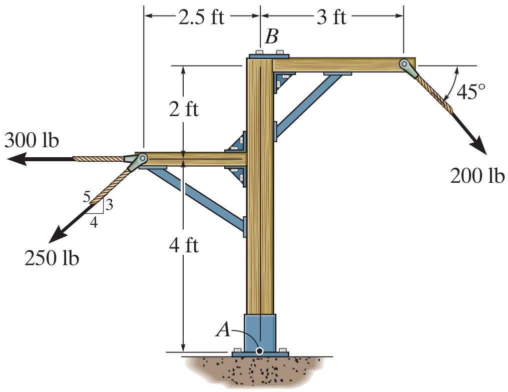
4–117. Replace the loading acting on the beam by a single resultant force. Specify where the force acts, measured from end *A*.

4–118. Replace the loading acting on the beam by a single resultant force. Specify where the force acts, measured from *B*.



Probs. 4–117/118

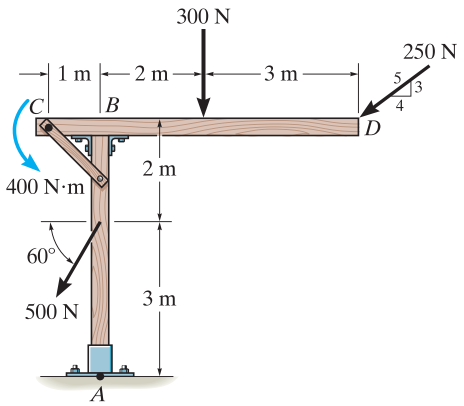
4–119. Replace the force system acting on the frame by a resultant force, and specify where its line of action intersects member *AB*, measured from point *A*.



Prob. 4–119

***4–120.** Replace the loading on the frame by a single resultant force. Specify where its line of action intersects member *AB*, measured from *A*.

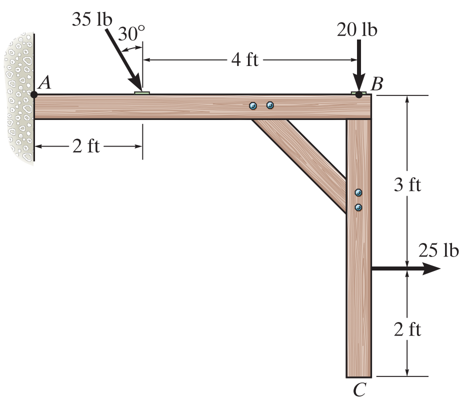
4–121. Replace the loading on the frame by a single resultant force. Specify where its line of action intersects member *CD*, measured from end *C*.



Probs. 4–120/121

4–122. Replace the force system acting on the frame by an equivalent resultant force and specify where the resultant's line of action intersects member *AB*, measured from point *A*.

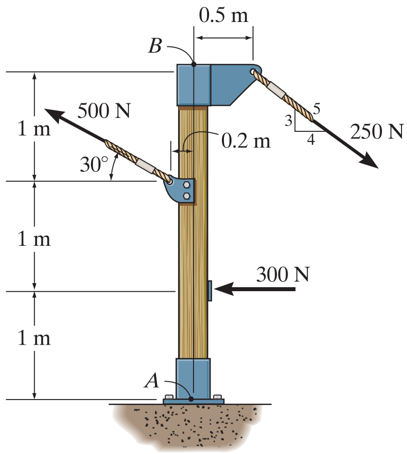
4–123. Replace the force system acting on the frame by an equivalent resultant force and specify where the resultant's line of action intersects member *BC*, measured from point *B*.



Probs. 4–122/123

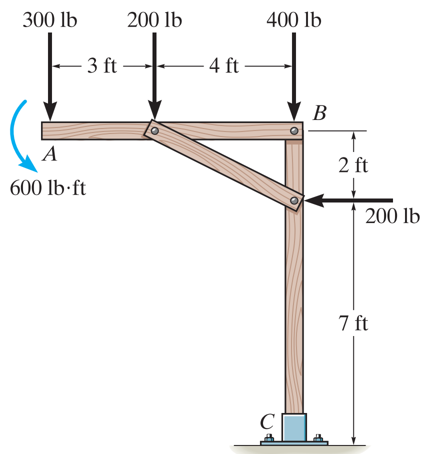
***4–124.** Replace the force system acting on the post by a resultant force, and specify where its line of action intersects the post *AB* measured from point *A*.

4–125. Replace the force system acting on the post by a resultant force, and specify where its line of action intersects the post *AB* measured from point *B*.



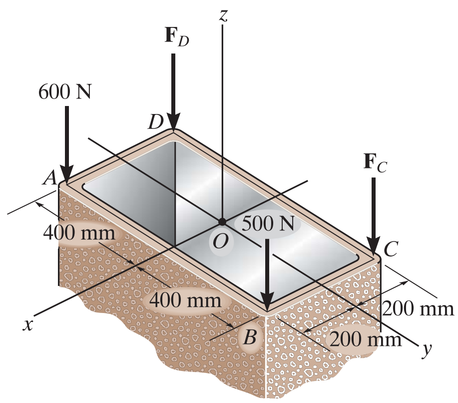
Probs. 4–124/125

4–126. Replace the loading on the frame by a single resultant force. Specify where its line of action intersects member *AB*, measured from *A*.



Prob. 4–126

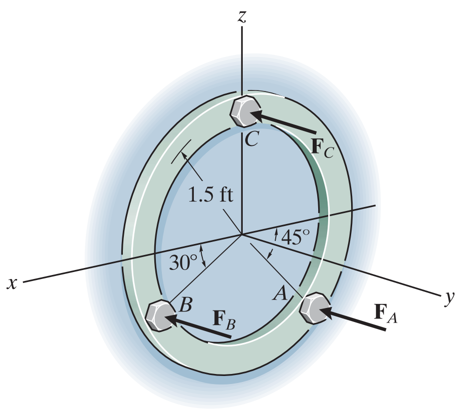
4–127. The tube supports the four parallel forces. Determine the magnitudes of forces $\mathbf{F}_C$ and $\mathbf{F}_D$ acting at C and D so that the equivalent resultant force of the force system acts through the midpoint O of the tube.



Prob. 4–127

*4–128. Three parallel bolting forces act on the circular plate. Determine the resultant force, and specify its location (x, z) on the plate. $F_A = 200$ lb, $F_B = 100$ lb, and $F_C = 400$ lb.

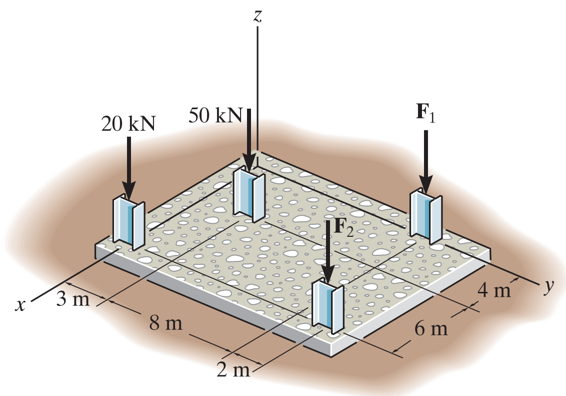
4–129. The three parallel bolting forces act on the circular plate. If the force at A has a magnitude of $F_A = 200$ lb, determine the magnitudes of $\mathbf{F}_B$ and $\mathbf{F}_C$ so that the resultant force $\mathbf{F}_R$ of the system has a line of action that coincides with the y axis. *Hint:* This requires $\Sigma M_x = 0$ and $\Sigma M_z = 0$.



Probs. 4–128/129

4–130. The building slab is subjected to four parallel column loadings. Determine the equivalent resultant force and specify its location (x, y) on the slab. Take $F_1 = 30$ kN, $F_2 = 40$ kN.

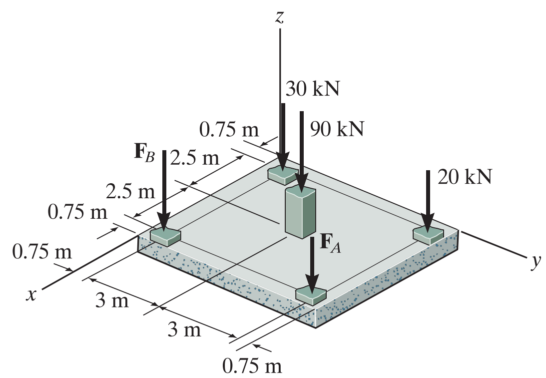
4–131. The building slab is subjected to four parallel column loadings. Determine the equivalent resultant force and specify its location (x, y) on the slab. Take $F_1 = 20$ kN, $F_2 = 50$ kN.



Probs. 4–130/131

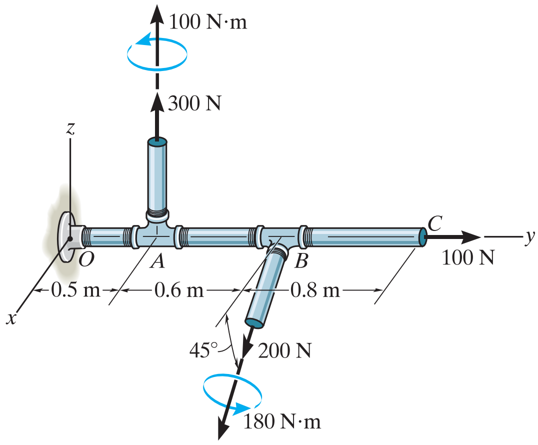
*4–132. If $F_A = 40$ kN and $F_B = 35$ kN, determine the magnitude of the resultant force and specify the location of its point of application (x, y) on the slab.

4–133. If the resultant force is required to act at the center of the slab, determine the magnitude of the column loadings $\mathbf{F}_A$ and $\mathbf{F}_B$ and the magnitude of the resultant force.



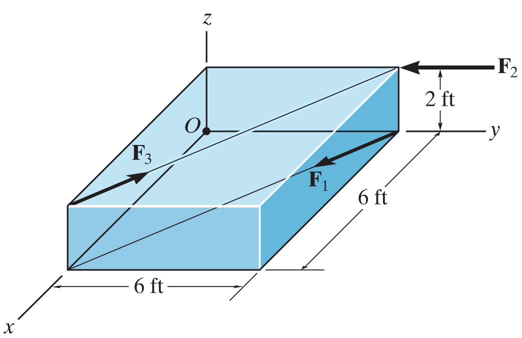
Probs. 4–132/133

4–134. Replace the two wrenches and the force, acting on the pipe assembly, by an equivalent resultant force and couple moment at point *O*.



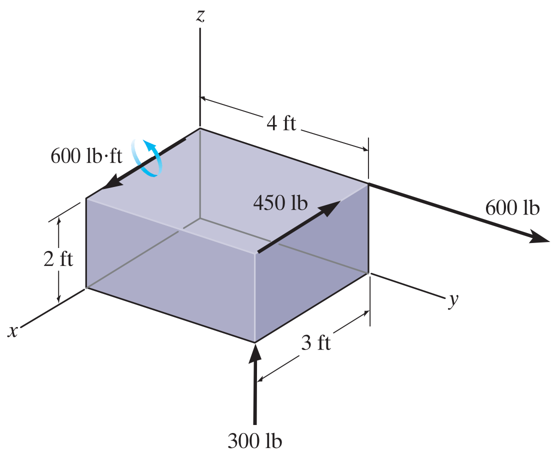
Prob. 4–134

4–135. The three forces acting on the block each have a magnitude of 10 lb. Replace this system by a wrench and specify the point where the wrench intersects the *z* axis, measured from point *O*.



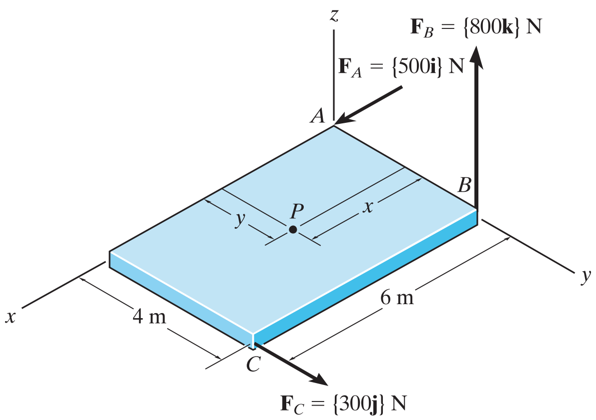
Prob. 4–135

***4–136.** Replace the force and couple moment system acting on the rectangular block by a wrench. Specify the magnitude of the force and couple moment of the wrench and where its line of action intersects the *x*–*y* plane.



Prob. 4–136

4–137. Replace the three forces acting on the plate by a wrench. Specify the magnitude of the force and couple moment for the wrench and the point *P*(*x*, *y*) where its line of action intersects the plate.



Prob. 4–137

4.9 Reduction of a Simple Distributed Loading

Sometimes, a body may be subjected to a loading that is distributed over its surface. For example, the pressure of the wind on the face of a sign, the pressure of water within a tank, or the weight of sand on the floor of a storage container, are all *distributed loadings*. The pressure exerted at each point on the surface indicates the intensity of the loading. It is measured using pascals Pa (or N/m^2) in SI units or lb/ft^2 in the U.S. Customary system.

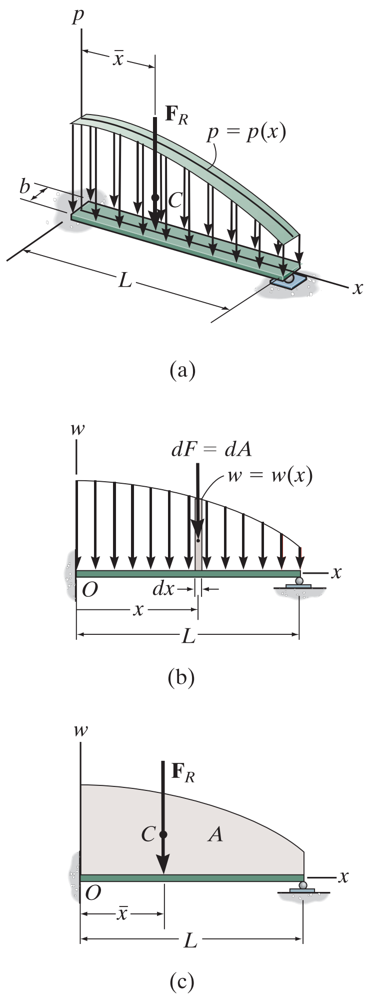


Fig. 4–48

Loading Along a Single Axis. The most common type of distributed loading encountered in engineering practice can be represented along a single axis.* For example, consider the beam (or plate) in Fig. 4–48*a* that has a constant width and is subjected to a pressure loading that varies only along the x axis. This loading can be described by the function $p = p(x)\ \text{N/m}^2$. It contains only one variable x, and for this reason, we can also represent it as a *coplanar distributed load*. To do so, we multiply the loading function by the width b m of the beam, so that $w(x) = p(x)b\ \text{N/m}$, Fig. 4–48*b*. Using the methods of Sec. 4.8, we can replace this coplanar parallel force system with a single equivalent resultant force $\mathbf{F}_R$ acting at a specific location on the beam, Fig. 4–48*c*.

Magnitude of Resultant Force. From Eq. 4–17 ($F_R = \Sigma F$), the magnitude of $\mathbf{F}_R$ is equivalent to the sum of all the forces in the system. In this case integration must be used since there is an infinite number of parallel forces $d\mathbf{F}$ acting on the beam, Fig. 4–48*b*. Since $d\mathbf{F}$ is acting on an element of length dx, and $w(x)$ is a force per unit length, then $dF = w(x)\,dx = dA$. In other words, the magnitude of $d\mathbf{F}$ is determined from the colored differential *area* dA under the loading curve. For the entire length L,

$$+\downarrow F_R = \Sigma F; \qquad F_R = \int_L w(x)\,dx = \int_A dA = A \qquad (4\text{–}19)$$

Therefore, the magnitude of the resultant force is equal to the area A under the loading diagram, Fig. 4–48*c*.

*The more general case of a surface loading acting on a body is considered in Sec. 9.5.

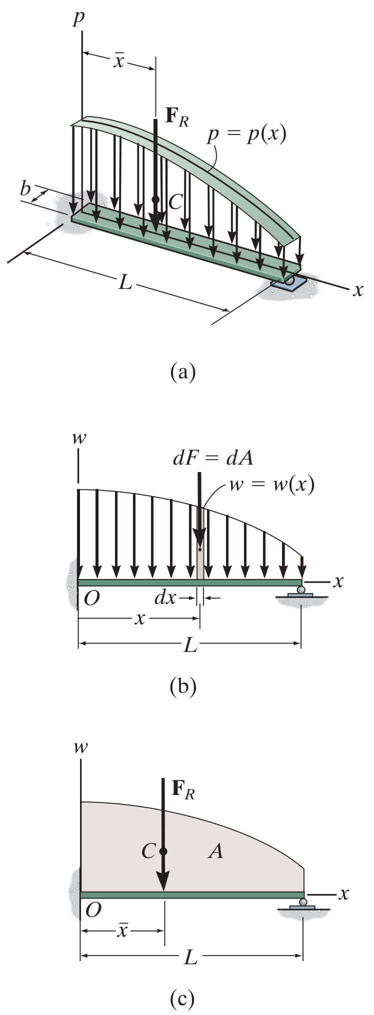
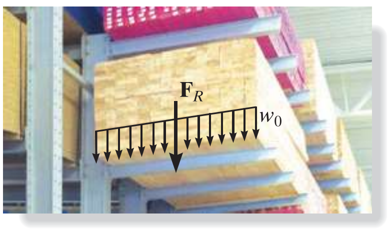


Fig. 4–48 (Repeated)



Each beam that supports this stack of lumber is subjected to a uniform loading of w_0. The resultant force is therefore equal to the area under the rectangular loading diagram. It acts through the centroid or geometric center of this area.

Location of Resultant Force. Applying Eq. 4–17 ($M_{R_O} = \Sigma M_O$), the location $\bar{x}$ of the line of action of $\mathbf{F}_R$ can be determined by equating the moments of the force resultant and the parallel force distribution about point O (the y axis). Since $d\mathbf{F}$ produces a moment of $x\,dF = xw(x)\,dx$ about O, Fig. 4–48b, then for the entire length, Fig. 4–48c,

$$\circlearrowleft + (M_R)_O = \Sigma M_O; \qquad -\bar{x}F_R = -\int_L xw(x)\,dx$$

Solving for $\bar{x}$, using Eq. 4–19, we have

$$\bar{x} = \frac{\int_L xw(x)\,dx}{\int_L w(x)\,dx} = \frac{\int_A x\,dA}{\int_A dA} \qquad (4\text{–}20)$$

This coordinate $\bar{x}$, locates the geometric center or *centroid* of the *area* under the distributed loading. *In other words, the resultant force has a line of action which passes through the centroid C (geometric center) of the area under the loading diagram*, Fig. 4–48c. Detailed treatment of the integration techniques for finding the location of the centroid for areas is given in Chapter 9. In many cases, however, the distributed-loading diagram is in the shape of a rectangle, triangle, or some other simple geometric form. The centroid location for such common shapes does not have to be determined from the above equation but can be obtained directly from the tabulation given on the inside back cover.

Once $\bar{x}$ is determined, $\mathbf{F}_R$ by symmetry passes through point ($\bar{x}$, 0) on the surface of the beam, Fig. 4–48a. *Therefore, in this case the resultant force has a magnitude equal to the volume under the loading curve $p = p(x)$ and a line of action which passes through the centroid (geometric center) of this volume.*

Important Points

- Coplanar distributed loadings are defined by using a loading function $w = w(x)$ that indicates the intensity of the loading along the length of a member. This intensity is measured in N/m or lb/ft.
- The external effects caused by a coplanar distributed load acting on a body can be represented by a single resultant force.
- This resultant force is equivalent to the *area* under the loading diagram, and has a line of action that passes through the *centroid* or geometric center of this area.

EXAMPLE 4.21

Determine the magnitude and location of the equivalent resultant force acting on the shaft in Fig. 4–49*a*.

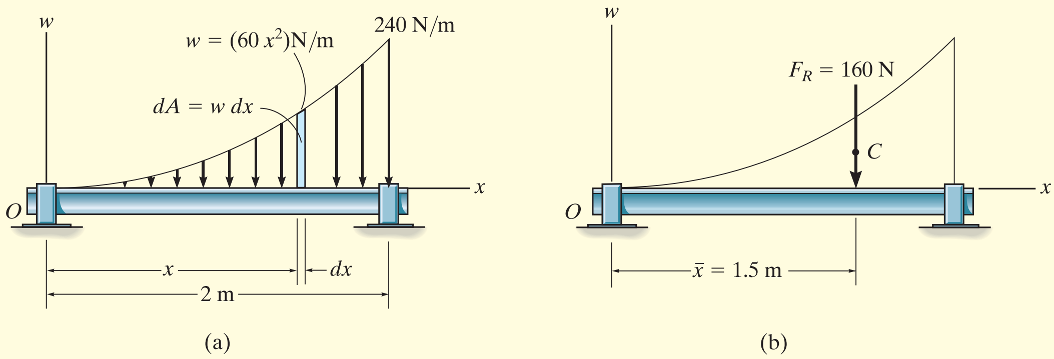


Fig. 4–49

SOLUTION

Since $w = w(x)$ is given, this problem will be solved by integration.

The differential element has an area $dA = w\,dx = 60x^2\,dx$. Applying Eq. 4–19,

$+\downarrow F_R = \Sigma F;$

$$F_R = \int_A dA = \int_0^{2\text{ m}} 60x^2\,dx = 60\left(\frac{x^3}{3}\right)\Bigg|_0^{2\text{ m}} = 60\left(\frac{2^3}{3} - \frac{0^3}{3}\right)$$

$$= 160 \text{ N} \qquad \textit{Ans.}$$

The location $\bar{x}$ of $\mathbf{F}_R$ measured from O, Fig. 4–49*b*, is determined from Eq. 4–20.

$$\bar{x} = \frac{\int_A x\,dA}{\int_A dA} = \frac{\int_0^{2\text{ m}} x(60x^2)\,dx}{160 \text{ N}} = \frac{60\left(\frac{x^4}{4}\right)\Big|_0^{2\text{ m}}}{160 \text{ N}} = \frac{60\left(\frac{2^4}{4} - \frac{0^4}{4}\right)}{160 \text{ N}}$$

$$= 1.5 \text{ m} \qquad \textit{Ans.}$$

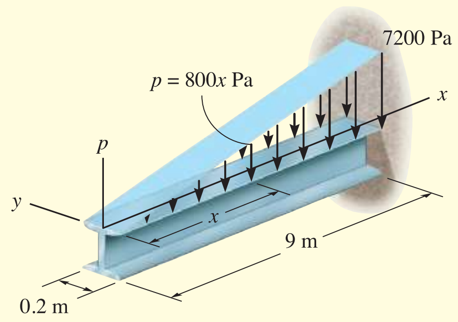
NOTE: These results can be checked by using the table on the inside back cover, where it is shown that formula for an exparabolic area of length *a*, height *b*, and shape shown in Fig. 4–49*a*, we have

$$A = \frac{ab}{3} = \frac{2\text{ m}(240 \text{ N/m})}{3} = 160 \text{ N and } \bar{x} = \frac{3}{4}a = \frac{3}{4}(2\text{ m}) = 1.5 \text{ m}$$

4

EXAMPLE 4.22

A distributed loading of $p = (800x)$ Pa acts over the top surface of the beam shown in Fig. 4–50*a*. Determine the magnitude and location of the equivalent resultant force.

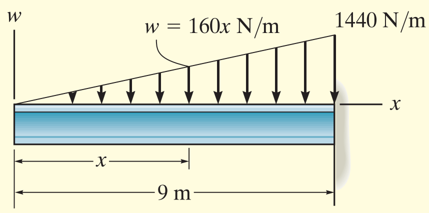


(a)

SOLUTION

Since the loading intensity is uniform along the width of the beam (the y axis), the loading can be viewed in two dimensions as shown in Fig. 4–50*b*. Here

$$w = (800x\ \text{N/m}^2)(0.2\ \text{m})$$
$$= (160x)\ \text{N/m}$$



(b)

At $x = 9$ m, note that $w = 1440$ N/m. Although we may again apply Eqs. 4–19 and 4–20 as in the previous example, it is simpler to use the table on the inside back cover.

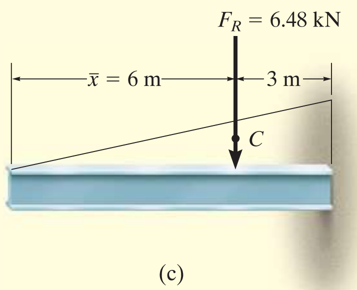
The magnitude of the resultant force is equivalent to the area of the triangle.

$$F_R = \tfrac{1}{2}(9\ \text{m})(1440\ \text{N/m}) = 6480\ \text{N} = 6.48\ \text{kN} \qquad \textit{Ans.}$$

The line of action of $\mathbf{F}_R$ passes through the *centroid* C of this triangle. Hence,

$$\bar{x} = 9\ \text{m} - \tfrac{1}{3}(9\ \text{m}) = 6\ \text{m} \qquad \textit{Ans.}$$

The results are shown in Fig. 4–50*c*.



(c)

Fig. 4–50

NOTE: We may also view the resultant $\mathbf{F}_R$ as *acting* through the *centroid* of the *volume* of the loading diagram $p = p(x)$ in Fig. 4–50*a*. Hence $\mathbf{F}_R$ intersects the x–y plane at the point (6 m, 0). Furthermore, the magnitude of $\mathbf{F}_R$ is equal to the volume under the loading diagram; i.e.,

$$F_R = V = \tfrac{1}{2}(7200\ \text{N/m}^2)(9\ \text{m})(0.2\ \text{m}) = 6.48\ \text{kN} \qquad \textit{Ans.}$$

EXAMPLE 4.23

The granular material exerts the distributed loading on the beam as shown in Fig. 4–51*a*. Determine the magnitude and location of the equivalent resultant of this load.

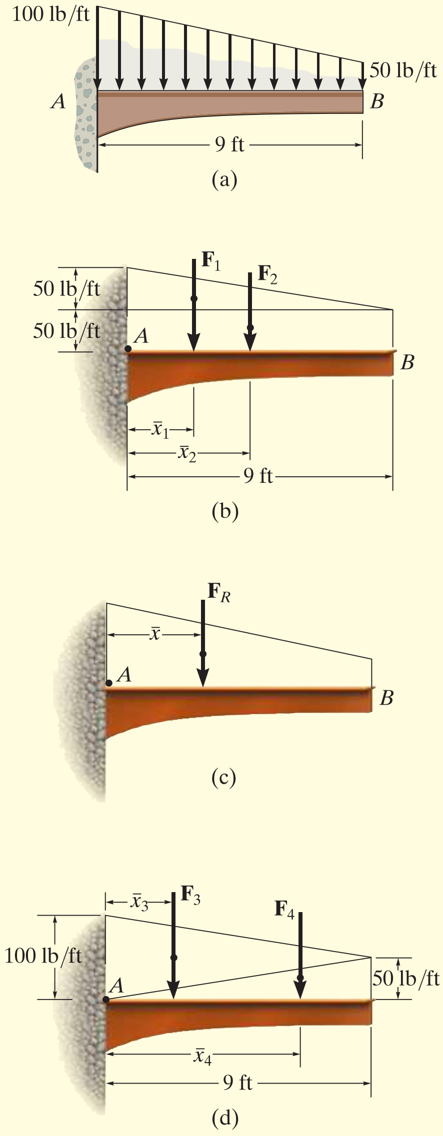


Fig. 4–51

SOLUTION

The area of the loading diagram is a *trapezoid*, and therefore the solution can be obtained directly from the area and centroid formulas for a trapezoid listed on the inside back cover. Since these formulas are not easily remembered, instead we will solve this problem by using "composite" areas. Here we will divide the trapezoidal loading into a rectangular and triangular loading as shown in Fig. 4–51*b*. The magnitude of the force represented by each of these loadings is equal to its associated *area*,

$$F_1 = \tfrac{1}{2}(9 \text{ ft})(50 \text{ lb/ft}) = 225 \text{ lb}$$

$$F_2 = (9 \text{ ft})(50 \text{ lb/ft}) = 450 \text{ lb}$$

The lines of action of these parallel forces act through the respective *centroids* of their associated areas and therefore intersect the beam at

$$\bar{x}_1 = \tfrac{1}{3}(9 \text{ ft}) = 3 \text{ ft}$$

$$\bar{x}_2 = \tfrac{1}{2}(9 \text{ ft}) = 4.5 \text{ ft}$$

The two parallel forces $\mathbf{F}_1$ and $\mathbf{F}_2$ can be reduced to a single resultant $\mathbf{F}_R$. The magnitude of $\mathbf{F}_R$ is

$$+\downarrow F_R = \Sigma F; \qquad F_R = 225 + 450 = 675 \text{ lb} \qquad \textit{Ans.}$$

We can find the location of $\mathbf{F}_R$ with reference to point A, Fig. 4–51*b* and 4–51*c*. We require

$$\circlearrowleft + (M_R)_A = \Sigma M_A; \qquad \bar{x}(675) = 3(225) + 4.5(450)$$

$$\bar{x} = 4 \text{ ft} \qquad \textit{Ans.}$$

NOTE: The trapezoidal area in Fig. 4–51*a* can also be divided into two triangular areas as shown in Fig. 4–51*d*. In this case

$$F_3 = \tfrac{1}{2}(9 \text{ ft})(100 \text{ lb/ft}) = 450 \text{ lb}$$

$$F_4 = \tfrac{1}{2}(9 \text{ ft})(50 \text{ lb/ft}) = 225 \text{ lb}$$

and

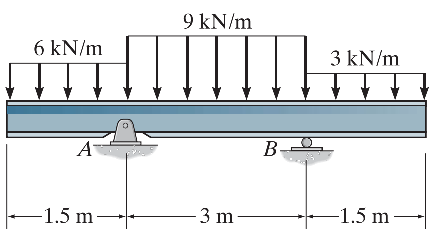
$$\bar{x}_3 = \tfrac{1}{3}(9 \text{ ft}) = 3 \text{ ft}$$

$$\bar{x}_4 = 9 \text{ ft} - \tfrac{1}{3}(9 \text{ ft}) = 6 \text{ ft}$$

Using these results, show that again $F_R = 675$ lb and $\bar{x} = 4$ ft.

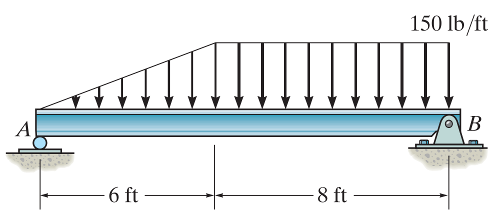
FUNDAMENTAL PROBLEMS

F4–37. Determine the resultant force and specify where it acts on the beam measured from A.



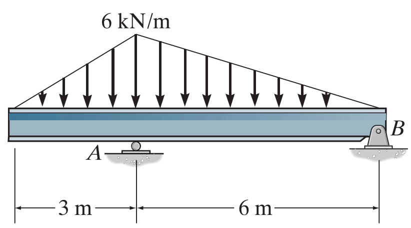
F4–37

F4–38. Determine the resultant force and specify where it acts on the beam measured from A.



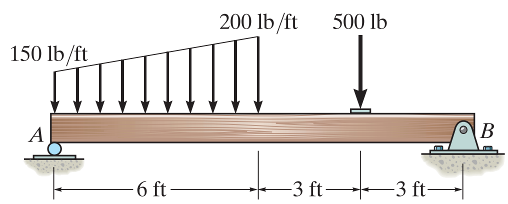
F4–38

F4–39. Determine the resultant force and specify where it acts on the beam measured from A.



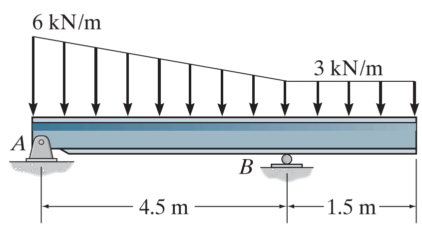
F4–39

F4–40. Determine the resultant force and specify where it acts on the beam measured from A.



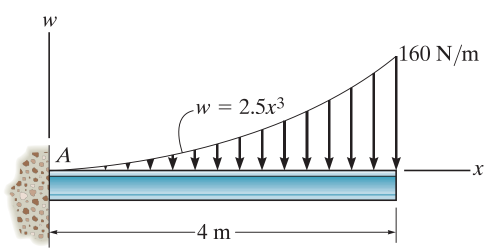
F4–40

F4–41. Determine the resultant force and specify where it acts on the beam measured from A.



F4–41

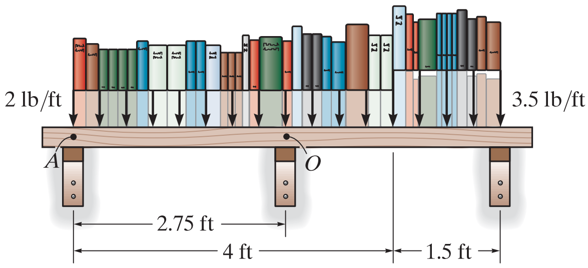
F4–42. Determine the resultant force and specify where it acts on the beam measured from A.



F4–42

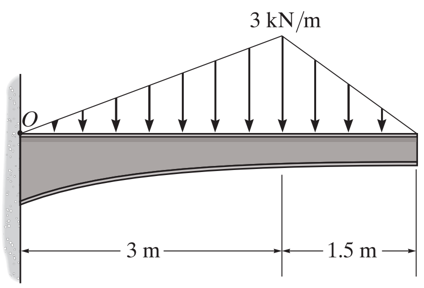
PROBLEMS

4–138. The loading on the bookshelf is distributed as shown. Determine the magnitude of the equivalent resultant location, measured from point *O*.



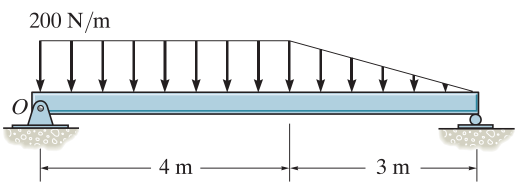
Prob. 4–138

4–139. Replace the distributed loading with an equivalent resultant force, and specify its location on the beam measured from point *O*.



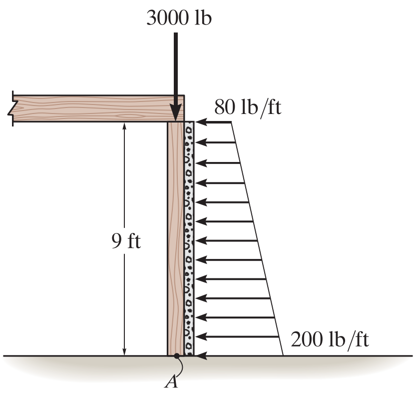
Prob. 4–139

***4–140.** Replace the loading by an equivalent force and couple moment acting at point *O*.



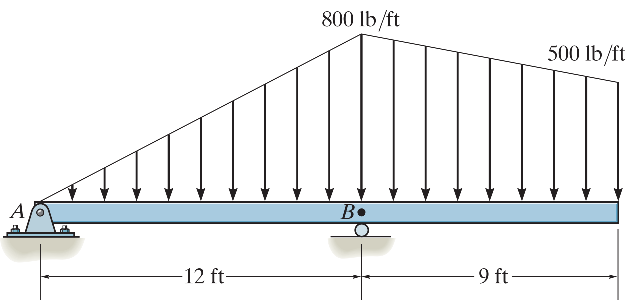
Prob. 4–140

4–141. The column is used to support the floor which exerts a force of 3000 lb on the top of the column. The effect of soil pressure along its side is distributed as shown. Replace this loading by an equivalent resultant force and specify where it acts along the column, measured from its base *A*.



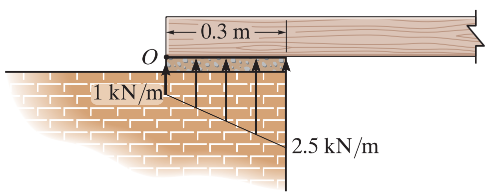
Prob. 4–141

4–142. Replace the loading by an equivalent resultant force and specify its location on the beam, measured from point *B*.



Prob. 4–142

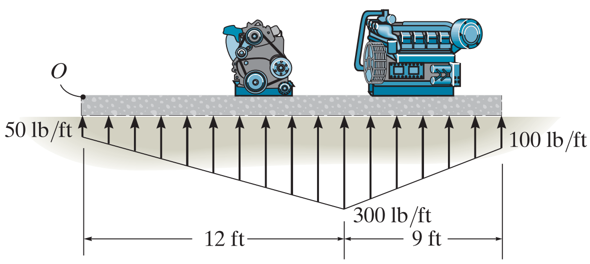
4–143. The masonry support creates the loading distribution acting on the end of the beam. Simplify this load to a single resultant force and specify its location measured from point *O*.



Prob. 4–143

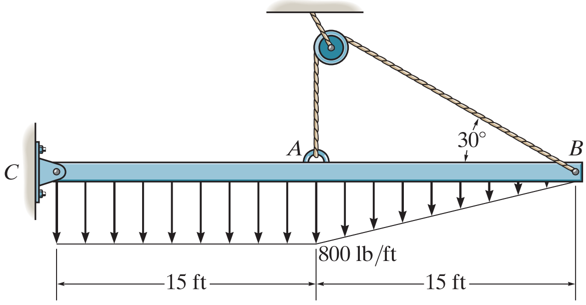
4

***4–144.** The distribution of soil loading on the bottom of a building slab is shown. Replace this loading by an equivalent resultant force and specify its location, measured from point O.



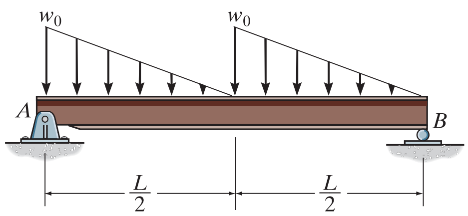
Prob. 4–144

4–145. Replace the distributed loading by an equivalent resultant force, and specify its location on the beam, measured from the pin at C.



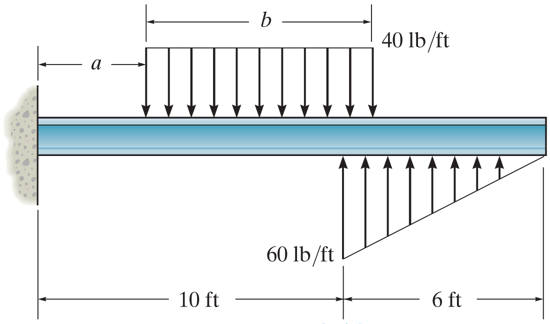
Prob. 4–145

4–146. Replace the distributed loading with an equivalent resultant force, and specify its location on the beam measured from point A.



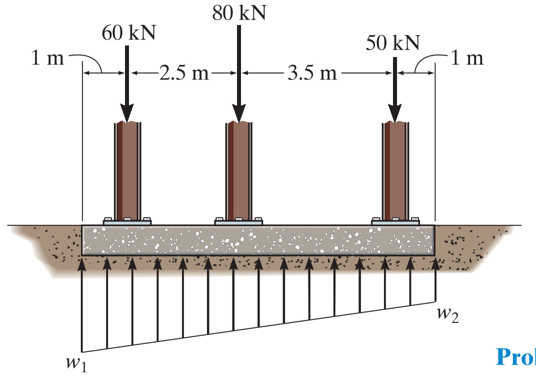
Prob. 4–146

4–147. The beam is subjected to the distributed loading. Determine the length b of the uniform load and its position a on the beam such that the resultant force and couple moment acting on the beam are zero.



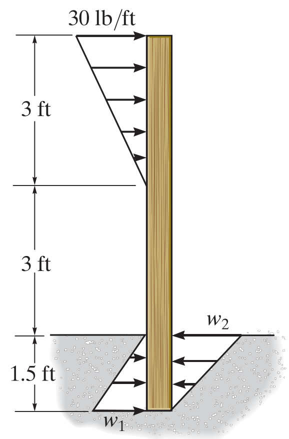
Prob. 4–147

***4–148.** If the soil exerts a trapezoidal distribution of load on the bottom of the footing, determine the intensities w_1 and w_2 of this distribution needed to support the column loadings.



Prob. 4–148

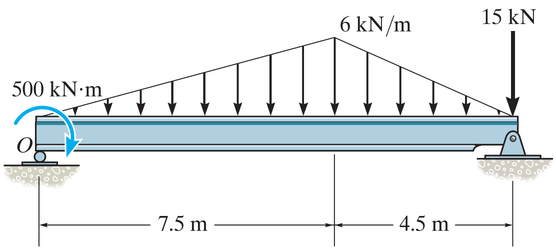
4–149. The post is embedded into a concrete footing so that it is fixed supported. If the reaction of the concrete on the post can be approximated by the distributed loading shown, determine the intensity of w_1 and w_2 so that the resultant force and couple moment on the post due to the loadings are both zero.



Prob. 4–149

4–150. Replace the loading by an equivalent force and couple moment acting at point O.

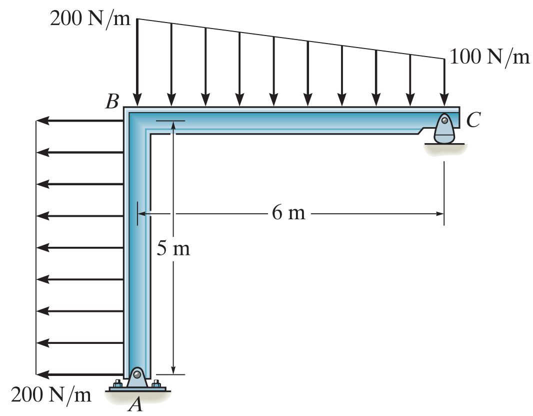
4–151. Replace the loading by a single resultant force, and specify the location of the force measured from point O.



Probs. 4–150/151

4–154. Replace the distributed loading by an equivalent resultant force and specify where its line of action intersects member AB, measured from A.

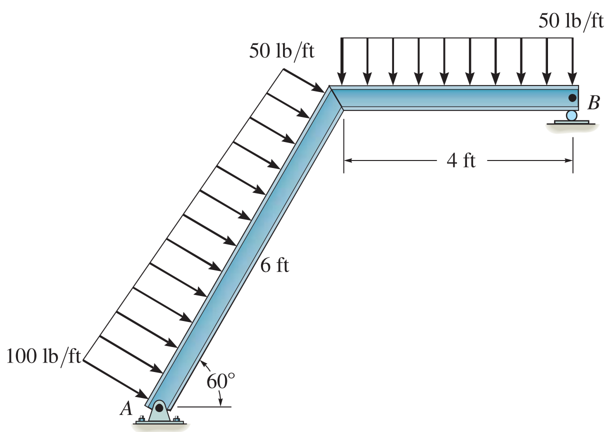
4–155. Replace the distributed loading by an equivalent resultant force and specify where its line of action intersects member BC, measured from C.



Probs. 4–154/155

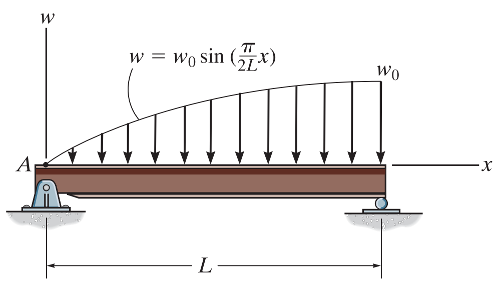
***4–152.** Replace the loading by an equivalent resultant force and couple moment at point A.

4–153. Replace the loading by an equivalent resultant force and couple moment acting at point B.



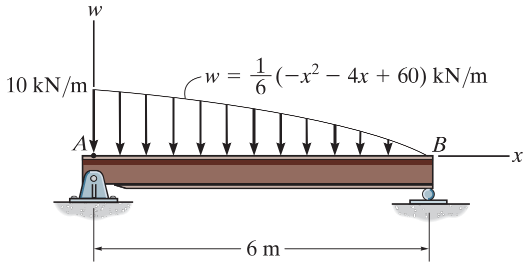
Probs. 4–152/153

***4–156.** Replace the distributed loading with an equivalent resultant force, and specify its location on the beam measured from point A.



Prob. 4–156

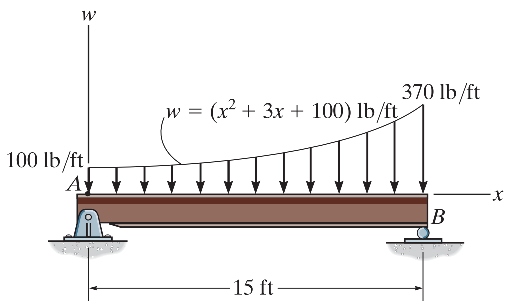
4–157. Replace the distributed loading with an equivalent resultant force, and specify its location on the beam measured from point A.



Prob. 4–157

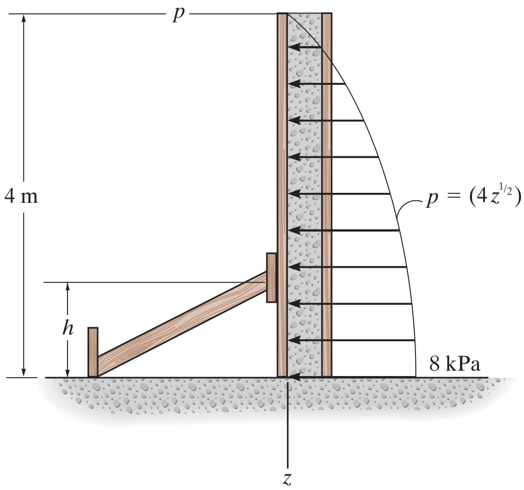
4

4–158. Replace the distributed loading with an equivalent resultant force, and specify its location on the beam measured from point A.



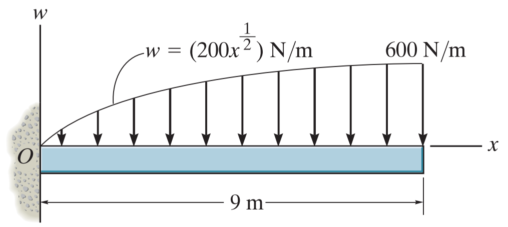
Prob. 4–158

4–159. Wet concrete exerts a pressure distribution along the wall of the form. Determine the resultant force of this distribution and specify the height h where the bracing strut should be placed so that it lies through the line of action of the resultant force. The wall has a width of 5 m.



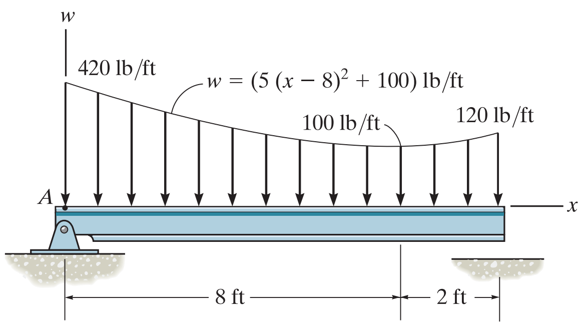
Prob. 4–159

***4–160.** Replace the loading by an equivalent force and couple moment acting at point O.



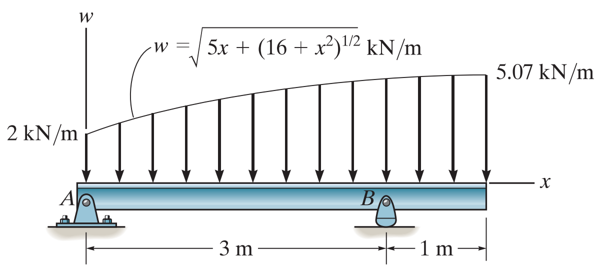
Prob. 4–160

4–161. Determine the magnitude of the equivalent resultant force of the distributed load and specify its location on the beam measured from point A.



Prob. 4–161

■**4–162.** Determine the equivalent resultant force of the distributed loading and its location, measured from point A. Evaluate the integrals using a numerical method.



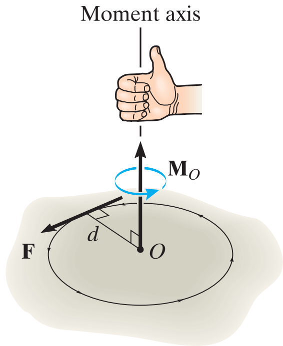
Prob. 4–162

CHAPTER REVIEW

Moment of Force—Scalar Definition

A force produces a turning effect or moment about a point O that does not lie on its line of action. In scalar form, the moment *magnitude* is the product of the force and the moment arm or perpendicular distance from point O to the line of action of the force.

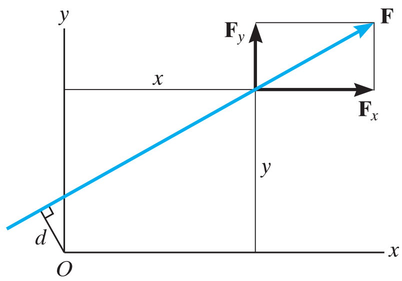
$$M_O = Fd$$



The *direction* of the moment is defined using the right-hand rule. $\mathbf{M}_O$ always acts along an axis perpendicular to the plane containing $\mathbf{F}$ and d, and passes through the point O.

Rather than finding d, it is normally easier to resolve the force into its x and y components, determine the moment of each component about the point, and then sum the results. This is called the principle of moments.

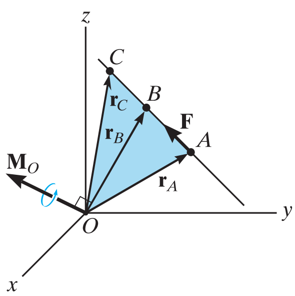
$$M_O = Fd = F_x y - F_y x$$



Moment of a Force—Vector Definition

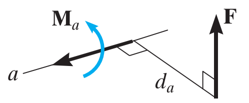
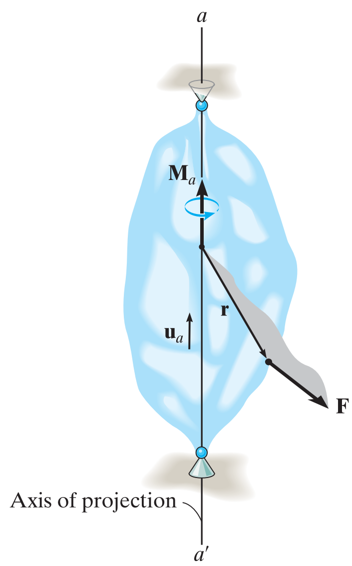
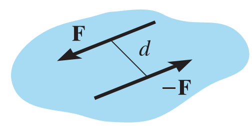
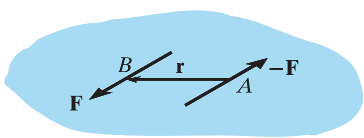
Since three-dimensional geometry is generally more difficult to visualize, the vector cross product should be used to determine the moment. Here $\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$, where $\mathbf{r}$ is a position vector that extends from point O to any point A, B, or C on the line of action of $\mathbf{F}$.

$$\mathbf{M}_O = \mathbf{r}_A \times \mathbf{F} = \mathbf{r}_B \times \mathbf{F} = \mathbf{r}_C \times \mathbf{F}$$



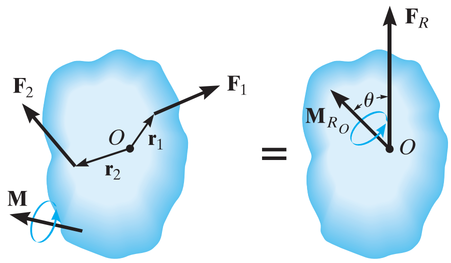
If the position vector $\mathbf{r}$ and force $\mathbf{F}$ are expressed as Cartesian vectors, then the cross product results from the expansion of a determinant.

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

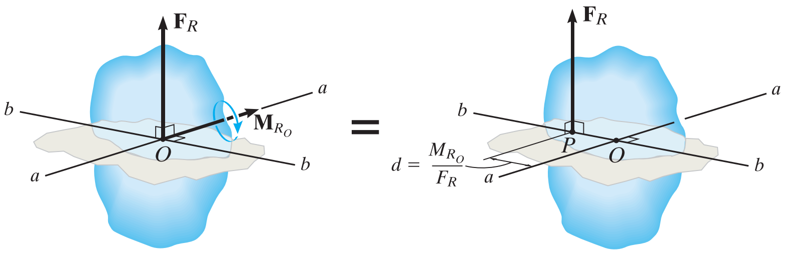
Moment about an Axis If the moment of a force **F** is to be determined about an arbitrary axis *a*, then for a scalar solution the moment arm, or shortest distance d_a from the line of action of the force to the axis must be used. This distance is perpendicular to both the axis and the force line of action.		
Note that when the line of action of **F** intersects the axis then the moment of **F** about the axis is zero. Also, when the line of action of **F** is parallel to the axis, the moment of **F** about the axis is zero. In three dimensions, the scalar triple product should be used. Here $\mathbf{u}_a$ is the unit vector that specifies the direction of the axis, and **r** is a position vector that is directed from any point on the axis to any point on the line of action of the force. If M_a is calculated as a negative scalar, then the sense of direction of $\mathbf{M}_a$ is opposite to $\mathbf{u}_a$. $$M_a = Fd_a$$	$$M_a = \mathbf{u}_a \cdot (\mathbf{r} \times \mathbf{F}) = \begin{vmatrix} u_{a_x} & u_{a_y} & u_{a_z} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$	
Couple Moment A couple consists of two equal but opposite forces that act a perpendicular distance *d* apart. Couples tend to produce a rotation without translation.	$$M = Fd$$	
The magnitude of the couple moment is $M = Fd$, and its direction is established using the right-hand rule.		
If the vector cross product is used to determine the moment of a couple, then **r** extends from any point on the line of action of one of the forces to any point on the line of action of the other force **F** that is used in the cross product.	$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$	

Simplification of a Force and Couple System

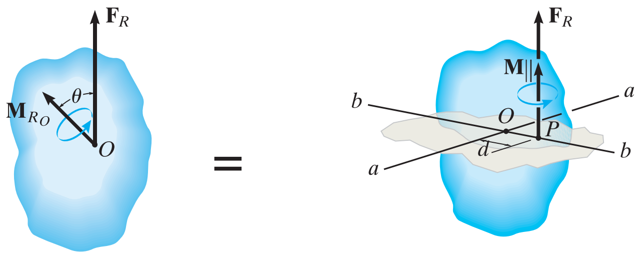
Any system of forces and couples can be reduced to a single resultant force and resultant couple moment acting at a point. The resultant force is the sum of all the forces in the system, $\mathbf{F}_R = \Sigma \mathbf{F}$, and the resultant couple moment is equal to the sum of all the moments of the forces about the point and couple moments. $\mathbf{M}_{R_O} = \Sigma \mathbf{M}_O + \Sigma \mathbf{M}$.



Further simplification to a single resultant force is possible provided the force system is concurrent, coplanar, or parallel. To find the location of the resultant force from a point, it is necessary to equate the moment of the resultant force about the point to the moment of the forces and couples in the system about the same point.

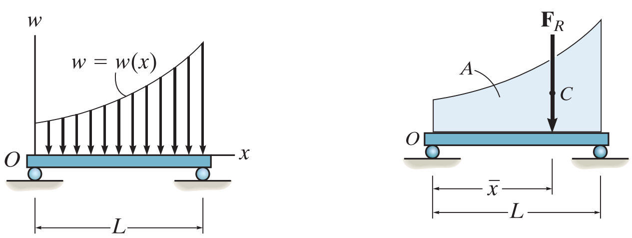


If the resultant force and couple moment at a point are not perpendicular to one another, then this system can be reduced to a wrench, which consists of the resultant force and collinear couple moment.



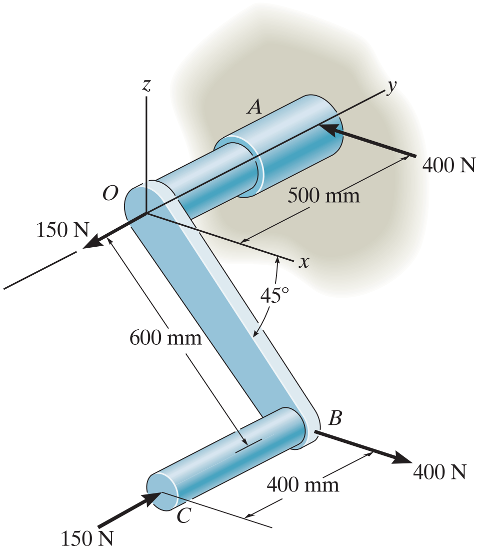
Coplanar Distributed Loading

A simple distributed loading can be represented by its resultant force, which is equivalent to the *area* under the loading curve. This resultant has a line of action that passes through the *centroid* or geometric center of the area or volume under the loading diagram.



REVIEW PROBLEMS

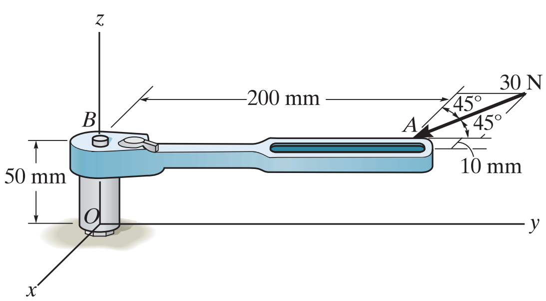
4–163. Determine the resultant couple moment of the two couples that act on the assembly. Member OB lies in the x–z plane.



Prob. 4–163

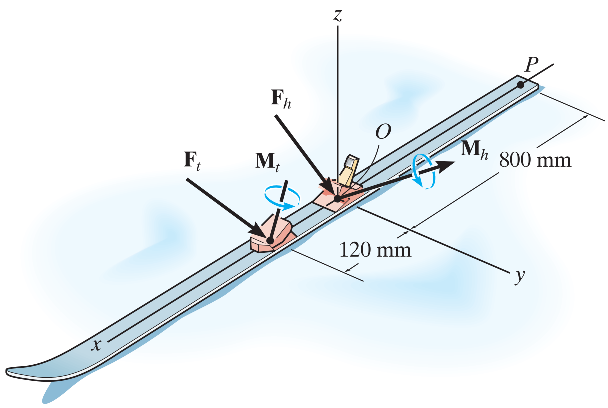
***4–164.** The horizontal 30-N force acts on the handle of the wrench. What is the magnitude of the moment of this force about the z axis?

4–165. The horizontal 30-N force acts on the handle of the wrench. Determine the moment of this force about point O. Specify the coordinate direction angles α, β, γ of the moment axis.



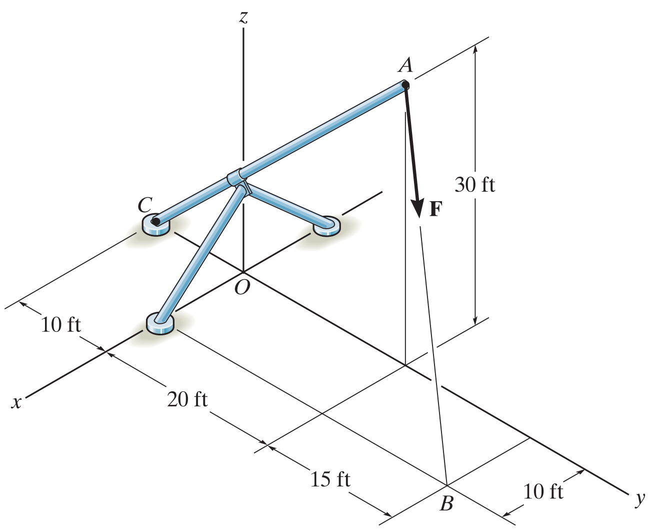
Probs. 4–164/165

4–166. The forces and couple moments that are exerted on the toe and heel plates of a snow ski are $\mathbf{F}_t = \{-50\mathbf{i} + 80\mathbf{j} - 158\mathbf{k}\}$ N, $\mathbf{M}_t = \{-6\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}\}$ N·m, and $\mathbf{F}_h = \{-20\mathbf{i} + 60\mathbf{j} - 250\mathbf{k}\}$ N, $\mathbf{M}_h = \{-20\mathbf{i} + 8\mathbf{j} + 3\mathbf{k}\}$ N·m, respectively. Replace this system by an equivalent force and couple moment acting at point P. Express the results in Cartesian vector form.



Prob. 4–166

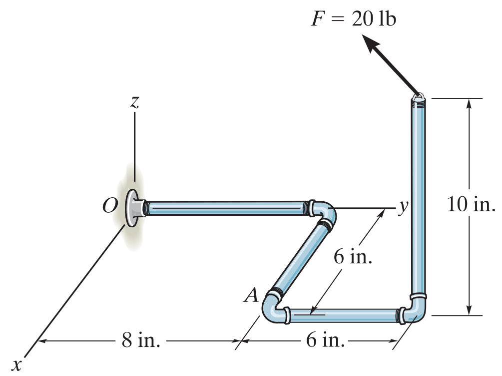
4–167. Replace the force $\mathbf{F}$ having a magnitude of $F = 50$ lb and acting at point A by an equivalent force and couple moment at point C.



Prob. 4–167

*4–168.** Determine the coordinate direction angles α, β, γ of **F**, which is applied to the end A of the pipe assembly, so that the moment of **F** about O is zero.

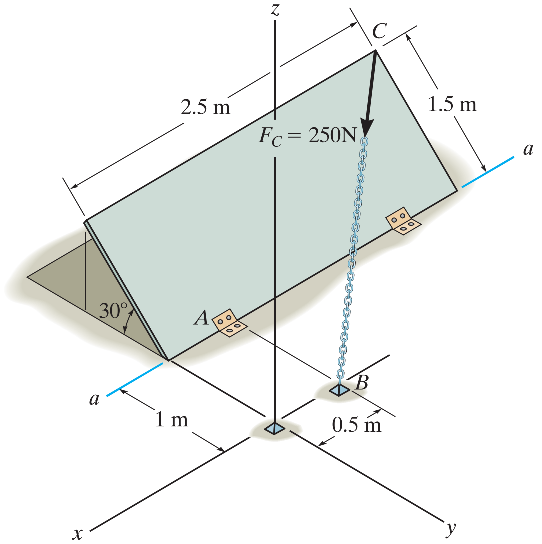
4–169. Determine the moment of the force **F** about point O. The force has coordinate direction angles of $\alpha = 60°$, $\beta = 120°$, $\gamma = 45°$. Express the result as a Cartesian vector.



Probs. 4–168/169

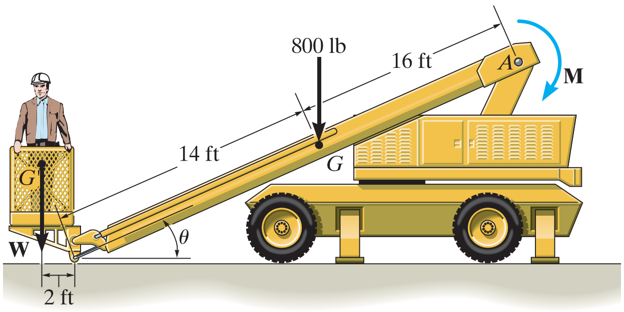
4–170. Determine the moment of the force $\mathbf{F}_c$ about the door hinge at A. Express the result as a Cartesian vector.

4–171. Determine the magnitude of the moment of the force $\mathbf{F}_c$ about the hinged axis *aa* of the door.



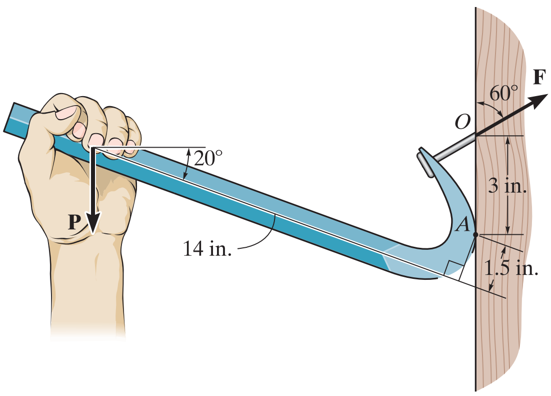
Probs. 4–170/171

***4–172.** The boom has a length of 30 ft, a weight of 800 lb, and mass center at G. If the maximum moment that can be developed by the motor at A is $M = 20(10^3)$ lb · ft, determine the maximum load W, having a mass center at G', that can be lifted. Take $\theta = 30°$.



Prob. 4–172

4–173. If it takes a force of $F = 125$ lb to pull the nail out, determine the smallest vertical force **P** that must be applied to the handle of the crowbar. *Hint:* This requires the moment of **F** about point A to be equal to the moment of **P** about A. Why?



Prob. 4–173