6.6 Frames and Machines

This large crane is a typical example of a framework.

Common tools such as these pliers act as simple machines. Here the applied force on the handles creates a much larger force at the jaws.

Frames and machines are two types of structures which are often composed of pin-connected *multiforce members*, i.e., members that are subjected to more than two forces. *Frames* are used to support loads, whereas *machines* contain moving parts and are designed to transmit and alter the effect of forces. Provided a frame or machine contains no more supports or members than are necessary to prevent its collapse, the forces acting at the joints and supports can be determined by applying the equations of equilibrium to each of its members. Once these forces are obtained, it is then possible to *design* the size of the members, connections, and supports using the theory of mechanics of materials and an appropriate engineering design code.

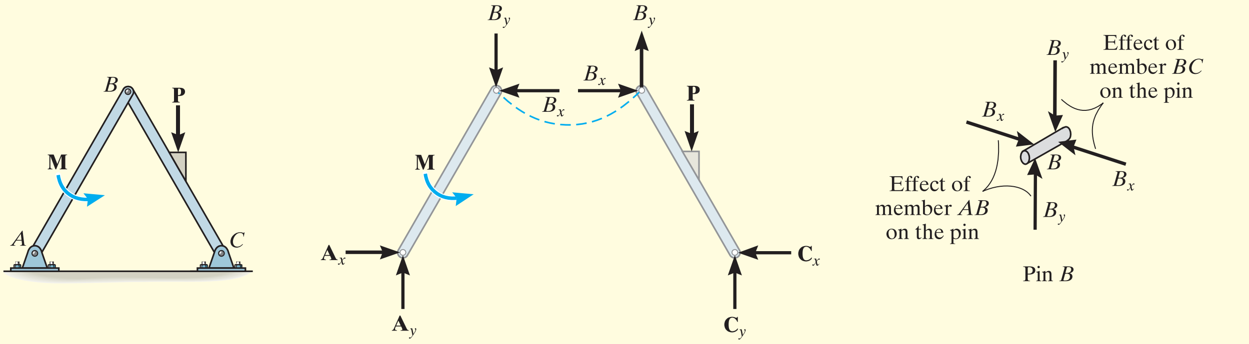
Free-Body Diagrams. In order to determine the forces acting at the joints and supports of a frame or machine, the structure must be disassembled and the free-body diagrams of its parts must be drawn. The following important points *must* be observed:

- Isolate each part by drawing its *outlined shape*. Then show all the forces and/or couple moments that act on the part. Make sure to *label* or *identify* each known and unknown force and couple moment with reference to an established x, y coordinate system. Also, indicate any dimensions used for taking moments. Most often the equations of equilibrium are easier to apply if the forces are represented by their rectangular components. As usual, the sense of an unknown force or couple moment can be assumed.
- Identify all the two-force members in the structure and represent their free-body diagrams as having two equal but opposite collinear forces acting at their points of application. (See Sec. 5.4.) By recognizing the two-force members, we can avoid solving an unnecessary number of equilibrium equations.
- Forces common to *any* two *contacting* members act with equal magnitudes but opposite sense on the respective members. If the two members are treated as a *"system" of connected members*, then these forces are *"internal"* and are *not shown* on the *free-body diagram of the system*; however, if the free-body diagram of *each member* is drawn, the forces are *"external"* and *must* be shown as equal in magnitude and opposite in direction on each of the two free-body diagrams.

The following examples graphically illustrate how to draw the free-body diagrams of a dismembered frame or machine. In all cases, the weight of the members is neglected.

EXAMPLE 6.9

For the frame shown in Fig. 6–21*a*, draw the free-body diagram of (a) each member, (b) the pins at *B* and *A*, and (c) the two members connected together.

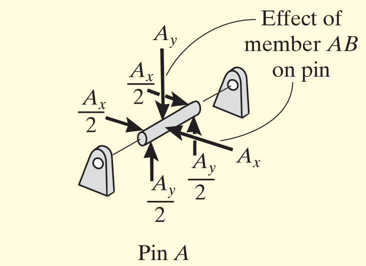


(a) (b) (c)

SOLUTION

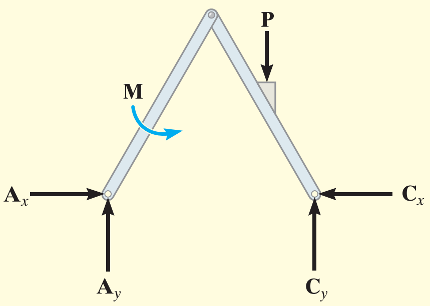
Part (a). By inspection, members *BA* and *BC* are *not* two-force members. Instead, as shown on the free-body diagrams, Fig. 6–21*b*, *BC* is subjected to a force from each of the pins at *B* and *C* and the external force **P**. Likewise, *AB* is subjected to a force from each of the pins at *A* and *B* and the external couple moment **M**. The pin forces are represented by their *x* and *y* components.

Part (b). The pin at *B* is subjected to only *two forces*, i.e., the force of member *BC* and the force of member *AB*. For *equilibrium* these forces (or their respective components) must be equal but opposite, Fig. 6–21*c*. Realize that Newton's third law is applied between the pin and its connected members, i.e., the effect of the pin on the two members, Fig. 6–21*b*, and the equal but opposite effect of the two members on the pin, Fig. 6–21*c*. In the same manner, there are three forces on pin *A*, Fig. 6–21*d*, caused by the force components of member *AB* and each of the two pin leafs.



(d)

Part (c). The free-body diagram of both members connected together, yet removed from the supporting pins at *A* and *C*, is shown in Fig. 6–21*e*. The force components $\mathbf{B}_x$ and $\mathbf{B}_y$ are *not shown* on this diagram since they are *internal* forces (Fig. 6–21*b*) and therefore cancel out. Also, to be consistent when later applying the equilibrium equations, the unknown force components at *A* and *C* must act in the *same sense* as those shown in Fig. 6–21*b*.

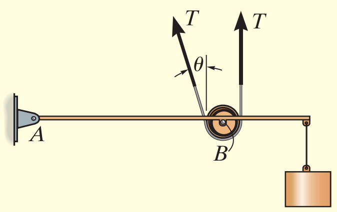


(e)

Fig. 6–21

EXAMPLE 6.10

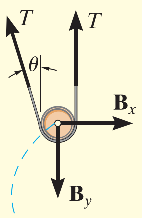
A constant tension in the conveyor belt is maintained by using the device shown in Fig. 6–22*a*. Draw the free-body diagrams of the frame and the cylinder (or pulley) that the belt surrounds. The suspended block has a weight of W.



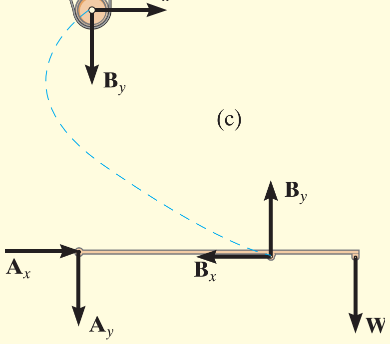
(b)

(a)

Fig. 6–22



(c)



(d)

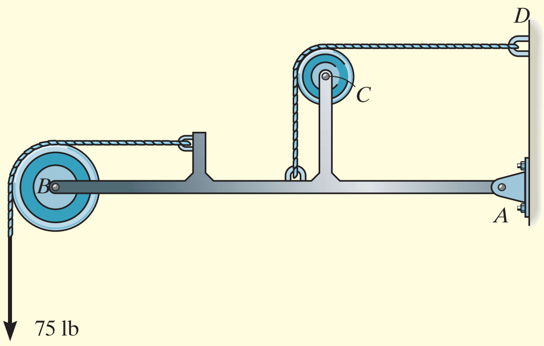
SOLUTION

The idealized model of the device is shown in Fig. 6–22*b*. Here the angle θ is assumed to be known. From this model, the free-body diagrams of the pulley and frame are shown in Figs. 6–22*c* and 6–22*d*, respectively. Note that the force components $\mathbf{B}_x$ and $\mathbf{B}_y$ that the pin at B exerts on the pulley must be equal but opposite to the ones acting on the frame. See Fig. 6–21*c* of Example 6.9.

6

EXAMPLE 6.11

For the frame shown in Fig. 6–23*a*, draw the free-body diagrams of (a) the entire frame including the pulleys and cords, (b) the frame without the pulleys and cords, and (c) each of the pulleys.



(a)

SOLUTION

Part (a). When the entire frame including the pulleys and cords is considered, the interactions at the points where the pulleys and cords are connected to the frame become pairs of *internal* forces which cancel each other and therefore are not shown on the free-body diagram, Fig. 6–23*b*.

Part (b). When the cords and pulleys are removed, their effect *on the frame* must be shown, Fig. 6–23*c*.

Part (c). The force components $\mathbf{B}_x$, $\mathbf{B}_y$, $\mathbf{C}_x$, $\mathbf{C}_y$ of the pins on the pulleys, Fig. 6–23*d*, are equal but opposite to the force components exerted by the pins on the frame, Fig. 6–23*c*. See Example 6.9.

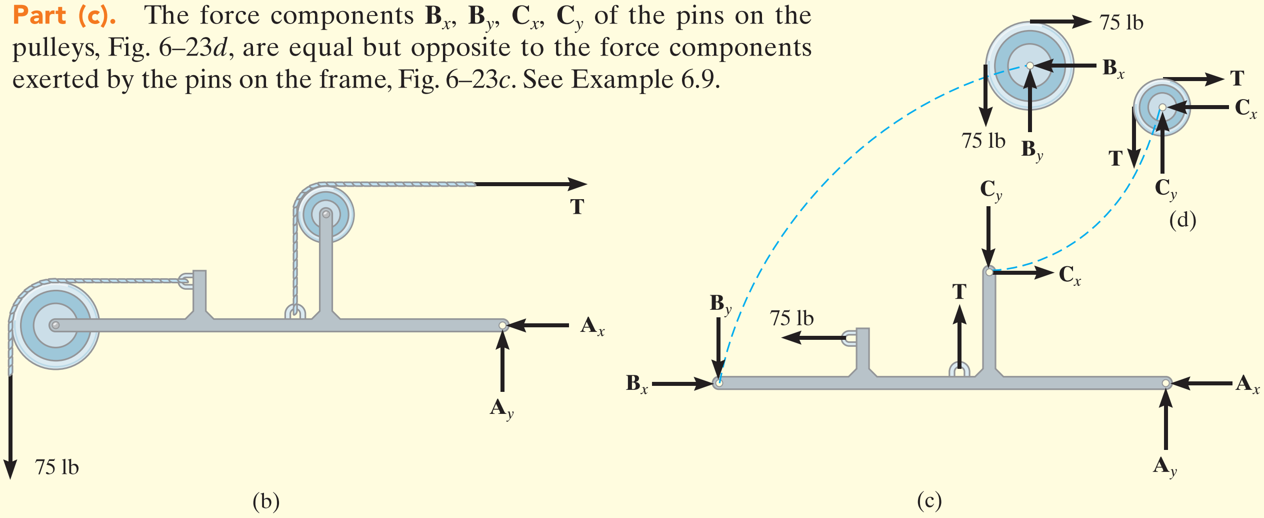


Fig. 6–23

EXAMPLE 6.12

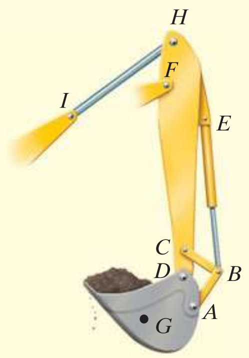
(a)

Fig. 6–24

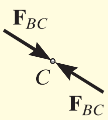
Draw the free-body diagrams of the members of the backhoe, shown in the photo, Fig. 6–24*a*. The bucket and its contents have a weight W.

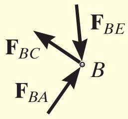
SOLUTION

The idealized model of the assembly is shown in Fig. 6–24*b*. By inspection, members AB, BC, BE, and HI are all two-force members since they are pin connected at their end points and no other forces act on them. The free-body diagrams of the bucket and the stick are shown in Fig. 6–24*c*. Note that pin C is subjected to only two forces, whereas the pin at B is subjected to three forces, Fig. 6–24*d*. The free-body diagram of the entire assembly is shown in Fig. 6–24*e*.

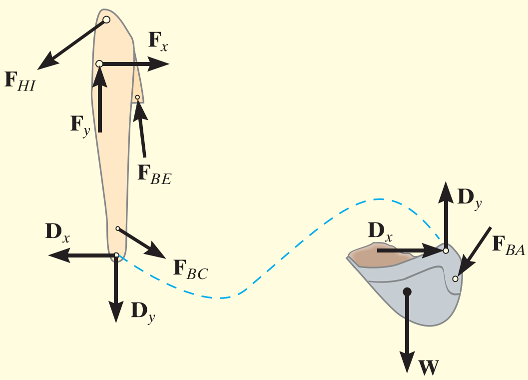


(b)

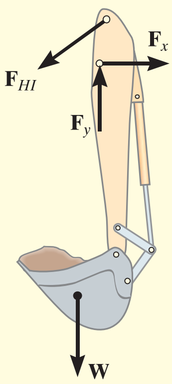




(d)



(c)



(e)

EXAMPLE 6.13

Draw the free-body diagram of each part of the smooth piston and link mechanism used to crush recycled cans, Fig. 6–25*a*.

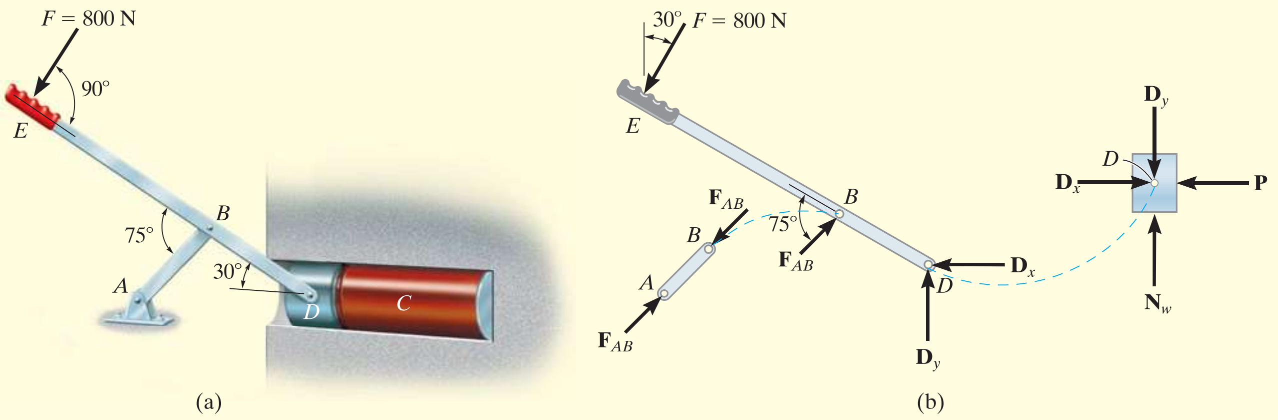
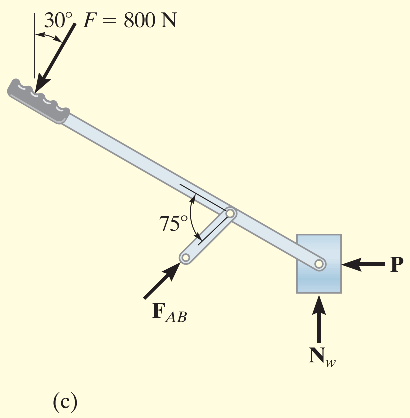


Fig. 6–25

SOLUTION

By inspection, member AB is a two-force member. The free-body diagrams of the three parts are shown in Fig. 6–25*b*. Since the pins at B and D *connect only two parts together*, the forces there are shown as equal but opposite on the separate free-body diagrams of their connected members. In particular, four components of force act on the piston: $\mathbf{D}_x$ and $\mathbf{D}_y$ represent the effect of the pin (or lever EBD), $\mathbf{N}_w$ is the *resultant force* of the wall support, and $\mathbf{P}$ is the resultant compressive force caused by the can C. The directional sense of each of the unknown forces is assumed, and the correct sense will be established after the equations of equilibrium are applied.

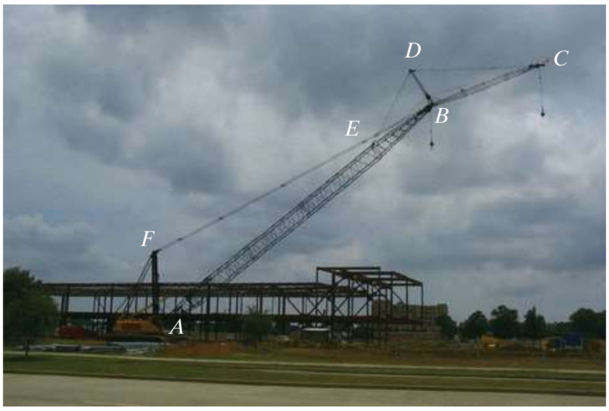


NOTE: A free-body diagram of the entire assembly is shown in Fig. 6–25*c*. Here the forces between the components are internal and are not shown on the free-body diagram.

Before proceeding, it is highly recommended that you cover the solutions to the previous examples and attempt to draw the requested free-body diagrams. When doing so, make sure the work is neat and that all the forces and couple moments are properly labeled. When finished, challenge yourself and solve the following four problems.

CONCEPTUAL PROBLEMS

P6–1. Draw the free-body diagrams of each of the crane boom segments AB, BC, and BD. Only the weights of AB and BC are significant. Assume A and B are pins.



P6–1

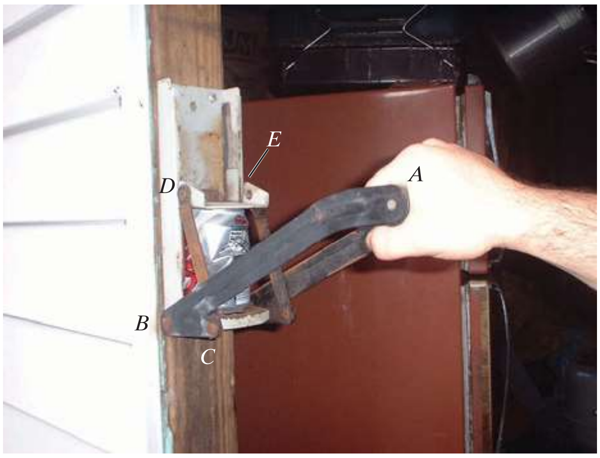
P6–2. Draw the free-body diagrams of the boom $ABCD$ and the stick $EDFGH$ of the backhoe. The weights of these two members are significant. Neglect the weights of all the other members, and assume all indicated points of connection are pins.

P6–2

P6–3. Draw the free-body diagrams of the boom $ABCDF$ and the stick FGH of the bucket lift. Neglect the weights of the members. The bucket weighs W. The two–force members are BI, CE, DE and GE. Assume all indicated points of connection are pins.

P6–3

P6–4. To operate the can crusher one pushes down on the lever arm ABC which rotates about the fixed pin at B. This moves the side links CD downward, which causes the guide plate E to also move downward and thereby crush the can. Draw the free-body diagrams of the lever, side link, and guide plate. Make up some reasonable numbers and do an equilibrium analysis to show how much an applied vertical force at the handle is magnified when it is transmitted to the can. Assume all points of connection are pins and the guides for the plate are smooth.



P6–4

Procedure for Analysis

The joint reactions on frames or machines (structures) composed of multiforce members can be determined using the following procedure.

Free-Body Diagram.

- Draw the free-body diagram of the entire frame or machine, a portion of it, or each of its members. The choice should be made so that it leads to the most direct solution of the problem.
- When the free-body diagram of a group of members of a frame or machine is drawn, the forces between the connected parts of this group are internal forces and are not shown on the free-body diagram of the group.
- Forces common to two members which are in contact act with equal magnitude but opposite sense on the respective free-body diagrams of the members.
- Two-force members, regardless of their shape, have equal but opposite collinear forces acting at the ends of the member.
- In many cases it is possible to tell by inspection the proper sense of the unknown forces acting on a member; however, if this seems difficult, the sense can be assumed.
- Remember that a couple moment is a free vector and can act at any point on the free-body diagram. Also, a force is a sliding vector and can act at any point along its line of action.

Equations of Equilibrium.

- Count the number of unknowns and compare it to the total number of equilibrium equations that are available. In two dimensions, there are three equilibrium equations that can be written for each member.
- Sum moments about a point that lies at the intersection of the lines of action of as many of the unknown forces as possible.
- If the solution of a force or couple moment magnitude is found to be negative, it means the sense of the force is the reverse of that shown on the free-body diagram.

EXAMPLE 6.14

Determine the tension in the cables and also the force **P** required to support the 600-N force using the frictionless pulley system shown in Fig. 6–26*a*.

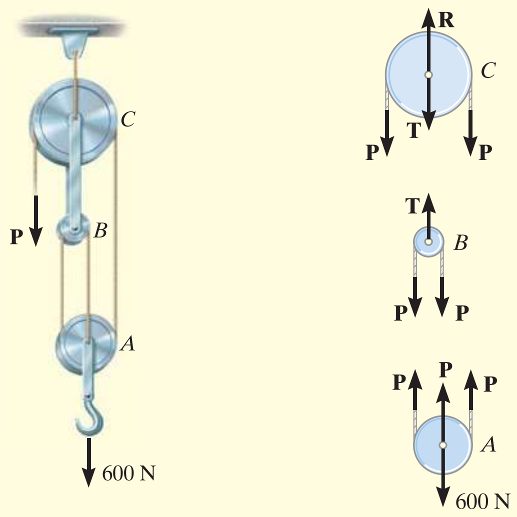


Fig. 6–26

SOLUTION

Free-Body Diagram. A free-body diagram of each pulley *including* its pin and a portion of the contacting cable is shown in Fig. 6–26*b*. Since the cable is *continuous*, it has a *constant tension* P acting throughout its length. The link connection between pulleys B and C is a two-force member, and therefore it has an unknown tension T acting on it. Notice that the *principle of action, equal but opposite reaction* must be carefully observed for forces **P** and **T** when the *separate* free-body diagrams are drawn.

Equations of Equilibrium. The three unknowns are obtained as follows:

Pulley A

$+\uparrow \Sigma F_y = 0;$ $\quad 3P - 600\text{ N} = 0 \quad P = 200\text{ N}$ *Ans.*

Pulley B

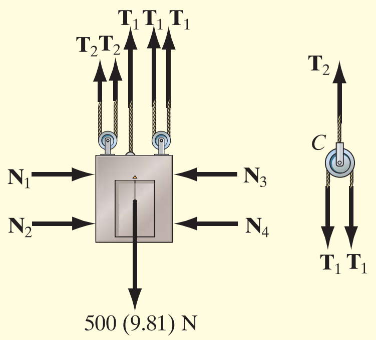
$+\uparrow \Sigma F_y = 0;$ $\quad T - 2P = 0 \quad T = 400\text{ N}$ *Ans.*

Pulley C

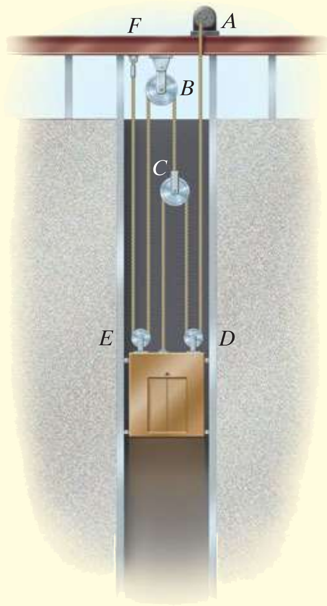
$+\uparrow \Sigma F_y = 0;$ $\quad R - 2P - T = 0 \quad R = 800\text{ N}$ *Ans.*

EXAMPLE 6.15

A 500-kg elevator car in Fig. 6–27*a* is being hoisted by motor *A* using the pulley system shown. If the car is traveling with a constant speed, determine the force developed in the two cables. Neglect the mass of the cable and pulleys.



(b)



(a)

Fig. 6–27

6

SOLUTION

Free-Body Diagram. We can solve this problem using the free-body diagrams of the elevator car and pulley *C*, Fig. 6–27*b*. The tensile forces developed in the cables are denoted as T_1 and T_2.

Equations of Equilibrium. For pulley *C*,

$$+\uparrow \Sigma F_y = 0; \qquad T_2 - 2T_1 = 0 \qquad \text{or} \qquad T_2 = 2T_1 \qquad (1)$$

For the elevator car,

$$+\uparrow \Sigma F_y = 0; \qquad 3T_1 + 2T_2 - 500(9.81)\text{ N} = 0 \qquad (2)$$

Substituting Eq. (1) into Eq. (2) yields

$$3T_1 + 2(2T_1) - 500(9.81)\text{ N} = 0$$

$$T_1 = 700.71\text{ N} = 701\text{ N} \qquad \textit{Ans.}$$

Substituting this result into Eq. (1),

$$T_2 = 2(700.71)\text{ N} = 1401\text{ N} = 1.40\text{ kN} \qquad \textit{Ans.}$$

EXAMPLE 6.16

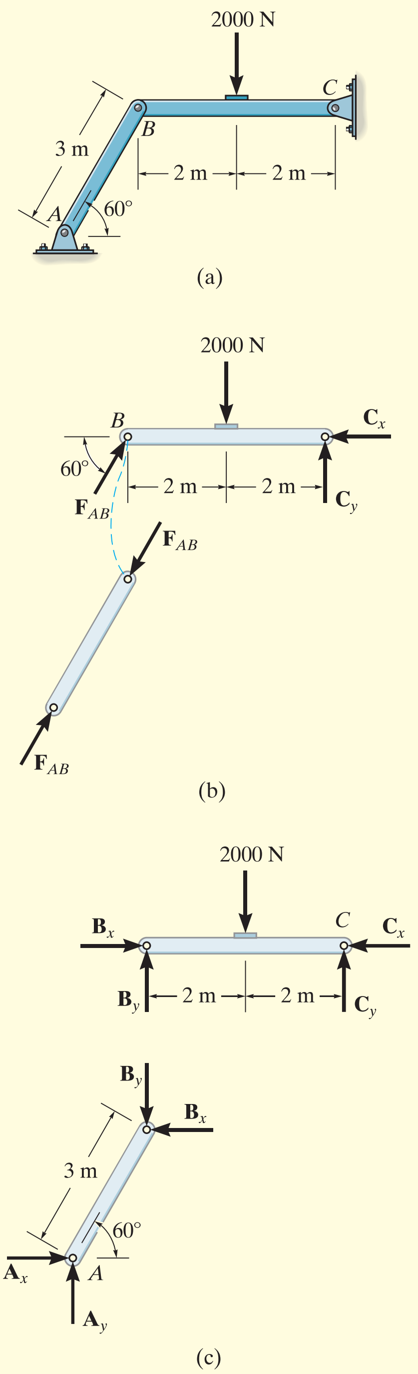


Fig. 6–28

Determine the horizontal and vertical components of force which the pin at C exerts on member BC of the frame in Fig. 6–28*a*.

SOLUTION I

Free-Body Diagrams. By inspection it can be seen that AB is a two-force member. The free-body diagrams are shown in Fig. 6–28*b*.

Equations of Equilibrium. The *three unknowns* can be determined by applying the three equations of equilibrium to member CB.

$$\circlearrowleft + \Sigma M_C = 0; \quad 2000\text{ N}(2\text{ m}) - (F_{AB}\sin 60°)(4\text{ m}) = 0 \quad F_{AB} = 1154.7\text{ N}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 1154.7\cos 60°\text{ N} - C_x = 0 \quad C_x = 577\text{ N} \qquad \textit{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \quad 1154.7\sin 60°\text{ N} - 2000\text{ N} + C_y = 0$$

$$C_y = 1000\text{ N} \qquad \textit{Ans.}$$

SOLUTION II

Free-Body Diagrams. If one does not recognize that AB is a two-force member, then more work is involved in solving this problem. The free-body diagrams are shown in Fig. 6–28*c*.

Equations of Equilibrium. The *six unknowns* are determined by applying the three equations of equilibrium to each member.

Member AB

$$\circlearrowleft + \Sigma M_A = 0; \quad B_x(3\sin 60°\text{ m}) - B_y(3\cos 60°\text{ m}) = 0 \qquad (1)$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad A_x - B_x = 0 \qquad (2)$$

$$+\uparrow \Sigma F_y = 0; \quad A_y - B_y = 0 \qquad (3)$$

Member BC

$$\circlearrowleft + \Sigma M_C = 0; \quad 2000\text{ N}(2\text{ m}) - B_y(4\text{ m}) = 0 \qquad (4)$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad B_x - C_x = 0 \qquad (5)$$

$$+\uparrow \Sigma F_y = 0; \quad B_y - 2000\text{ N} + C_y = 0 \qquad (6)$$

The results for C_x and C_y can be determined by solving these equations in the following sequence: 4, 1, 5, then 6. The results are

$$B_y = 1000\text{ N}$$

$$B_x = 577\text{ N}$$

$$C_x = 577\text{ N} \qquad \textit{Ans.}$$

$$C_y = 1000\text{ N} \qquad \textit{Ans.}$$

By comparison, Solution I is simpler since the requirement that F_{AB} in Fig. 6–28*b* be equal, opposite, and collinear at the ends of member AB automatically satisfies Eqs. 1, 2, and 3 above and therefore eliminates the need to write these equations. ***As a result, save yourself some time and effort by always identifying the two-force members before starting the analysis!***

6

EXAMPLE 6.17

The compound beam shown in Fig. 6–29*a* is pin connected at *B*. Determine the components of reaction at its supports. Neglect its weight and thickness.

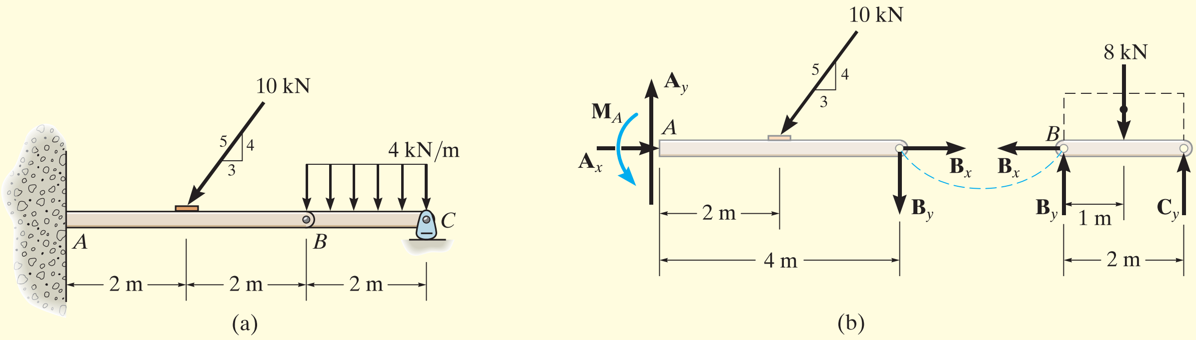


Fig. 6–29

SOLUTION

Free-Body Diagrams. By inspection, if we consider a free-body diagram of the *entire beam ABC*, there will be three unknown reactions at *A* and one at *C*. These four unknowns cannot all be obtained from the three available equations of equilibrium, and so for the solution it will become necessary to dismember the beam into its two segments, as shown in Fig. 6–29*b*.

Equations of Equilibrium. The six unknowns are determined as follows:

Segment BC

$$\overset{+}{\leftarrow} \Sigma F_x = 0; \qquad B_x = 0$$

$$\circlearrowleft + \Sigma M_B = 0; \qquad -8\text{ kN}(1\text{ m}) + C_y(2\text{ m}) = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad B_y - 8\text{ kN} + C_y = 0$$

Segment AB

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad A_x - (10\text{ kN})\left(\tfrac{3}{5}\right) + B_x = 0$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad M_A - (10\text{ kN})\left(\tfrac{4}{5}\right)(2\text{ m}) - B_y(4\text{ m}) = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y - (10\text{ kN})\left(\tfrac{4}{5}\right) - B_y = 0$$

Solving each of these equations successively, using previously calculated results, we obtain

$A_x = 6\text{ kN}$ $\quad A_y = 12\text{ kN}$ $\quad M_A = 32\text{ kN}\cdot\text{m}$ *Ans.*

$B_x = 0$ $\quad B_y = 4\text{ kN}$

$C_y = 4\text{ kN}$ *Ans.*

6

EXAMPLE 6.18

The two planks in Fig. 6–30*a* are connected together by cable *BC* and a smooth spacer *DE*. Determine the reactions at the smooth supports *A* and *F*, and also find the force developed in the cable and spacer.

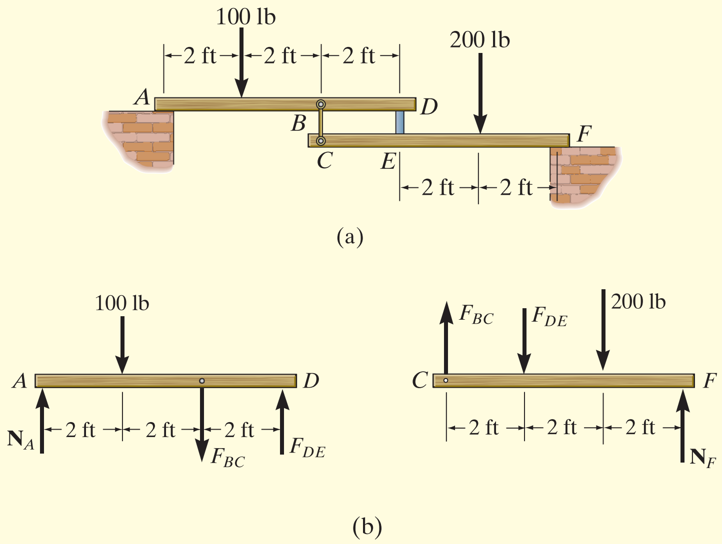


Fig. 6–30

6

SOLUTION

Free-Body Diagrams. The free-body diagram of each plank is shown in Fig. 6–30*b*. It is important to apply Newton's third law to the interaction forces F_{BC} and F_{DE} as shown.

Equations of Equilibrium. For plank *AD*,

$$\circlearrowleft + \Sigma M_A = 0; \qquad F_{DE}(6\text{ ft}) - F_{BC}(4\text{ ft}) - 100\text{ lb}\,(2\text{ ft}) = 0$$

For plank *CF*,

$$\circlearrowleft + \Sigma M_F = 0; \qquad F_{DE}(4\text{ ft}) - F_{BC}(6\text{ ft}) + 200\text{ lb}\,(2\text{ ft}) = 0$$

Solving simultaneously,

$$F_{DE} = 140\text{ lb} \qquad F_{BC} = 160\text{ lb} \qquad \textit{Ans.}$$

Using these results, for plank *AD*,

$$+\uparrow \Sigma F_y = 0; \qquad N_A + 140\text{ lb} - 160\text{ lb} - 100\text{ lb} = 0$$

$$N_A = 120\text{ lb} \qquad \textit{Ans.}$$

And for plank *CF*,

$$+\uparrow \Sigma F_y = 0; \qquad N_F + 160\text{ lb} - 140\text{ lb} - 200\text{ lb} = 0$$

$$N_F = 180\text{ lb} \qquad \textit{Ans.}$$

NOTE: Draw the free-body diagram of the system of *both* planks and apply $\Sigma M_A = 0$ to determine N_F. Then use the free-body diagram of *CEF* to determine F_{DE} and F_{BC}.

EXAMPLE 6.19

The 75-kg man in Fig. 6–31*a* attempts to lift the 40-kg uniform beam off the roller support at *B*. Determine the tension developed in the cable attached to *B* and the normal reaction of the man on the beam when this is about to occur.

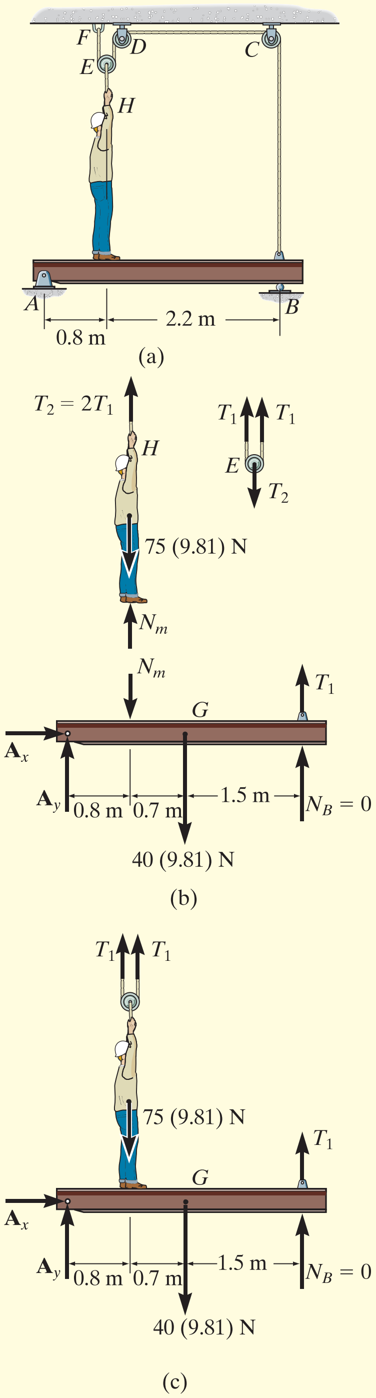


Fig. 6–31

SOLUTION

Free-Body Diagrams. The tensile force in the cable will be denoted as T_1. The free-body diagrams of the pulley *E*, the man, and the beam are shown in Fig. 6–31*b*. Since the man must lift the beam off the roller *B* then $N_B = 0$. When drawing each of these diagrams, it is very important to apply Newton's third law.

Equations of Equilibrium. Using the free-body diagram of pulley *E*,

$$+\uparrow \Sigma F_y = 0; \qquad 2T_1 - T_2 = 0 \quad \text{or} \quad T_2 = 2T_1 \qquad (1)$$

Referring to the free-body diagram of the man using this result,

$$+\uparrow \Sigma F_y = 0 \qquad N_m + 2T_1 - 75(9.81)\text{ N} = 0 \qquad (2)$$

Summing moments about point *A* on the beam,

$$\circlearrowleft + \Sigma M_A = 0; \quad T_1(3\text{ m}) - N_m(0.8\text{ m}) - [40(9.81)\text{ N}]\,(1.5\text{ m}) = 0 \qquad (3)$$

Solving Eqs. 2 and 3 simultaneously for T_1 and N_m, then using Eq. (1) for T_2, we obtain

$$T_1 = 256\text{ N} \qquad N_m = 224\text{ N} \qquad T_2 = 512\text{ N}$$ *Ans.*

SOLUTION II

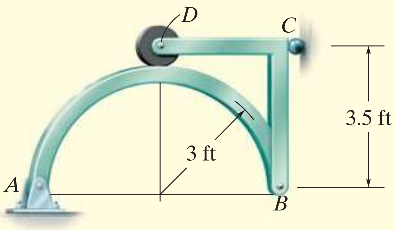
A direct solution for T_1 can be obtained by considering the beam, the man, and pulley *E* as a *single system*. The free-body diagram is shown in Fig. 6–31*c*. Thus,

$$\circlearrowleft + \Sigma M_A = 0; \qquad 2T_1(0.8\text{ m}) - [75(9.81)\text{ N}](0.8\text{ m})$$
$$- [40(9.81)\text{ N}](1.5\text{ m}) + T_1(3\text{ m}) = 0$$
$$T_1 = 256\text{ N}$$ *Ans.*

With this result Eqs. 1 and 2 can then be used to find N_m and T_2.

EXAMPLE 6.20

The smooth disk shown in Fig. 6–32*a* is pinned at *D* and has a weight of 20 lb. Neglecting the weights of the other members, determine the horizontal and vertical components of reaction at pins *B* and *D*.



(a)

SOLUTION

Free-Body Diagrams. The free-body diagrams of the entire frame and each of its members are shown in Fig. 6–32*b*.

Equations of Equilibrium. The eight unknowns can of course be obtained by applying the eight equilibrium equations to each member—three to member *AB*, three to member *BCD*, and two to the disk. (Moment equilibrium is automatically satisfied for the disk.) If this is done, however, all the results can be obtained only from a simultaneous solution of some of the equations. (Try it and find out.) To avoid this situation, it is best first to determine the three support reactions on the *entire* frame; then, using these results, the remaining five equilibrium equations can be applied to two other parts in order to solve successively for the other unknowns.

Entire Frame

$\circlearrowleft + \Sigma M_A = 0; \quad -20\text{ lb}(3\text{ ft}) + C_x(3.5\text{ ft}) = 0 \qquad C_x = 17.1\text{ lb}$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad A_x - 17.1\text{ lb} = 0 \qquad A_x = 17.1\text{ lb}$

$+\uparrow \Sigma F_y = 0; \quad A_y - 20\text{ lb} = 0 \qquad A_y = 20\text{ lb}$

Member AB

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 17.1\text{ lb} - B_x = 0 \qquad B_x = 17.1\text{ lb}$ *Ans.*

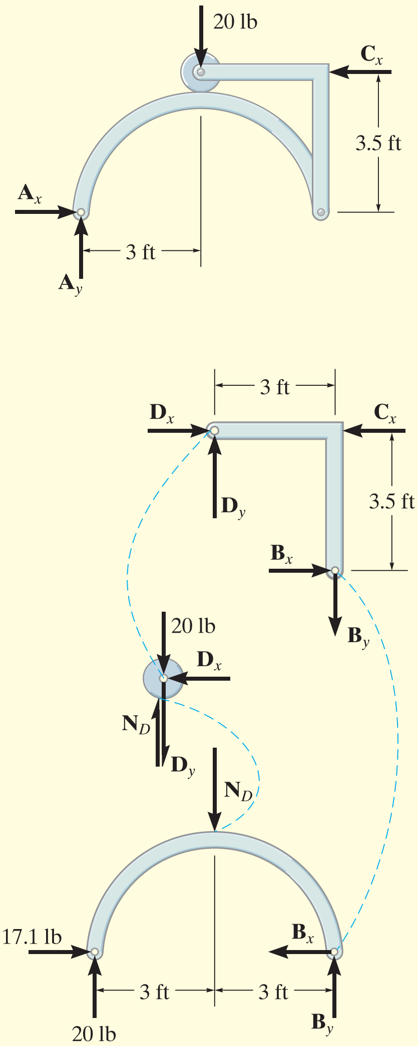
$\circlearrowleft + \Sigma M_B = 0; \quad -20\text{ lb}(6\text{ ft}) + N_D(3\text{ ft}) = 0 \qquad N_D = 40\text{ lb}$

$+\uparrow \Sigma F_y = 0; \quad 20\text{ lb} - 40\text{ lb} + B_y = 0 \qquad B_y = 20\text{ lb}$ *Ans.*

Disk

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad D_x = 0$ *Ans.*

$+\uparrow \Sigma F_y = 0; \quad 40\text{ lb} - 20\text{ lb} - D_y = 0 \qquad D_y = 20\text{ lb}$ *Ans.*



(b)

Fig. 6–32

EXAMPLE 6.21

The frame in Fig. 6–33*a* supports the 50-kg cylinder. Determine the horizontal and vertical components of reaction at *A* and the force at *C*.

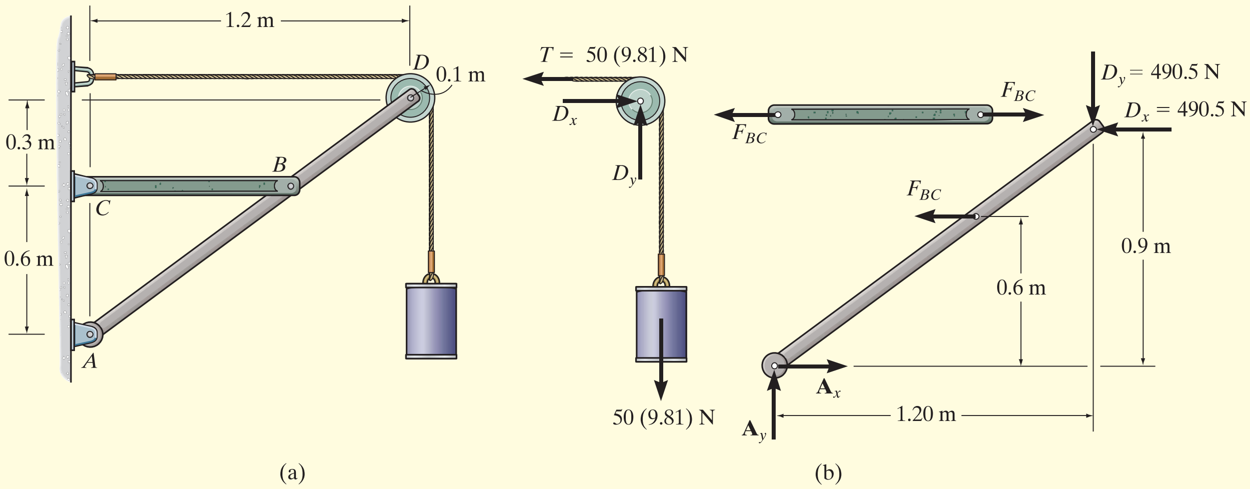


Fig. 6–33

SOLUTION

Free-Body Diagrams. The free-body diagram of pulley *D*, along with the cylinder and a portion of the cord (a system), is shown in Fig. 6–33*b*. Member *BC* is a two-force member as indicated by its free-body diagram. The free-body diagram of member *ABD* is also shown.

Equations of Equilibrium. We will begin by analyzing the equilibrium of the pulley. The moment equation of equilibrium is automatically satisfied with $T = 50(9.81)$ N, and so

$$\stackrel{+}{\rightarrow}\Sigma F_x = 0; \qquad D_x - 50(9.81)\text{ N} = 0 \quad D_x = 490.5\text{ N}$$

$$+\uparrow\Sigma F_y = 0; \qquad D_y - 50(9.81)\text{ N} = 0 \quad D_y = 490.5\text{ N} \qquad \textit{Ans.}$$

Using these results, F_{BC} can be determined by summing moments about point *A* on member *ABD*.

$$\circlearrowleft +\Sigma M_A = 0;\ F_{BC}(0.6\text{ m}) + 490.5\text{ N}(0.9\text{ m}) - 490.5\text{ N}(1.20\text{ m}) = 0$$

$$F_{BC} = 245.25\text{ N} \qquad \textit{Ans.}$$

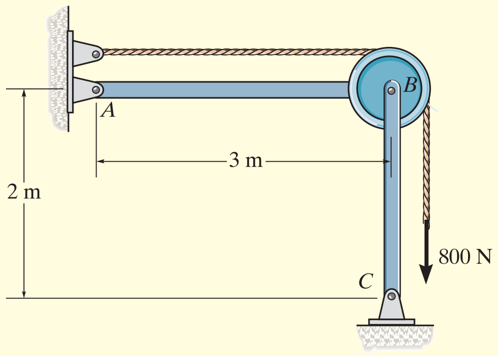
Now A_x and A_y can be determined by summing forces.

$$\stackrel{+}{\rightarrow}\Sigma F_x = 0; \quad A_x - 245.25\text{ N} - 490.5\text{ N} = 0 \quad A_x = 736\text{ N} \qquad \textit{Ans.}$$

$$+\uparrow\Sigma F_y = 0; \qquad A_y - 490.5\text{ N} = 0 \quad A_y = 490.5\text{ N} \qquad \textit{Ans.}$$

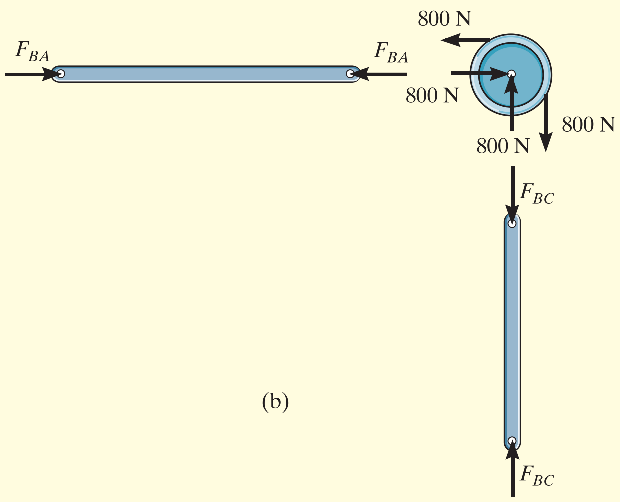
EXAMPLE 6.22

Determine the force the pins at A and B exert on the two-member frame shown in Fig. 6–34*a*.

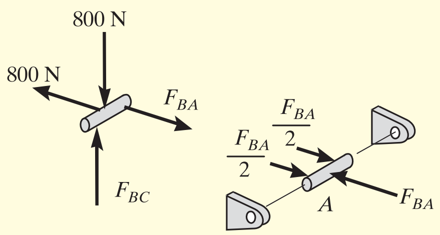


(a)

SOLUTION I



(b)



(c) (d)

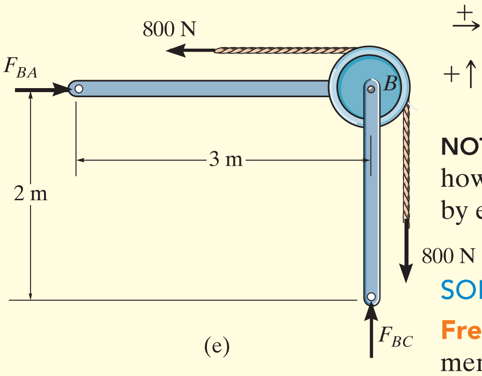
Free-Body Diagrams. By inspection AB and BC are two-force members. Their free-body diagrams, along with that of the pulley, are shown in Fig. 6–34*b*. In order to solve this problem we must also include the free-body diagram of the pin at B because this pin connects all *three members* together, Fig. 6–34*c*.

Equations of Equilibrium: Apply the equations of force equilibrium to pin B.

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad F_{BA} - 800 \text{ N} = 0; \quad F_{BA} = 800 \text{ N} \quad \textit{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \quad F_{BC} - 800 \text{ N} = 0; \quad F_{BC} = 800 \text{ N} \quad \textit{Ans.}$$

NOTE: The free-body diagram of the pin at A, Fig. 6–34*d*, indicates how the force F_{AB} is balanced by the force $(F_{AB}/2)$ exerted on the pin by each of the two pin leaves.



(e)

Fig. 6–34

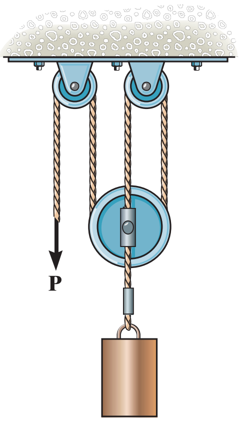
SOLUTION II

Free-Body Diagram. If we realize that AB and BC are two-force members, then the free-body diagram of the *entire frame* produces an easier solution, Fig. 6–34*e*. The force equations of equilibrium are the same as those above. Note that moment equilibrium will be satisfied, regardless of the radius of the pulley.

FUNDAMENTAL PROBLEMS

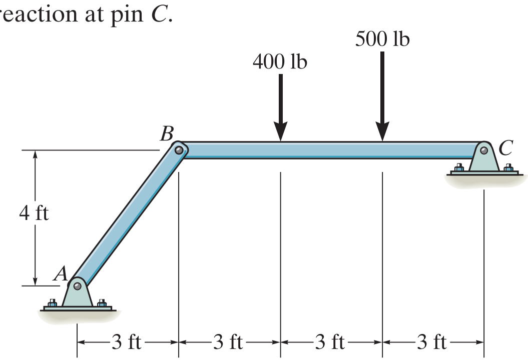
All problem solutions must include FBDs.

F6–13. Determine the force P needed to hold the 60-lb weight in equilibrium.



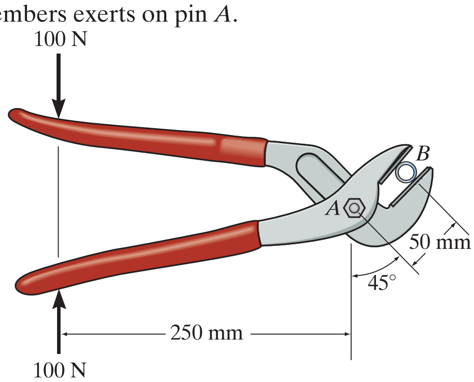
F6–13

F6–14. Determine the horizontal and vertical components of reaction at pin C.



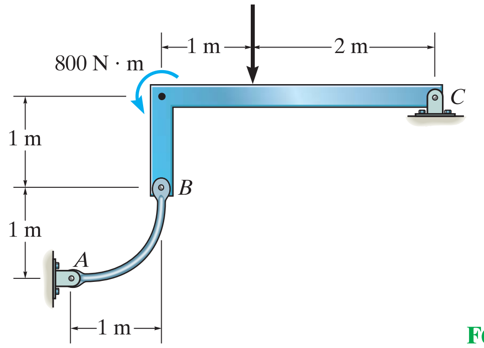
F6–14

F6–15. If a 100-N force is applied to the handles of the pliers, determine the clamping force exerted on the smooth pipe B and the magnitude of the resultant force that one of the members exerts on pin A.



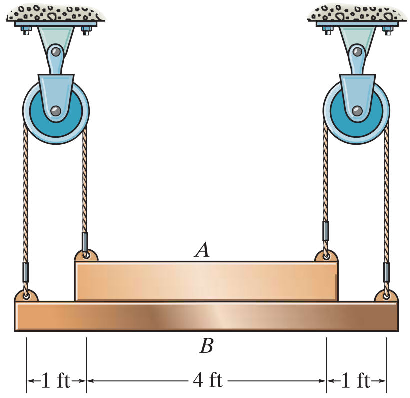
F6–15

F6–16. Determine the horizontal and vertical components of reaction at pin C.



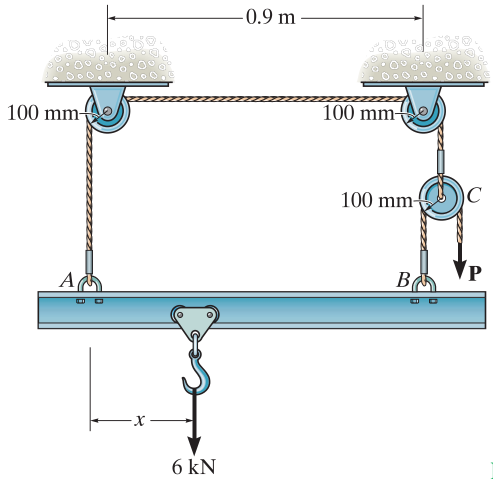
F6–16

F6–17. Determine the normal force that the 100-lb plate A exerts on the 30-lb plate B.



F6–17

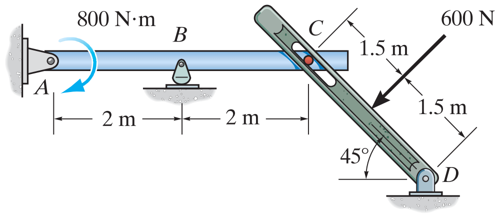
F6–18. Determine the force P needed to lift the load. Also, determine the proper placement x of the hook for equilibrium. Neglect the weight of the beam.



F6–18

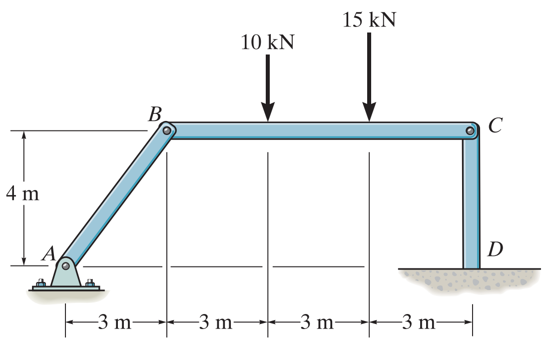
6

F6–19. Determine the components of reaction at A and B.



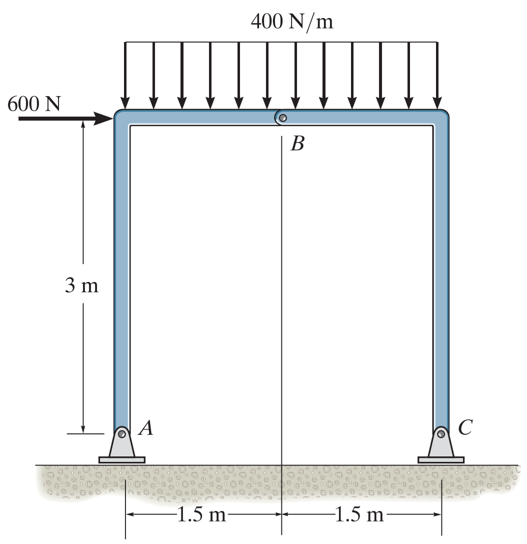
F6–19

F6–20. Determine the components of reaction at D.



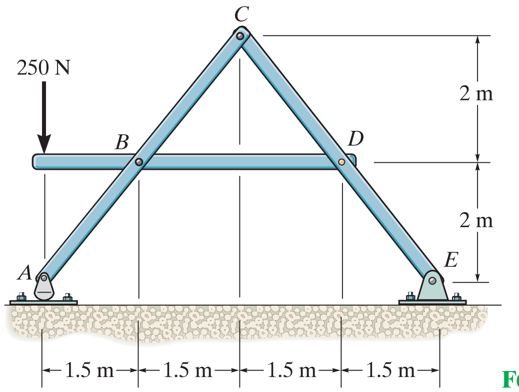
F6–20

F6–21. Determine the components of reaction at A and C.



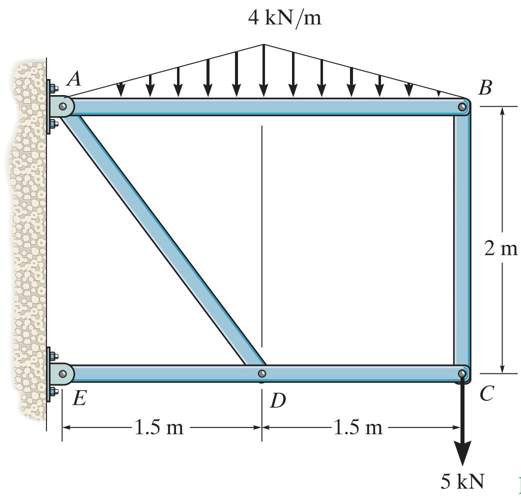
F6–21

F6–22. Determine the components of reaction at C.



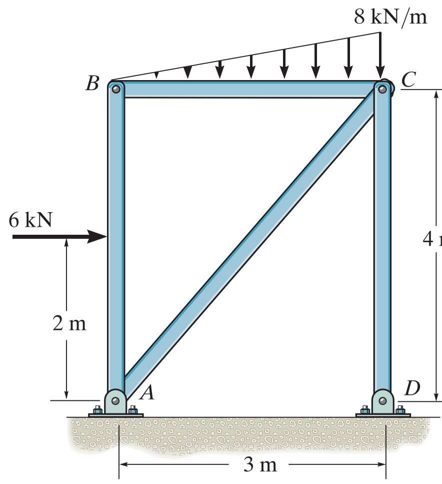
F6–22

F6–23. Determine the components of reaction at E.



F6–23

F6–24. Determine the components of reaction at D and the components of reaction the pin at A exerts on member BA.

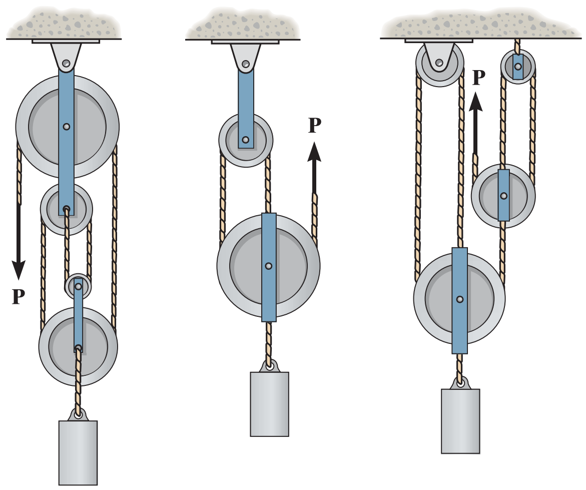


F6–24

PROBLEMS

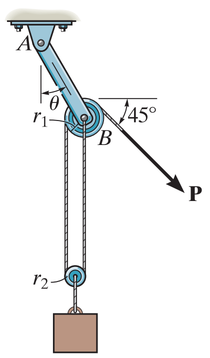
All problem solutions must include FBDs.

6–61. In each case, determine the force **P** required to maintain equilibrium. The block weighs 100 lb.



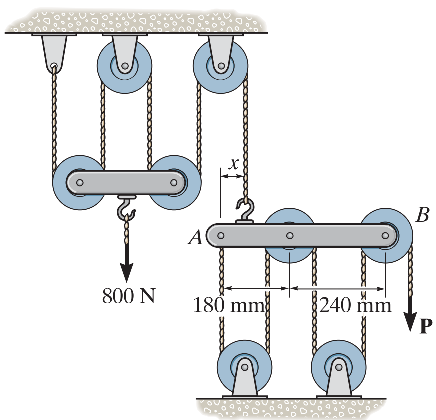
Prob. 6–61

6–62. Determine the force P on the cord, and the angle θ that the pulley-supporting link AB makes with the vertical. Neglect the mass of the pulleys and the link. The block has a weight of 200 lb and the cord is attached to the pin at B. The pulleys have radii of $r_1 = 2$ in. and $r_2 = 1$ in.



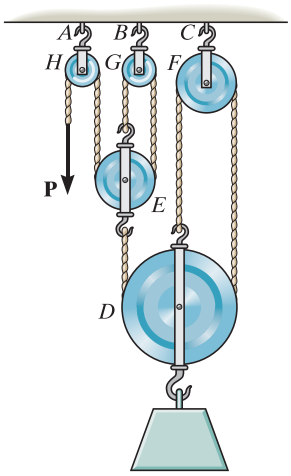
Prob. 6–62

6–63. The principles of a *differential chain block* are indicated schematically in the figure. Determine the magnitude of force **P** needed to support the 800-N force. Also, find the distance x where the cable must be attached to bar AB so the bar remains horizontal. All pulleys have a radius of 60 mm.



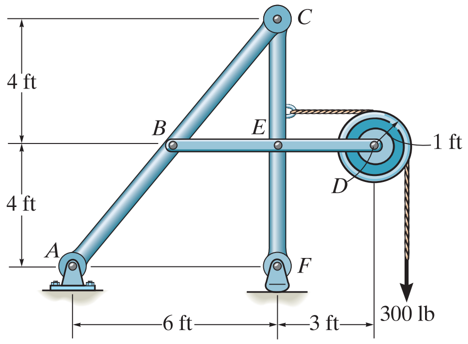
Prob. 6–63

***6–64.** Determine the force P needed to support the 20-kg mass using the *Spanish Burton rig*. Also, what are the reactions at the supporting hooks A, B, and C?



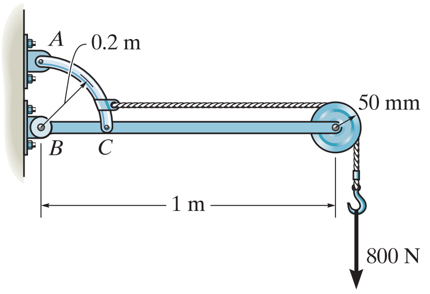
Prob. 6–64

6–65. Determine the horizontal and vertical components of force at C which member ABC exerts on member CEF.



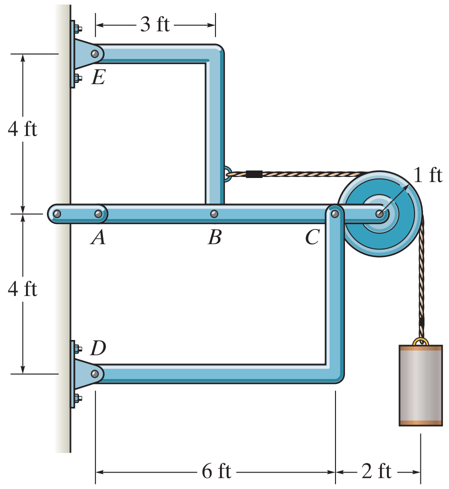
Prob. 6–65

6–66. Determine the horizontal and vertical components of force that the pins at A, B, and C exert on their connecting members.



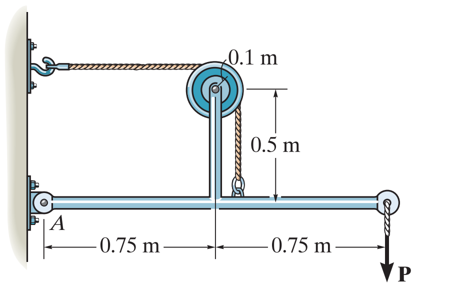
Prob. 6–66

6–67. Determine the horizontal and vertical components of force at pins D and E, and the force on the short link at A. The suspended cylinder has a weight of 80 lb.



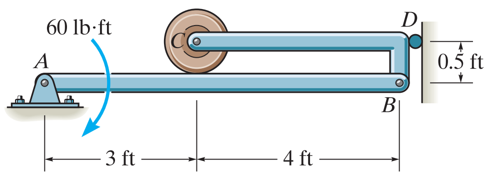
Prob. 6–67

***6–68.** Determine the greatest force P that can be applied to the frame if the largest force resultant acting at A can have a magnitude of 2 kN.



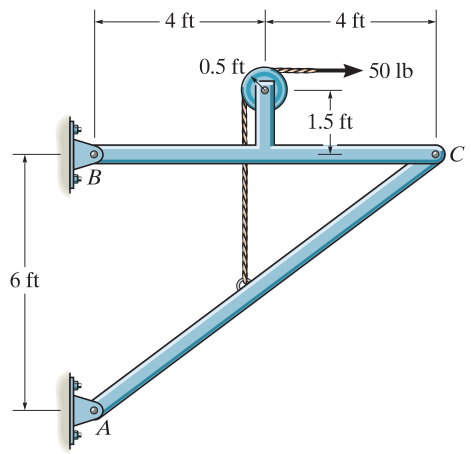
Prob. 6–68

6–69. Determine the force that the smooth roller C exerts on member AB. Also, what are the horizontal and vertical components of reaction at pin A? Neglect the weight of the frame and roller.



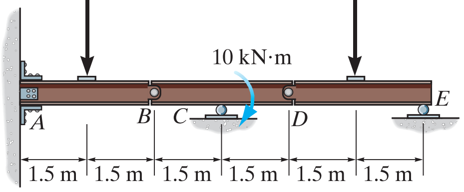
Prob. 6–69

6–70. Determine the horizontal and vertical components of force at pins B and C.



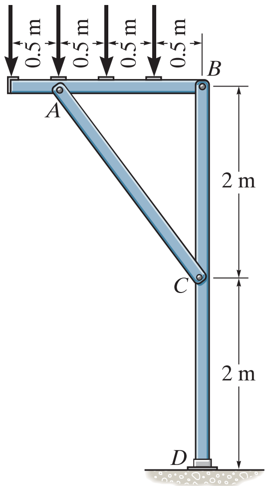
Prob. 6–70

6–71. Determine the support reactions at A, C, and E on the compound beam which is pin connected at B and D.



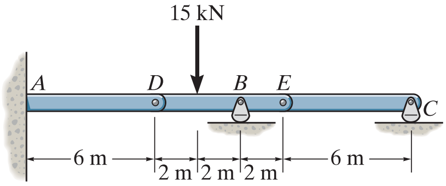
Prob. 6–71

***6–72.** Determine the horizontal and vertical components of force at pins A, B, and C, and the reactions at the fixed support D of the three-member frame.



Prob. 6–72

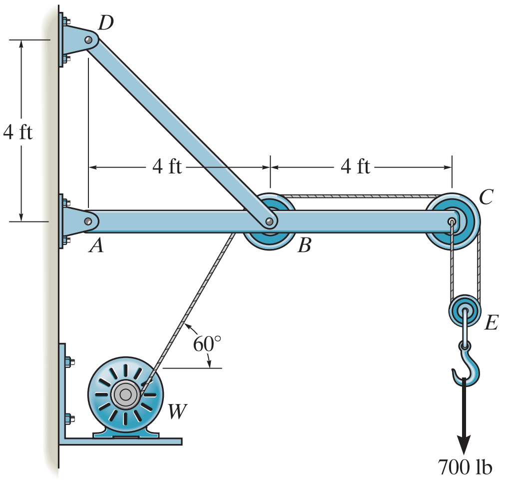
6–73. The compound beam is fixed at A and supported by a rocker at B and C. There are hinges (pins) at D and E. Determine the reactions at the supports.



Prob. 6–73

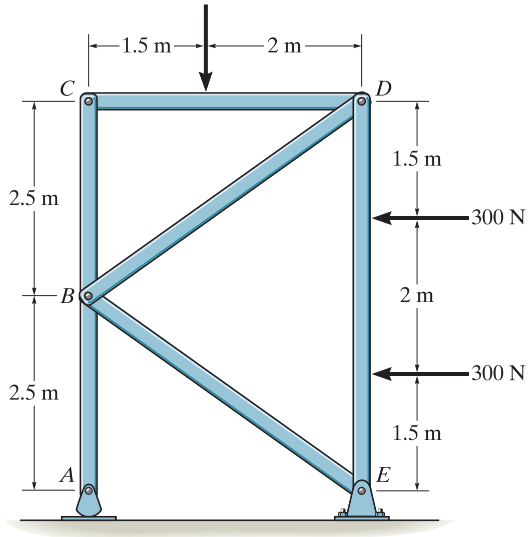
6–74. The wall crane supports a load of 700 lb. Determine the horizontal and vertical components of reaction at the pins A and D. Also, what is the force in the cable at the winch W?

6–75. The wall crane supports a load of 700 lb. Determine the horizontal and vertical components of reaction at the pins A and D. Also, what is the force in the cable at the winch W? The jib ABC has a weight of 100 lb and member BD has a weight of 40 lb. Each member is uniform and has a center of gravity at its center.



Probs. 6–74/75

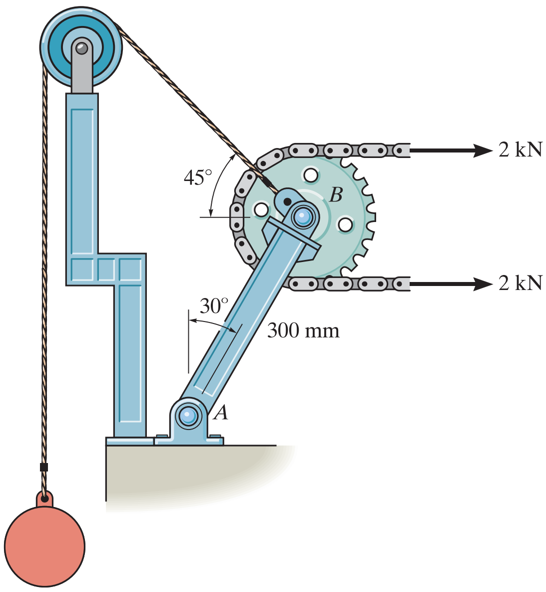
***6–76.** Determine the horizontal and vertical components of force which the pins at A, B, and C exert on member ABC of the frame.



Prob. 6–76

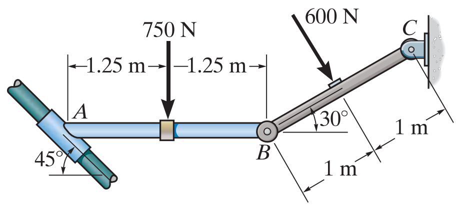
6

6–77. Determine the required mass of the suspended cylinder if the tension in the chain wrapped around the freely turning gear is to be 2 kN. Also, what is the magnitude of the resultant force on pin *A*?



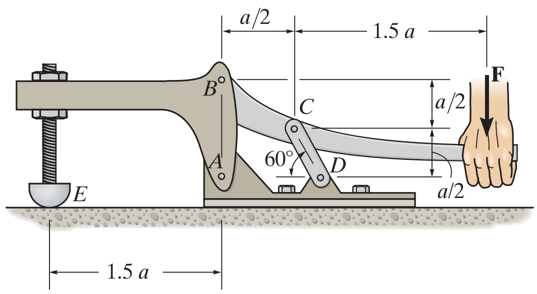
Prob. 6–77

6–78. Determine the reactions on the collar at *A* and the pin at *C*. The collar fits over a smooth rod, and rod *AB* is fixed connected to the collar.



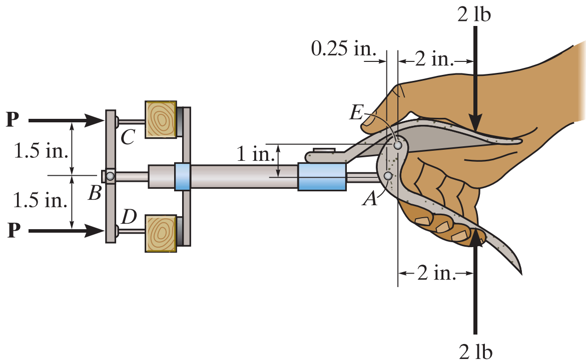
Prob. 6–78

6–79. The toggle clamp is subjected to a force **F** at the handle. Determine the vertical clamping force acting at *E*.



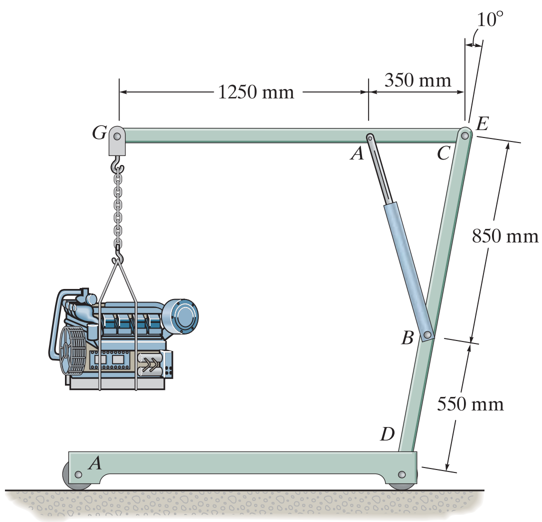
Prob. 6–79

***6–80.** When a force of 2 lb is applied to the handles of the brad squeezer, it pulls in the smooth rod *AB*. Determine the force **P** exerted on each of the smooth brads at *C* and *D*.



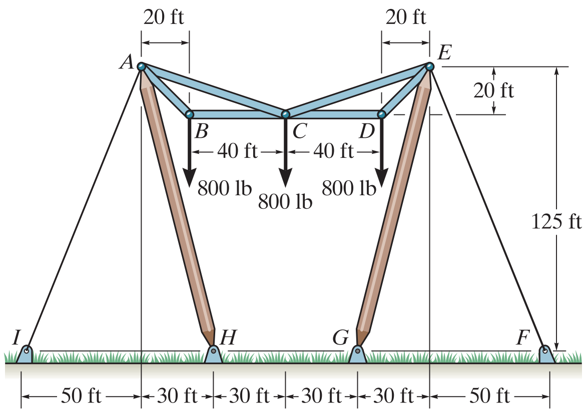
Prob. 6–80

6–81. The engine hoist is used to support the 200-kg engine. Determine the force acting in the hydraulic cylinder *AB*, the horizontal and vertical components of force at the pin *C*, and the reactions at the fixed support *D*.



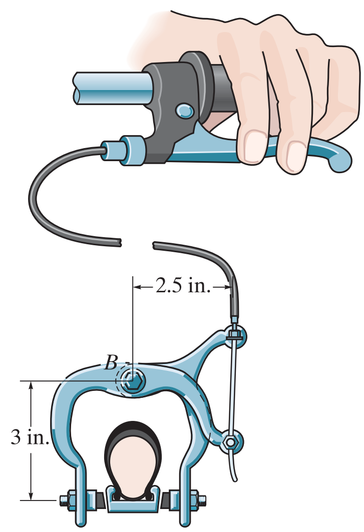
Prob. 6–81

6–82. The three power lines exert the forces shown on the pin-connected members at joints *B*, *C*, and *D*, which in turn are pin connected to the poles *AH* and *EG*. Determine the force in the guy cable *AI* and the pin reaction at the support *H*.



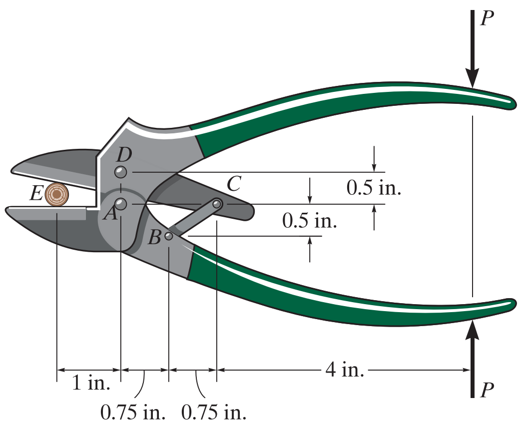
Prob. 6–82

6–83. By squeezing on the hand brake of the bicycle, the rider subjects the brake cable to a tension of 50 lb. If the caliper mechanism is pin connected to the bicycle frame at *B*, determine the normal force each brake pad exerts on the rim of the wheel. Is this the force that stops the wheel from turning? Explain.



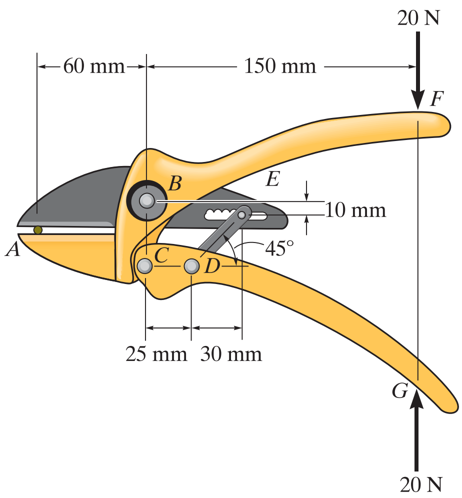
Prob. 6–83

***6–84.** Determine the required force *P* that must be applied at the blade of the pruning shears so that the blade exerts a normal force of 20 lb on the twig at *E*.



Prob. 6–84

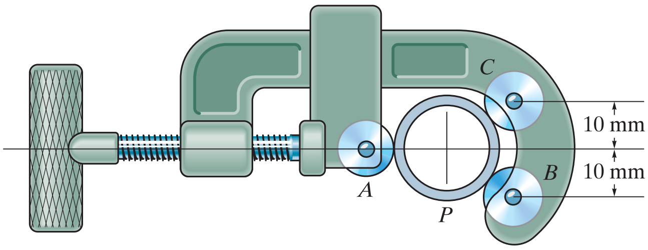
6–85. The pruner multiplies blade-cutting power with the compound leverage mechanism. If a 20-N force is applied to the handles, determine the cutting force generated at *A*. Assume that the contact surface at *A* is smooth.



Prob. 6–85

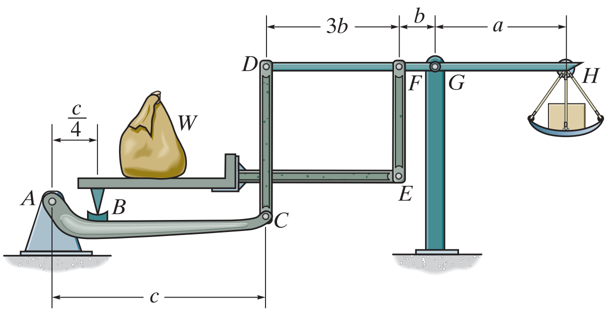
6

6–86. The pipe cutter is clamped around the pipe P. If the wheel at A exerts a normal force of $F_A = 80$ N on the pipe, determine the normal forces of wheels B and C on the pipe. Also compute the pin reaction on the wheel at C. The three wheels each have a radius of 7 mm and the pipe has an outer radius of 10 mm.



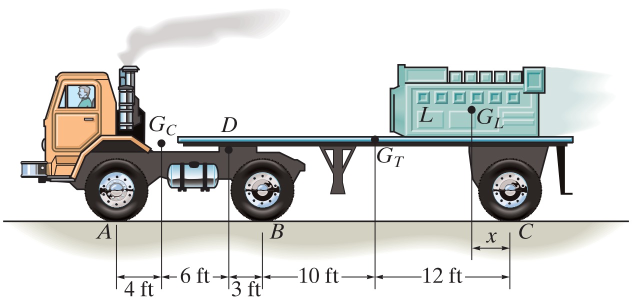
Prob. 6–86

***6–88.** Show that the weight W_1 of the counterweight at H required for equilibrium is $W_1 = (b/a)W$, and so it is independent of the placement of the load W on the platform.



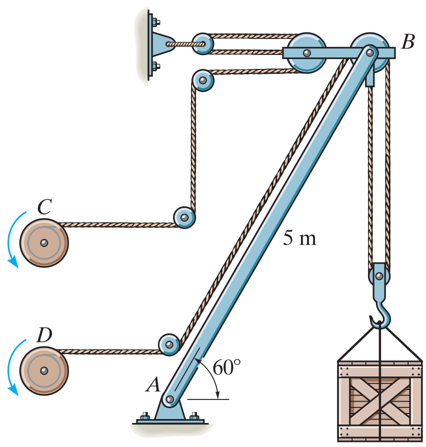
Prob. 6–88

6–87. The flat-bed trailer has a weight of 7000 lb and center of gravity at G_T. It is pin connected to the cab at D. The cab has a weight of 6000 lb and center of gravity at G_C. Determine the range of values x for the position of the 2000-lb load L so that no axle is subjected to more than 5500 lb. The load has a center of gravity at G_L.



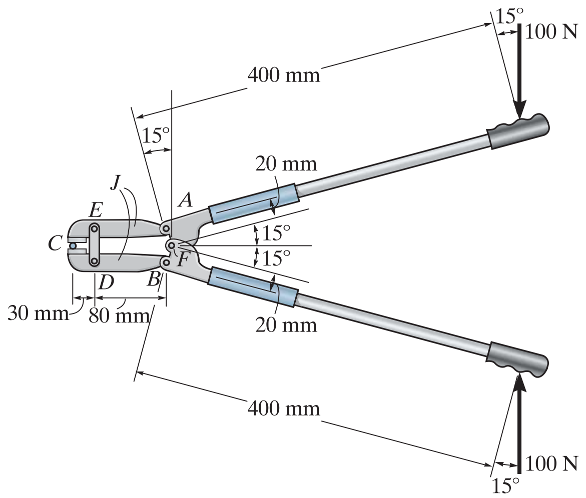
Prob. 6–87

6–89. The derrick is pin connected to the pivot at A. Determine the largest mass that can be supported by the derrick if the maximum force that can be sustained by the pin at A is 18 kN.



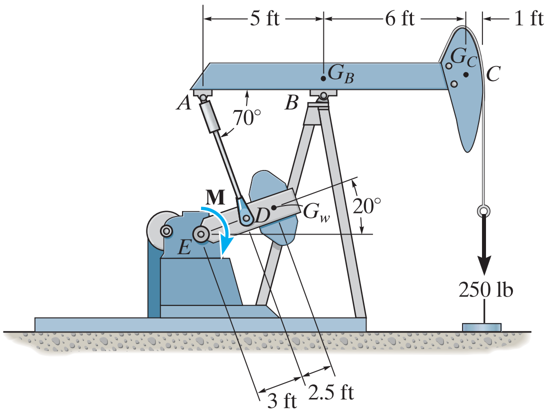
Prob. 6–89

6–90. Determine the force that the jaws J of the metal cutters exert on the smooth cable C if 100-N forces are applied to the handles. The jaws are pinned at E and A, and D and B. There is also a pin at F.



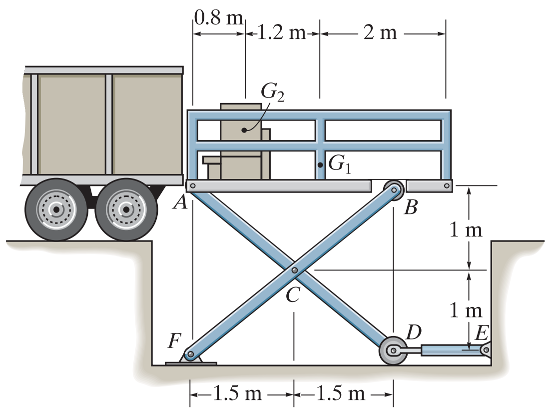
Prob. 6–90

6–91. The pumping unit is used to recover oil. When the walking beam ABC is horizontal, the force acting in the wireline at the well head is 250 lb. Determine the torque **M** which must be exerted by the motor in order to overcome this load. The horse-head C weighs 60 lb and has a center of gravity at G_C. The walking beam ABC has a weight of 130 lb and a center of gravity at G_B, and the counterweight has a weight of 200 lb and a center of gravity at G_W. The pitman, AD, is pin connected at its ends and has negligible weight.



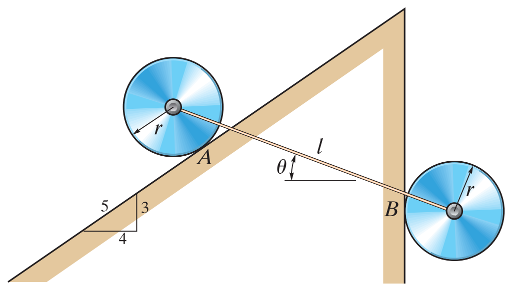
Prob. 6–91

***6–92.** The scissors lift consists of *two* sets of cross members and *two* hydraulic cylinders, DE, symmetrically located on *each side* of the platform. The platform has a uniform mass of 60 kg, with a center of gravity at G_1. The load of 85 kg, with center of gravity at G_2, is centrally located between each side of the platform. Determine the force in each of the hydraulic cylinders for equilibrium. Rollers are located at B and D.



Prob. 6–92

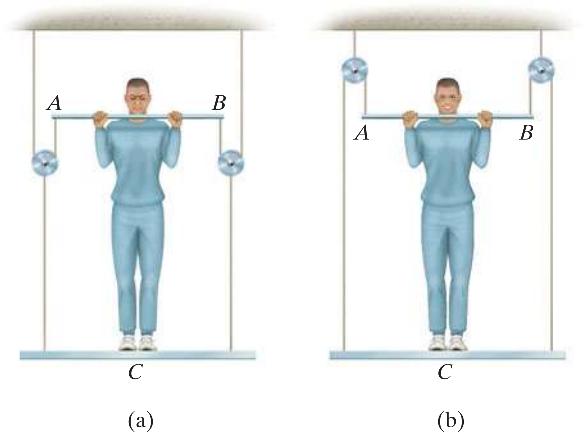
6–93. The two disks each have a mass of 20 kg and are attached at their centers by an elastic cord that has a stiffness of $k = 2\ \text{kN/m}$. Determine the stretch of the cord when the system is in equilibrium and the angle θ of the cord.



Prob. 6–93

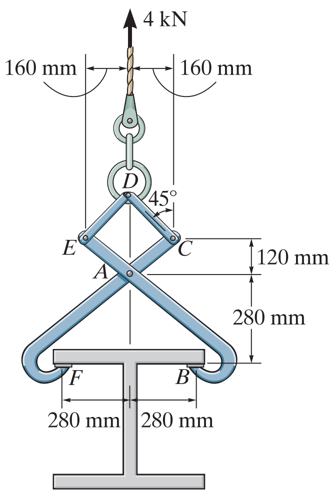
6–94. A man having a weight of 175 lb attempts to hold himself using one of the two methods shown. Determine the total force he must exert on bar AB in each case and the normal reaction he exerts on the platform at C. Neglect the weight of the platform.

6–95. A man having a weight of 175 lb attempts to hold himself using one of the two methods shown. Determine the total force he must exert on bar AB in each case and the normal reaction he exerts on the platform at C. The platform has a weight of 30 lb.



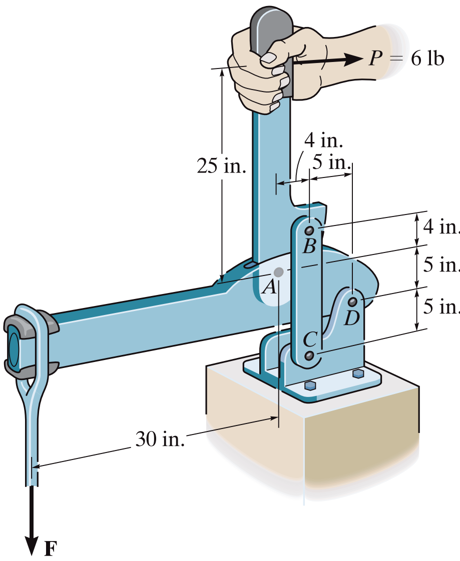
Probs. 6–94/95

*6–96. The double link grip is used to lift the beam. If the beam weighs 4 kN, determine the horizontal and vertical components of force acting on the pin at A and the horizontal and vertical components of force that the flange of the beam exerts on the jaw at B.



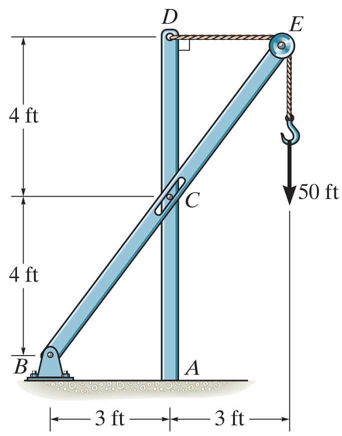
Prob. 6–96

6–97. If a force of $P = 6$ lb is applied perpendicular to the handle of the mechanism, determine the magnitude of force **F** for equilibrium. The members are pin connected at A, B, C, and D.



Prob. 6–97

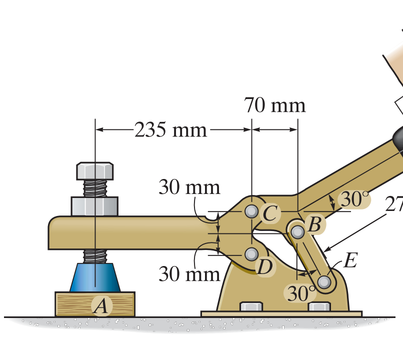
6–98. Determine the horizontal and vertical components of force at pin B and the normal force the pin at C exerts on the smooth slot. Also, determine the moment and horizontal and vertical reactions of force at A. There is a pulley at E.



Prob. 6–98

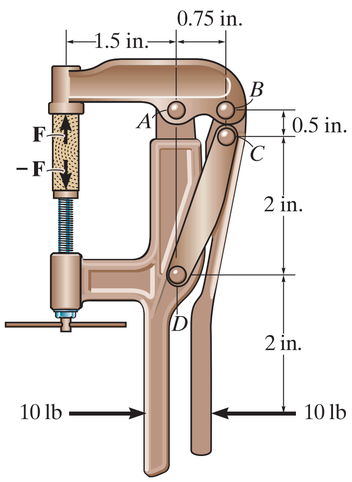
6–99. If a clamping force of 300 N is required at A, determine the amount of force **F** that must be applied to the handle of the toggle clamp.

***6–100.** If a force of $F = 350$ N is applied to the handle of the toggle clamp, determine the resulting clamping force at A.



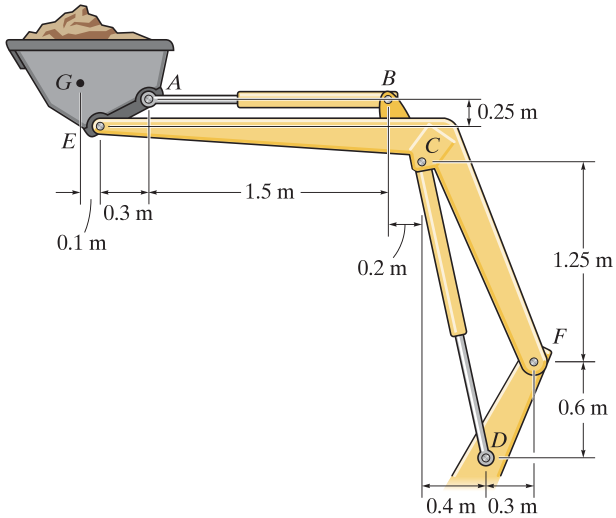
Probs. 6–99/100

6–101. If a force of 10 lb is applied to the grip of the clamp, determine the compressive force F that the wood block exerts on the clamp.



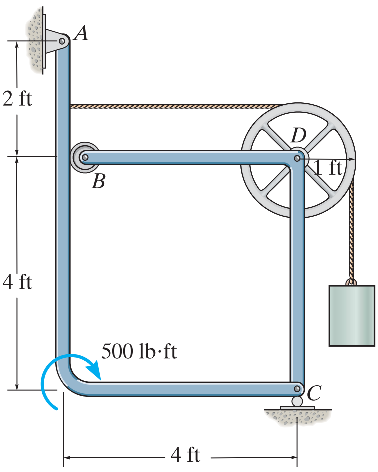
Prob. 6–101

6–102. The tractor boom supports the uniform mass of 500 kg in the bucket which has a center of mass at G. Determine the force in each hydraulic cylinder AB and CD and the resultant force at pins E and F. The load is supported equally on each side of the tractor by a similar mechanism.



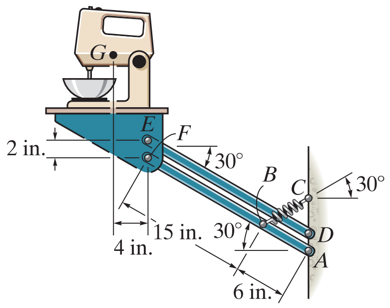
Prob. 6–102

6–103. The two-member frame supports the 200-lb cylinder and 500-lb · ft couple moment. Determine the force of the roller at B on member AC and the horizontal and vertical components of force which the pin at C exerts on member CB and the pin at A exerts on member AC. The roller C does not contact member CB.



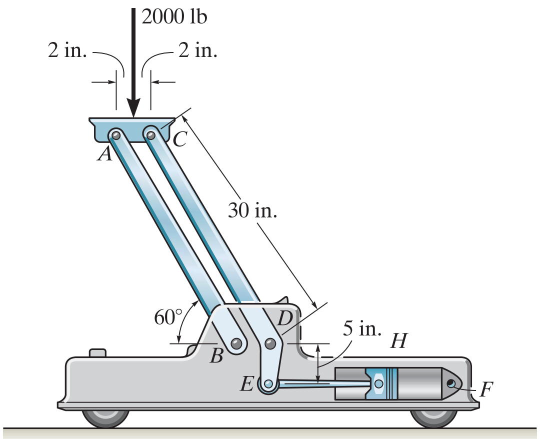
Prob. 6–103

***6–104.** The mechanism is used to hide kitchen appliances under a cabinet by allowing the shelf to rotate downward. If the mixer weighs 10 lb, is centered on the shelf, and has a mass center at G, determine the stretch in the spring necessary to hold the shelf in the equilibrium position shown. There is a similar mechanism on each side of the shelf, so that each mechanism supports 5 lb of the load. The springs each have a stiffness of $k = 4$ lb/in.



Prob. 6–104

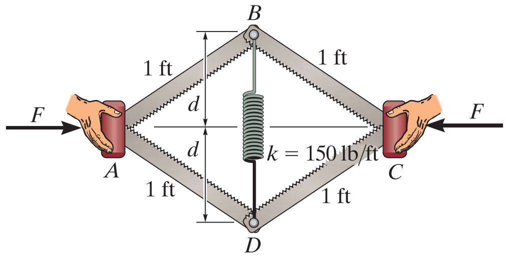
6–105. The linkage for a hydraulic jack is shown. If the load on the jack is 2000 lb, determine the pressure acting on the fluid when the jack is in the position shown. All lettered points are pins. The piston at H has a cross-sectional area of $A = 2\text{ in}^2$. *Hint:* First find the force F acting along link EH. The pressure in the fluid is $p = F/A$.



Prob. 6–105

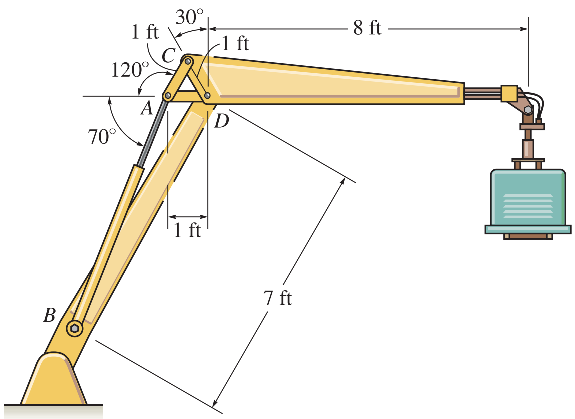
6–106. If $d = 0.75$ ft and the spring has an unstretched length of 1 ft, determine the force F required for equilibrium.

■**6–107.** If a force of $F = 50$ lb is applied to the pads at A and C, determine the smallest dimension d required for equilibrium if the spring has an unstretched length of 1 ft.



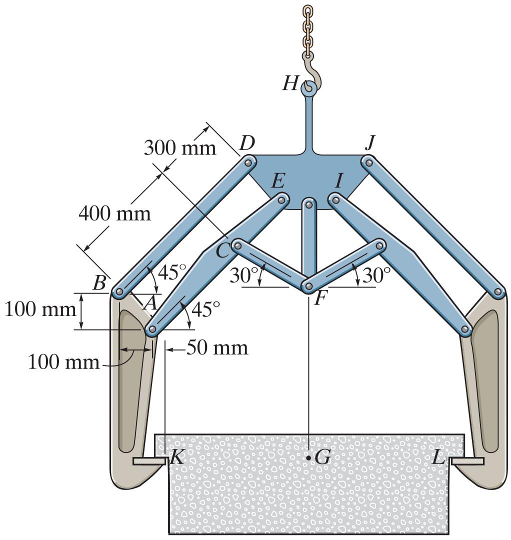
Probs. 6–106/107

***6–108.** The hydraulic crane is used to lift the 1400-lb load. Determine the force in the hydraulic cylinder AB and the force in links AC and AD when the load is held in the position shown.



Prob. 6–108

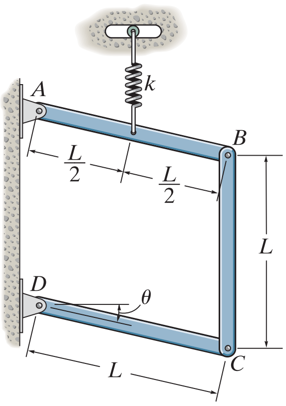
6–109. The symmetric coil tong supports the coil which has a mass of 800 kg and center of mass at G. Determine the horizontal and vertical components of force the linkage exerts on plate $DEIJH$ at points D and E. The coil exerts only vertical reactions at K and L.



Prob. 6–109

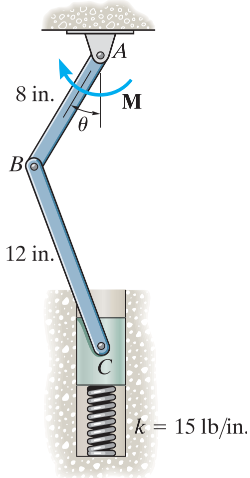
6–110. If each of the three uniform links of the mechanism has a length $L = 3$ ft and weight of $W = 10$ lb, determine the angle θ for equilibrium. The spring has a stiffness of $k = 20$ lb/in. It always remains vertical due to the roller guide and is unstretched when $\theta = 0$.

6–111. If each of the three uniform links of the mechanism has a length L and weight W, determine the angle θ for equilibrium. The spring, which always remains vertical, is unstretched when $\theta = 0°$.



Probs. 6–110/111

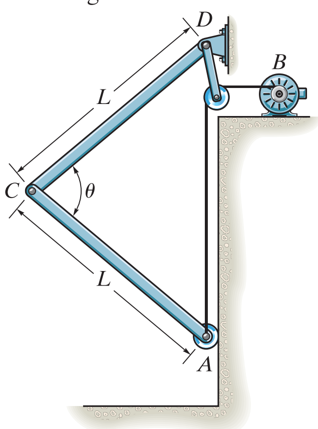
***6–112.** The piston C moves vertically between the two smooth walls. If the spring has a stiffness of $k = 15$ lb/in., and is unstretched when $\theta = 0°$, determine the couple $\mathbf{M}$ that must be applied to AB to hold the mechanism in equilibrium when $\theta = 30°$.



Prob. 6–112

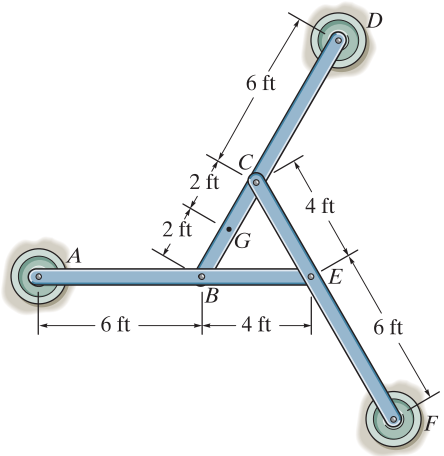
6–113. The aircraft-hangar door opens and closes slowly by means of a motor, which draws in the cable AB. If the door is made in two sections (bifold) and each section has a uniform weight of 300 lb and height $L = 10$ ft, determine the force on the cable when $\theta = 90°$. The sections are pin connected at C and D and the bottom is attached to a roller that travels along the vertical track.

6–114. The aircraft-hangar door opens and closes slowly by means of a motor which draws in the cable AB. If the door is made in two sections (bifold) and each section has a uniform weight W and height L, determine the force in the cable as a function of the door's position θ. The sections are pin connected at C and D and the bottom is attached to a roller that travels along the vertical track.



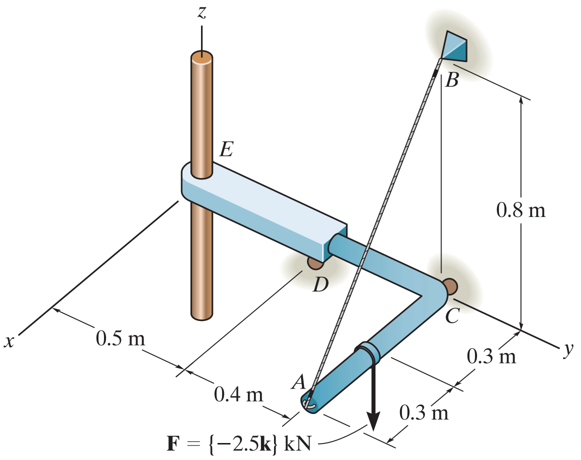
Probs. 6–113/114

6–115. The three pin-connected members shown in the *top view* support a downward force of 60 lb at *G*. If only vertical forces are supported at the connections *B, C, E* and pad supports *A, D, F*, determine the reactions at each pad.



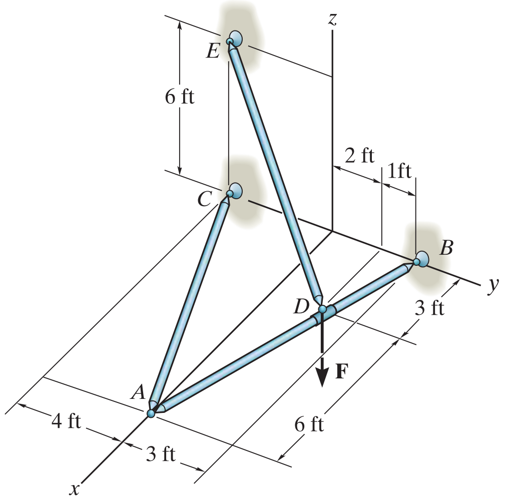
Prob. 6–115

***6–116.** The structure is subjected to the loading shown. Member *AD* is supported by a cable *AB* and roller at *C* and fits through a smooth circular hole at *D*. Member *ED* is supported by a roller at *D* and a pole that fits in a smooth snug circular hole at *E*. Determine the *x, y, z* components of reaction at *E* and the tension in cable *AB*.



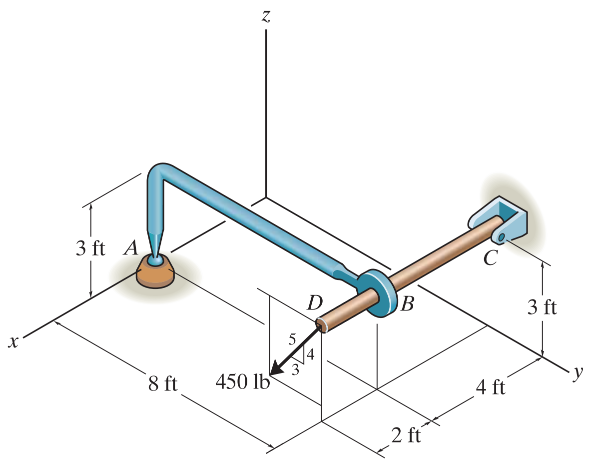
Prob. 6–116

6–117. The three-member frame is connected at its ends using ball-and-socket joints. Determine the *x, y, z* components of reaction at *B* and the tension in member *ED*. The force acting at *D* is $\mathbf{F} = \{135\mathbf{i} + 200\mathbf{j} - 180\mathbf{k}\}$ lb.



Prob. 6–117

6–118. The structure is subjected to the force of 450 lb which lies in a plane parallel to the *y–z* plane. Member *AB* is supported by a ball-and-socket joint at *A* and fits through a snug hole at *B*. Member *CD* is supported by a pin at *C*. Determine the *x, y, z* components of reaction at *A* and *C*.

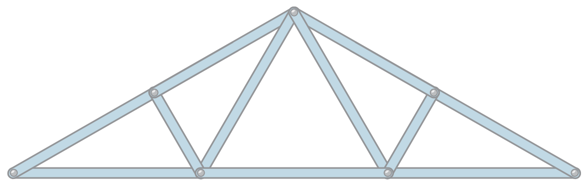


Prob. 6–118

CHAPTER REVIEW

Simple Truss

A simple truss consists of triangular elements connected together by pinned joints. The forces within its members can be determined by assuming the members are all two-force members, connected concurrently at each joint. The members are either in tension or compression, or carry no force.



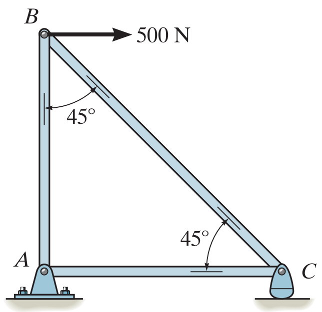
Roof truss

Method of Joints

The method of joints states that if a truss is in equilibrium, then each of its joints is also in equilibrium. For a plane truss, the concurrent force system at each joint must satisfy force equilibrium.

$$\Sigma F_x = 0$$

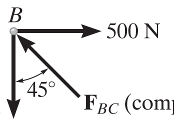
$$\Sigma F_y = 0$$



To obtain a numerical solution for the forces in the members, select a joint that has a free-body diagram with at most two unknown forces and one known force. (This may require first finding the reactions at the supports.)

Once a member force is determined, use its value and apply it to an adjacent joint.

Remember that forces that are found to *pull* on the joint are *tensile forces*, and those that *push* on the joint are *compressive forces*.

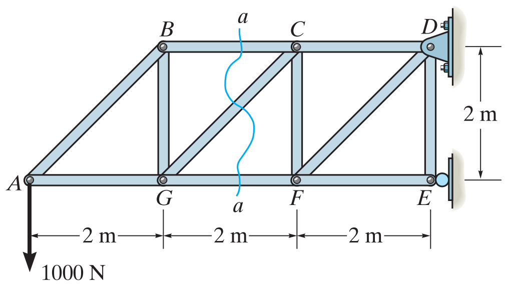


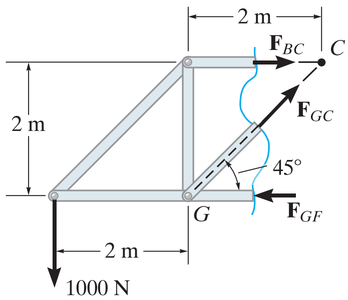
To avoid a simultaneous solution of two equations, set one of the coordinate axes along the line of action of one of the unknown forces and sum forces perpendicular to this axis. This will allow a direct solution for the other unknown.

The analysis can also be simplified by first identifying all the zero-force members.

Method of Sections

The method of sections states that if a truss is in equilibrium, then each segment of the truss is also in equilibrium. Pass a section through the truss and the member whose force is to be determined. Then draw the free-body diagram of the sectioned part having the least number of forces on it.





Sectioned members subjected to *pulling* are in *tension*, and those that are subjected to *pushing* are in *compression*.

Three equations of equilibrium are available to determine the unknowns.

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$
$$\Sigma M_O = 0$$

If possible, sum forces in a direction that is perpendicular to two of the three unknown forces. This will yield a direct solution for the third force.

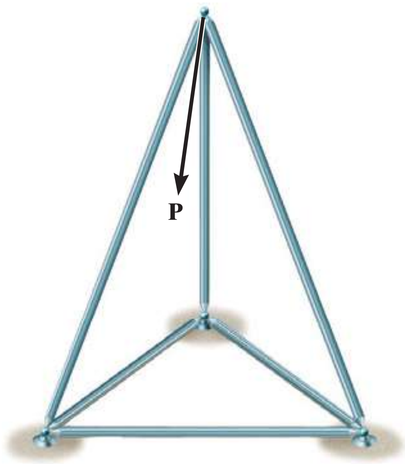
$$+\uparrow \Sigma F_y = 0$$
$$-1000 \text{ N} + F_{GC} \sin 45^\circ = 0$$
$$F_{GC} = 1.41 \text{ kN (T)}$$

Sum moments about the point where the lines of action of two of the three unknown forces intersect, so that the third unknown force can be determined directly.

$$\circlearrowleft + \Sigma M_C = 0$$
$$1000 \text{ N}(4 \text{ m}) - F_{GF}(2 \text{ m}) = 0$$
$$F_{GF} = 2 \text{ kN (C)}$$

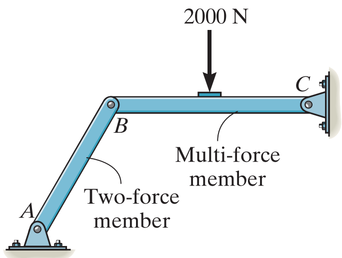
Space Truss

A space truss is a three-dimensional truss built from tetrahedral elements, and is analyzed using the same methods as for plane trusses. The joints are assumed to be ball-and-socket connections.



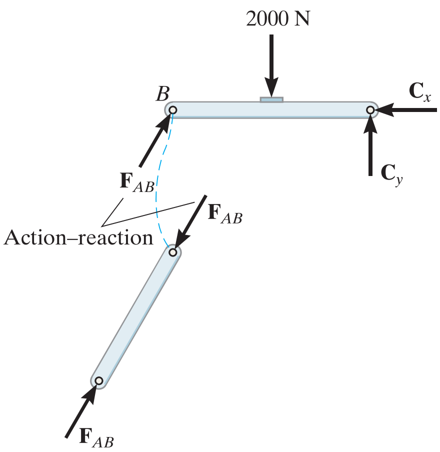
Frames and Machines

Frames and machines are structures that contain one or more multiforce members, that is, members with three or more forces or couples acting on them. Frames are designed to support loads, and machines transmit and alter the effect of forces.



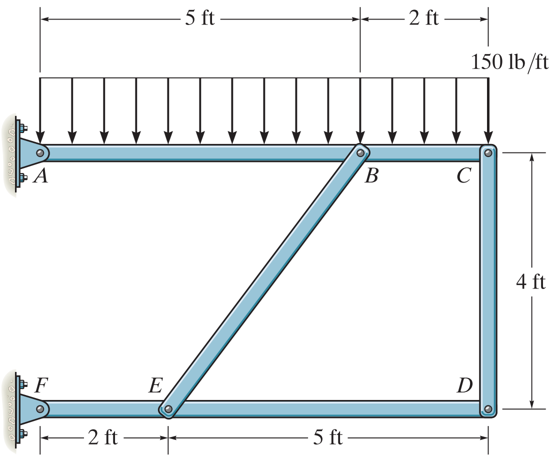
The forces acting at the joints of a frame or machine can be determined by drawing the free-body diagrams of each of its members or parts. The principle of action–reaction should be carefully observed when indicating these forces on the free-body diagram of each adjacent member or pin. For a coplanar force system, there are three equilibrium equations available for each member.

To simplify the analysis, be sure to recognize all two-force members. They have equal but opposite collinear forces at their ends.



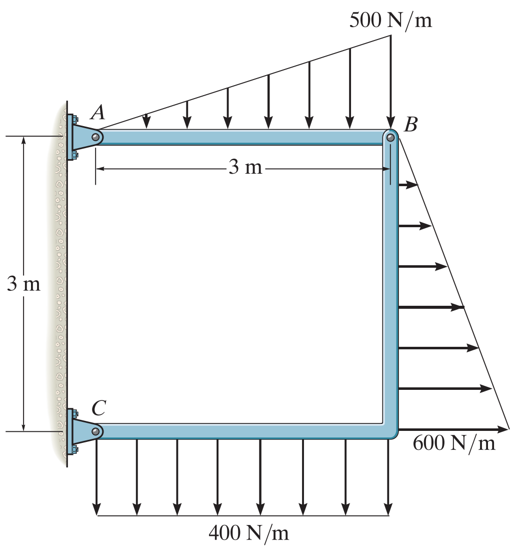
REVIEW PROBLEMS

6–119. Determine the resultant forces at pins *B* and *C* on member *ABC* of the four-member frame.



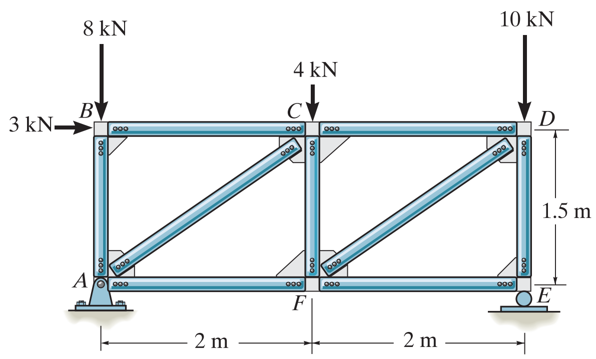
Prob. 6–119

6–121. Determine the horizontal and vertical components of force at pins *A* and *C* of the two-member frame.



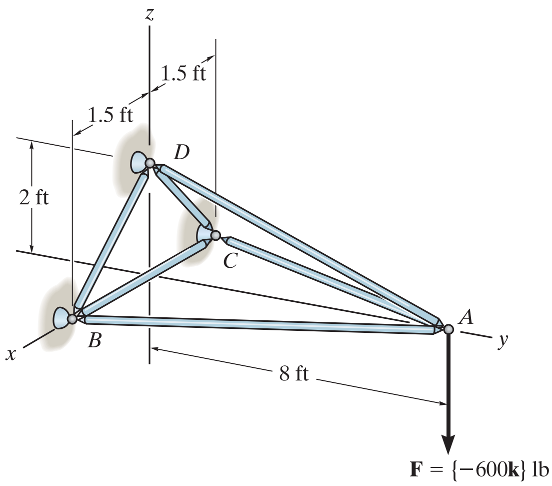
Prob. 6–121

***6–120.** Determine the force in each member of the truss and state if the members are in tension or compression.



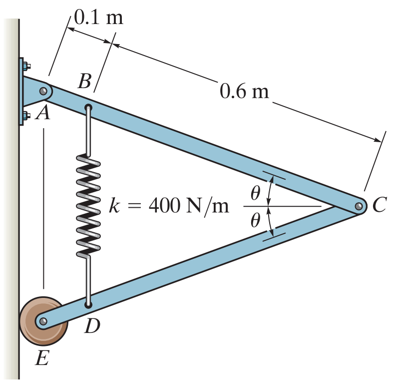
Prob. 6–120

6–122. Determine the force in members *AB*, *AD*, and *AC* of the space truss and state if the members are in tension or compression.



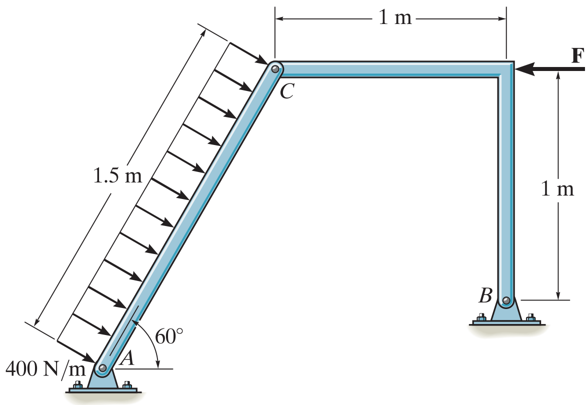
Prob. 6–122

6–123. The spring has an unstretched length of 0.3 m. Determine the mass m of each uniform link if the angle $\theta = 20^\circ$ for equilibrium.



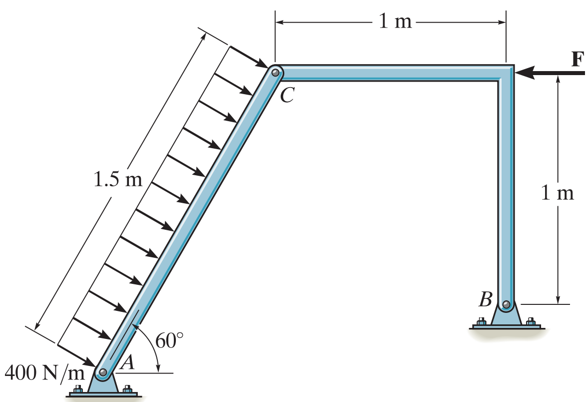
Prob. 6–123

6–125. Determine the horizontal and vertical components of force that pins A and B exert on the two-member frame. Set $F = 500$ N.



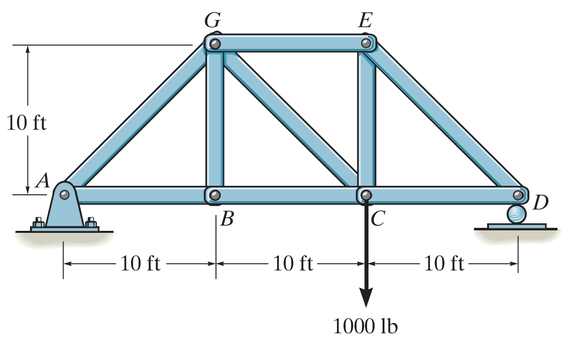
Prob. 6–125

***6–124.** Determine the horizontal and vertical components of force that the pins A and B exert on the two-member frame. Set $F = 0$.



Prob. 6–124

6–126. Determine the force in each member of the truss and state if the members are in tension or compression.



Prob. 6–126