

- b. At  $P = 2000$  kPa, from Table B.1.2, the given  $u$  of 2000 kJ/kg is greater than  $u_f$  (906.4) but less than  $u_g$  (2600.3). Therefore, this state is in the two-phase region with  $T = T_g = 212.4^\circ\text{C}$ , and

$$u = 2000 = 906.4 + x1693.8, \quad x = 0.6456$$

Then,

$$v = 0.001\,177 + 0.6456 \times 0.098\,45 = 0.064\,74\text{ m}^3/\text{kg}.$$

### In-Text Concept Questions

- e. Water is heated from 100 kPa,  $20^\circ\text{C}$  to 1000 kPa,  $200^\circ\text{C}$ . In one case, pressure is raised at  $T = C$ , then  $T$  is raised at  $P = C$ . In a second case, the opposite order is used. Does that make a difference for  ${}_1Q_2$  and  ${}_1W_2$ ?
- f. A rigid insulated tank  $A$  contains water at 400 kPa,  $800^\circ\text{C}$ . A pipe and valve connect this to another rigid insulated tank  $B$  of equal volume having saturated water vapor at 100 kPa. The valve is opened and stays open while the water in the two tanks comes to a uniform final state. Which two properties determine the final state?

## 5.4 PROBLEM ANALYSIS AND SOLUTION TECHNIQUE

At this point in our study of thermodynamics, we have progressed sufficiently far (that is, we have accumulated sufficient tools with which to work) that it is worthwhile to develop a somewhat formal technique or procedure for analyzing and solving thermodynamic problems. For the time being, it may not seem entirely necessary to use such a rigorous procedure for many of our problems, but we should keep in mind that as we acquire more analytical tools, the problems that we are capable of dealing with will become much more complicated. Thus, it is appropriate that we begin to practice this technique now in anticipation of these future problems.

Our problem analysis and solution technique is contained within the framework of the following questions that must be answered in the process of an orderly solution of a thermodynamic problem.

1. What is the control mass or control volume? Is it useful, or necessary, to choose more than one? It may be helpful to draw a sketch of the system at this point, illustrating all heat and work flows, and indicating forces such as external pressures and gravitation.
2. What do we know about the initial state (that is, which properties are known)?
3. What do we know about the final state?
4. What do we know about the process that takes place? Is anything constant or zero? Is there some known functional relation between two properties?
5. Is it helpful to draw a diagram of the information in steps 2 to 4 (for example, a  $T$ - $v$  or  $P$ - $v$  diagram)?
6. What is our thermodynamic model for the behavior of the substance (for example, steam tables, ideal gas, and so on)?

7. What is our analysis of the problem (that is, do we examine control surfaces for various work modes or use the first law or conservation of mass)?
8. What is our solution technique? In other words, from what we have done so far in steps 1–7, how do we proceed to find what is desired? Is a trial-and-error solution necessary?

It is not always necessary to write out all these steps, and in the majority of the examples throughout this book we will not do so. However, when faced with a new and unfamiliar problem, the student should always at least think through this set of questions to develop the ability to solve more challenging problems. In solving the following example, we will use this technique in detail.

**EXAMPLE 5.5** A vessel having a volume of  $5 \text{ m}^3$  contains  $0.05 \text{ m}^3$  of saturated liquid water and  $4.95 \text{ m}^3$  of saturated water vapor at  $0.1 \text{ MPa}$ . Heat is transferred until the vessel is filled with saturated vapor. Determine the heat transfer for this process.

*Control mass:* All the water inside the vessel.

*Sketch:* Fig. 5.5.

*Initial state:* Pressure, volume of liquid, volume of vapor; therefore, state 1 is fixed.

*Final state:* Somewhere along the saturated-vapor curve; the water was heated, so  $P_2 > P_1$ .

*Process:* Constant volume and mass; therefore, constant specific volume.

*Diagram:* Fig. 5.6.

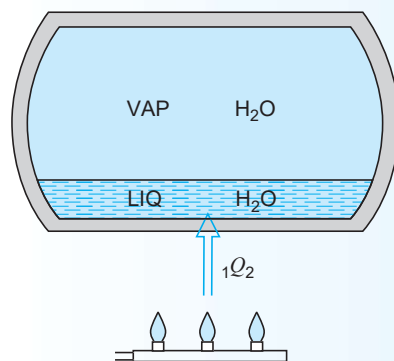
*Model:* Steam tables.

### Analysis

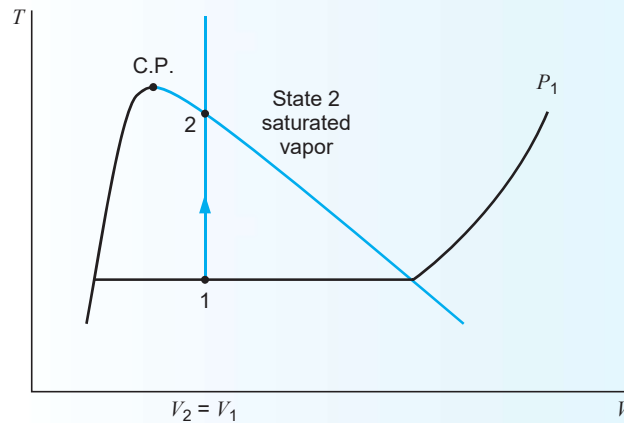
From the first law we have

$${}_1 Q_2 = U_2 - U_1 + m \frac{\mathbf{V}_2^2 - \mathbf{V}_1^2}{2} + mg(Z_2 - Z_1) + {}_1 W_2$$

From examining the control surface for various work modes, we conclude that the work for this process is zero. Furthermore, the system is not moving, so there is no change in kinetic energy. There is a small change in the center of mass of the system, but we will



**FIGURE 5.5** Sketch for Example 5.5.



**FIGURE 5.6** Diagram for Example 5.5.

assume that the corresponding change in potential energy (in kilojoules) is negligible. Therefore,

$${}_1Q_2 = U_2 - U_1$$

### Solution

The heat transfer will be found from the first law. State 1 is known, so  $U_1$  can be calculated. The specific volume at state 2 is also known (from state 1 and the process). Since state 2 is saturated vapor, state 2 is fixed, as is seen in Fig. 5.6. Therefore,  $U_2$  can also be found.

The solution proceeds as follows:

$$m_{1 \text{ liq}} = \frac{V_{\text{liq}}}{v_f} = \frac{0.05}{0.001043} = 47.94 \text{ kg}$$

$$m_{1 \text{ vap}} = \frac{V_{\text{vap}}}{v_g} = \frac{4.95}{1.6940} = 2.92 \text{ kg}$$

Then

$$\begin{aligned} U_1 &= m_{1 \text{ liq}} u_{1 \text{ liq}} + m_{1 \text{ vap}} u_{1 \text{ vap}} \\ &= 47.94(417.36) + 2.92(2506.1) = 27\,326 \text{ kJ} \end{aligned}$$

To determine  $u_2$  we need to know two thermodynamic properties, since this determines the final state. The properties we know are the quality,  $x = 100\%$ , and  $v_2$ , the final specific volume, which can readily be determined.

$$m = m_{1 \text{ liq}} + m_{1 \text{ vap}} = 47.94 + 2.92 = 50.86 \text{ kg}$$

$$v_2 = \frac{V}{m} = \frac{5.0}{50.86} = 0.09831 \text{ m}^3/\text{kg}$$

In Table B.1.2 we find, by interpolation, that at a pressure of 2.03 MPa,  $v_g = 0.09831 \text{ m}^3/\text{kg}$ . The final pressure of the steam is therefore 2.03 MPa. Then

$$u_2 = 2600.5 \text{ kJ/kg}$$

$$U_2 = mu_2 = 50.86(2600.5) = 132\,261 \text{ kJ}$$

$${}_1Q_2 = U_2 - U_1 = 132\,261 - 27\,326 = 104\,935 \text{ kJ}$$

**EXAMPLE 5.5E** A vessel having a volume of 100 ft<sup>3</sup> contains 1 ft<sup>3</sup> of saturated liquid water and 99 ft<sup>3</sup> of saturated water vapor at 14.7 lbf/in.<sup>2</sup>. Heat is transferred until the vessel is filled with saturated vapor. Determine the heat transfer for this process.

*Control mass:* All the water inside the vessel.

*Sketch:* Fig. 5.5.

*Initial state:* Pressure, volume of liquid, volume of vapor; therefore, state 1 is fixed.

*Final state:* Somewhere along the saturated-vapor curve; the water was heated, so  $P_2 > P_1$ .

*Process:* Constant volume and mass; therefore, constant specific volume.

*Diagram:* Fig. 5.6.

*Model:* Steam tables.

### Analysis

$$\text{First law: } {}_1Q_2 = U_2 - U_1 + m \frac{(\mathbf{V}_2^2 - \mathbf{V}_1^2)}{2} + mg(Z_2 - Z_1) + {}_1W_2$$

By examining the control surface for various work modes, we conclude that the work for this process is zero. Furthermore, the system is not moving, so there is no change in kinetic energy. There is a small change in the center of mass of the system, but we will assume that the corresponding change in potential energy is negligible (compared to other terms). Therefore,

$${}_1Q_2 = U_2 - U_1$$

### Solution

The heat transfer will be found from the first law. State 1 is known, so  $U_1$  can be calculated. Also, the specific volume at state 2 is known (from state 1 and the process). Since state 2 is saturated vapor, state 2 is fixed, as is seen in Fig. 5.6. Therefore,  $U_2$  can also be found.

The solution proceeds as follows:

$$m_{1 \text{ liq}} = \frac{V_{\text{liq}}}{v_f} = \frac{1}{0.01672} = 59.81 \text{ lbm}$$

$$m_{1 \text{ vap}} = \frac{V_{\text{vap}}}{v_g} = \frac{99}{26.80} = 3.69 \text{ lbm}$$

Then

$$\begin{aligned} U_1 &= m_{1 \text{ liq}} u_{1 \text{ liq}} + m_{1 \text{ vap}} u_{1 \text{ vap}} \\ &= 59.81(180.1) + 3.69(1077.6) = 14\,748 \text{ Btu} \end{aligned}$$

To determine  $u_2$  we need to know two thermodynamic properties, since this determines the final state. The properties we know are the quality,  $x = 100\%$ , and  $v_2$ , the final specific

volume, which can readily be determined.

$$m = m_{1 \text{ liq}} + m_{1 \text{ vap}} = 59.81 + 3.69 = 63.50 \text{ lbm}$$

$$v_2 = \frac{V}{m} = \frac{100}{63.50} = 1.575 \text{ ft}^3/\text{lbm}$$

In Table F7.1 of the steam tables we find, by interpolation, that at a pressure of 294 lbf/in.<sup>2</sup>,  $v_g = 1.575 \text{ ft}^3/\text{lbm}$ . The final pressure of the steam is therefore 294 lbf/in.<sup>2</sup>. Then

$$u_2 = 1117.0 \text{ Btu/lbm}$$

$$U_2 = mu_2 = 63.50(1117.0) = 70\,930 \text{ Btu}$$

$${}_1Q_2 = U_2 - U_1 = 70\,930 - 14\,748 = 56\,182 \text{ Btu}$$

## 5.5 THE THERMODYNAMIC PROPERTY ENTHALPY

In analyzing specific types of processes, we frequently encounter certain combinations of thermodynamic properties, which are therefore also properties of the substance undergoing the change of state. To demonstrate one such situation, let us consider a control mass undergoing a quasi-equilibrium constant-pressure process, as shown in Fig. 5.7. Assume that there are no changes in kinetic or potential energy and that the only work done during the process is that associated with the boundary movement. Taking the gas as our control mass and applying the first law, Eq. 5.11, we have, in terms of  $Q$ ,

$${}_1Q_2 = U_2 - U_1 + {}_1W_2$$

The work done can be calculated from the relation

$${}_1W_2 = \int_1^2 P dV$$

Since the pressure is constant,

$${}_1W_2 = P \int_1^2 dV = P(V_2 - V_1)$$

Therefore,

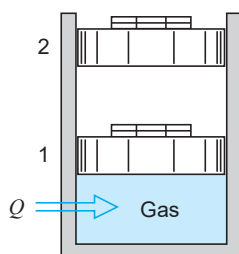
$$\begin{aligned} {}_1Q_2 &= U_2 - U_1 + P_2 V_2 - P_1 V_1 \\ &= (U_2 + P_2 V_2) - (U_1 + P_1 V_1) \end{aligned}$$

We find that, in this very restricted case, the heat transfer during the process is given in terms of the change in the quantity  $U + PV$  between the initial and final states. Because all these quantities are thermodynamic properties, that is, functions only of the state of the system, their combination must also have these same characteristics. Therefore, we find it convenient to define a new extensive property, the **enthalpy**,

$$H \equiv U + PV \quad (5.12)$$

or, per unit mass,

$$h \equiv u + Pv \quad (5.13)$$



**FIGURE 5.7** The constant-pressure quasi-equilibrium process.

As for internal energy, we could speak of specific enthalpy,  $h$ , and total enthalpy,  $H$ . However, we will refer to both as enthalpy, since the context will make it clear which is being discussed.

The heat transfer in a constant-pressure quasi-equilibrium process is equal to the change in enthalpy, which includes both the change in internal energy and the work for this particular process. This is by no means a general result. It is valid for this special case only because the work done during the process is equal to the difference in the  $PV$  product for the final and initial states. This would not be true if the pressure had not remained constant during the process.

The significance and use of enthalpy are not restricted to the special process just described. Other cases in which this same combination of properties  $u + Pv$  appears will be developed later, notably in Chapter 6 where we discuss control volume analyses. Our reason for introducing enthalpy at this time is that although the tables in Appendix B list values for internal energy, many other tables and charts of thermodynamic properties give values for enthalpy but not for internal energy. Therefore, it is necessary to calculate internal energy at a state using the tabulated values and Eq. 5.13:

$$u = h - Pv$$

Students often become confused about the validity of this calculation when analyzing system processes that do not occur at constant pressure, for which enthalpy has no physical significance. We must keep in mind that enthalpy, being a property, is a state or point function, and its use in calculating internal energy at the same state is not related to, or dependent on, any process that may be taking place.

Tabular values of internal energy and enthalpy, such as those included in Tables B.1 through B.7, are all relative to some arbitrarily selected base. In the steam tables, the internal energy of saturated liquid at 0.01°C is the reference state and is given a value of zero. For refrigerants, such as R-134a, R-410a, and ammonia, the reference state is arbitrarily taken as saturated liquid at -40°C. The enthalpy in this reference state is assigned the value of zero. Cryogenic fluids, such as nitrogen, have other arbitrary reference states chosen for enthalpy values listed in their tables. Because each of these reference states is arbitrarily selected, it is always possible to have negative values for enthalpy, as for saturated-solid water in Table B.1.5. When enthalpy and internal energy are given values relative to the same reference state, as they are in essentially all thermodynamic tables, the difference between internal energy and enthalpy at the reference state is equal to  $Pv$ . Since the specific volume of the liquid is very small, this product is negligible as far as the significant figures of the tables are concerned, but the principle should be kept in mind, for in certain cases it is significant.

In many thermodynamic tables, values of the specific internal energy  $u$  are not given. As mentioned earlier, these values can be readily calculated from the relation  $u = h - Pv$ , though it is important to keep the units in mind. As an example, let us calculate the internal energy  $u$  of superheated R-134a at 0.4 MPa, 70°C.

$$\begin{aligned} u &= h - Pv \\ &= 460.55 - 400 \times 0.06648 \\ &= 433.96 \text{ kJ/kg} \end{aligned}$$

The enthalpy of a substance in a saturation state and with a given quality is found in the same way as the specific volume and internal energy. The enthalpy of saturated liquid

has the symbol  $h_f$ , saturated vapor  $h_g$ , and the increase in enthalpy during vaporization  $h_{fg}$ . For a saturation state, the enthalpy can be calculated by one of the following relations:

$$h = (1 - x)h_f + xh_g$$

$$h = h_f + xh_{fg}$$

The enthalpy of compressed liquid water may be found from Table B.1.4. For substances for which compressed-liquid tables are not available, the enthalpy is taken as that of saturated liquid at the same temperature.

**EXAMPLE 5.6** A cylinder fitted with a piston has a volume of  $0.1 \text{ m}^3$  and contains  $0.5 \text{ kg}$  of steam at  $0.4 \text{ MPa}$ . Heat is transferred to the steam until the temperature is  $300^\circ\text{C}$ , while the pressure remains constant.

Determine the heat transfer and the work for this process.

*Control mass:* Water inside cylinder.

*Initial state:*  $P_1, V_1, m$ ; therefore,  $v_1$  is known, state 1 is fixed (at  $P_1, v_1$ , check steam tables—two-phase region).

*Final state:*  $P_2, T_2$ ; therefore, state 2 is fixed (superheated).

*Process:* Constant pressure.

*Diagram:* Fig. 5.8.

*Model:* Steam tables.

### Analysis

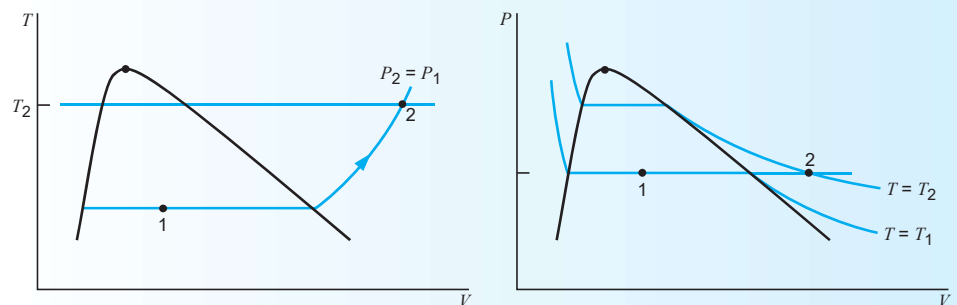
There is no change in kinetic energy or potential energy. Work is done by movement at the boundary. Assume the process to be quasi-equilibrium. Since the pressure is constant, we have

$${}_1W_2 = \int_1^2 P dV = P \int_1^2 dV = P(V_2 - V_1) = m(P_2v_2 - P_1v_1)$$

Therefore, the first law is, in terms of  $Q$ ,

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2$$

$$= m(u_2 - u_1) + m(P_2v_2 - P_1v_1) = m(h_2 - h_1)$$



**FIGURE 5.8** The constant-pressure quasi-equilibrium process.

**Solution**

There is a choice of procedures to follow. State 1 is known, so  $v_1$  and  $h_1$  (or  $u_1$ ) can be found. State 2 is also known, so  $v_2$  and  $h_2$  (or  $u_2$ ) can be found. Using the first law and the work equation, we can calculate the heat transfer and work. Using the enthalpies, we have

$$v_1 = \frac{V_1}{m} = \frac{0.1}{0.5} = 0.2 = 0.001\,084 + x_1 0.4614$$

$$x_1 = \frac{0.1989}{0.4614} = 0.4311$$

$$\begin{aligned} h_1 &= h_f + x_1 h_{fg} \\ &= 604.74 + 0.4311 \times 2133.8 = 1524.7 \text{ kJ/kg} \end{aligned}$$

$$h_2 = 3066.8 \text{ kJ/kg}$$

$${}_1Q_2 = 0.5(3066.8 - 1524.7) = 771.1 \text{ kJ}$$

$${}_1W_2 = mP(v_2 - v_1) = 0.5 \times 400(0.6548 - 0.2) = 91.0 \text{ kJ}$$

Therefore,

$$U_2 - U_1 = {}_1Q_2 - {}_1W_2 = 771.1 - 91.0 = 680.1 \text{ kJ}$$

The heat transfer could also have been found from  $u_1$  and  $u_2$ :

$$\begin{aligned} u_1 &= u_f + x_1 u_{fg} \\ &= 604.31 + 0.4311 \times 1949.3 = 1444.7 \text{ kJ/kg} \\ u_2 &= 2804.8 \text{ kJ/kg} \end{aligned}$$

and

$$\begin{aligned} {}_1Q_2 &= U_2 - U_1 + {}_1W_2 \\ &= 0.5(2804.8 - 1444.7) + 91.0 = 771.1 \text{ kJ} \end{aligned}$$

**EXAMPLE 5.7** Saturated-vapor R-134a is contained in a piston/cylinder at room temperature, 20°C, at which point the cylinder volume is 10 L. The external force restraining the piston is now reduced, allowing the system to expand to 40 L. We will consider two different situations:

- The cylinder is uninsulated. In addition, the external force is reduced very slowly as the process takes place. If the work done during the process is 8.0 kJ, how much heat is transferred?
- The cylinder is insulated. Also, the external force is reduced rapidly, causing the process to occur rapidly, such that the final pressure inside the cylinder is 150 kPa. What are the heat transfer and work for this process?

**a. Analysis**

Since the cylinder is not insulated, we assume that heat transfer is possible between the room at 20°C and the control mass, the R-134a. Further, since the process takes place very slowly, it is reasonable to assume that the temperature of R-134a remains constant at 20°C. Therefore,



*Initial state:* Temperature, quality ( $= 1.0$ ); state 1 known. Volume fixes mass.

*Process:* Constant temperature. Work given.

*Final state:* Temperature, specific volume; state known.

*Model:* R-134a tables.

There is no change in kinetic energy and negligible change in potential energy, so the first law reduces to

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2$$

### Solution

From Table B.5.1 at  $20^\circ\text{C}$ ,

$$x_1 = 1.0,$$

$$P_1 = P_g = 573 \text{ kPa}, \quad v_1 = v_g = 0.03606 \text{ m}^3/\text{kg}, \quad u_1 = u_g = 389.2 \text{ kJ/kg}$$

$$m = \frac{V_1}{v_1} = \frac{0.010}{0.03606} = 0.277 \text{ kg}$$

$$v_2 = v_1 \times \frac{V_2}{V_1} = 0.03606 \times \frac{0.040}{0.010} = 0.14424 \text{ m}^3/\text{kg}$$

From Table B.5.2 at  $T_2, v_2$ ,

$$P_2 = 163 \text{ kPa}, \quad u_2 = 395.8 \text{ kJ/kg}$$

Substituting into the first law,

$${}_1Q_2 = 0.277 \times (395.8 - 389.2) + 8.0 = 9.83 \text{ kJ}$$

### b. Analysis

Since the cylinder is insulated and the process takes place rapidly, it is reasonable to assume that the process is adiabatic, that is, heat transfer is zero. Thus,

*Initial state:* Temperature, quality ( $= 1.0$ ); state 1 known. Volume fixes mass.

*Process:* Adiabatic.  ${}_1Q_2 = 0$ .

*Final state:* Pressure, specific volume; state known.

*Model:* R-134a tables.

There is no change in kinetic energy and negligible change in potential energy, so the first law reduces to

$${}_1Q_2 = 0 = m(u_2 - u_1) + {}_1W_2$$

### Solution

The values for  $m, u_1$ , and  $v_2$  are the same as in part a.

From Table B.5.2 at  $P_2, v_2$ ,

$$T_2 = 3.3^\circ\text{C}, \quad u_2 = 383.4 \text{ kJ/kg}$$

Substituting into the first law,

$${}_1W_2 = 0.277 \times (389.2 - 383.4) = 1.6 \text{ kJ}$$