7.1

Two heat engines operate between the same two energy reservoirs and both receives the same Q_H. One engine is reversible and the other is not. What can you say about the two Q_L's?

The reversible heat engine can produce more work (has a higher efficiency) than the irreversible heat engine and due to the energy conservation it then gives out a smaller Q_L compared to the irreversible heat engine.

$$W_{rev} = Q_H - Q_{L\ rev} \quad > \quad W_{irrev} = Q_H - Q_{L\ irrev}$$

$$\Rightarrow \quad Q_{L\ rev} < Q_{L\ irrev}$$

7.2

Compare two domestic heat pumps (A and B) running with the same work input. If A is better than B which one heats the house most?

The statement that A is better means it has a higher COP and since

$$Q_{H\,A} = COP_A\ W \quad > \quad Q_{H\,B} = COP_B\ W$$

it can thus provide more heat to the house. The higher heat comes from the higher Q_L it is able to draw in.

7.3

Suppose we forget the model for heat transfer as $\dot{Q} = CA\ \Delta T$, can we draw some information about direction of Q from the second law?

One of the classical statements of the second law is the Clausius statement saying that you cannot have heat transfer from a lower temperature domain to a higher temperature domain without work input.

The opposite, namely a transfer of heat from a high temperature domain towards a lower temperature domain can happen (which is a heat engine with zero efficiency).

7.4

A combination of two heat engines is shown in Fig. P7.4. Find the overall thermal efficiency as a function of the two individual efficiencies.

The overall efficiency

$$\eta_{TH} = \dot{W}_{net} / \dot{Q}_H = (\dot{W}_1 + \dot{W}_2) / \dot{Q}_H = \eta_1 + \dot{W}_2 / \dot{Q}_H$$

For the second heat engine and the energy Eq. for the first heat engine

$$\dot{W}_2 = \eta_2 \, \dot{Q}_M = \eta_2 \, (1 - \eta_1) \, \dot{Q}_H$$

so the final result is

$$\eta_{TH} = \eta_1 + \eta_2 \, (1 - \eta_1)$$

7.5

Compare two heat engines receiving the same Q, one at 1200 K and the other at 1800 K; they both reject heat at 500 K. Which one is better?

The maximum efficiency for the engines are given by the Carnot heat engine efficiency as

$$\eta_{TH} = \dot{W}_{net} / \dot{Q}_H = 1 - \frac{T_L}{T_H}$$

Since they have the same low temperature the one with the highest T_H will have a higher efficiency and thus presumably better.

7.6

A car engine takes atmospheric air in at 20^{o}C, no fuel, and exhausts the air at –20^{o}C producing work in the process. What do the first and the second laws say about that?

Energy Eq.: $W = Q_H - Q_L$ = change in energy of air. **OK**

2nd law: Exchange energy with only one reservoir. **NOT OK**.
This is a violation of the statement of Kelvin-Planck.

Remark: You cannot create and maintain your own energy reservoir.

7.7

A combination of two refrigerator cycles is shown in Fig. P7.7. Find the overall COP as a function of COP_1 and COP_2.

The overall COP becomes

$$COP = \beta = \frac{\dot{Q}_L}{\dot{W}_{tot}} = \frac{\dot{Q}_L}{\dot{W}_1}\frac{\dot{W}_1}{\dot{W}_{tot}} = COP_1 \frac{\dot{W}_1}{\dot{W}_{tot}} = COP_1 \frac{1}{1 + \dot{W}_2/\dot{W}_1}$$

where we used $\dot{W}_{tot} = \dot{W}_1 + \dot{W}_2$. Use definition of COP_2 and energy equation for refrigerator 1 to eliminate $\dot{Q}_M$ and we have

$$\dot{W}_2 = \dot{Q}_M / COP_2 = (\dot{W}_1 + \dot{Q}_L) / COP_2$$

so then

$$\dot{W}_2 / \dot{W}_1 = (1 + \dot{Q}_L/\dot{W}_1) / COP_2 = (1 + COP_1) / COP_2$$

Finally substitute into the first equation and rearrange a little to get

$$COP = \beta = \frac{COP_1\ COP_2}{COP_1 + COP_2 + 1}$$

7.8

After you have returned from a car trip the car engine has cooled down and is thus back to the state in which it started. What happened to all the energy released in the burning of the gasoline? What happened to all the work the engine gave out?

Solution:
All the energy from the fuel generates heat and work out of the engine. The heat is directly dissipated in the atmosphere and the work is turned into kinetic energy and internal energy by all the frictional forces (wind resistance, rolling resistance, brake action). Eventually the kinetic energy is lost by braking the car so in the end all the energy is absorbed by the environment increasing its internal energy.

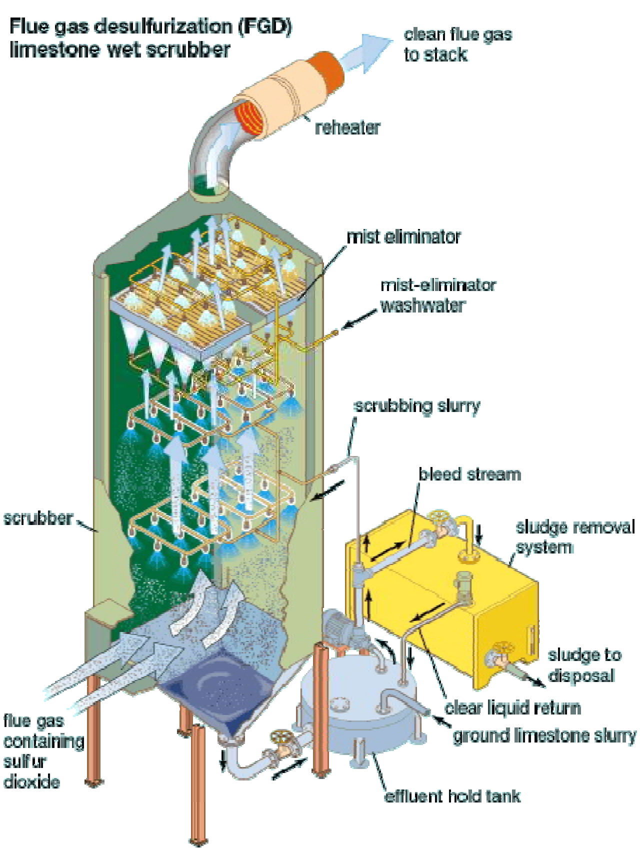
7.9

Does a reversible heat engine burning coal (which, in practice, cannot be done reversibly) have impacts on our world other than depletion of the coal reserve?

Solution:

When you burn coal you form carbon dioxide CO_2 which is a greenhouse gas. It absorbs energy over a wide spectrum of wavelengths and thus traps energy in the atmosphere that otherwise would go out into space.

Coal from various locations also has sulfur and other substances like heavy metals in it. The sulfur generates sulfuric acid (resulting in acid rain) in the atmosphere and can damage the forests.

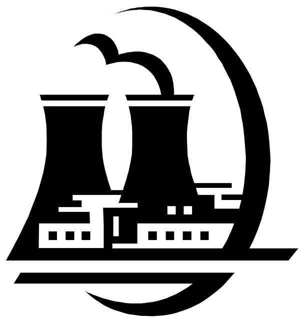


7.10

If the efficiency of a power plant goes up as the low temperature drops, why do power plants not just reject energy at say –40^{o}C?

In order to reject heat the ambient must be at the low temperature. Only if we moved the plant to the North Pole would we see such a low T.

Remark: You cannot create and maintain your own energy reservoir.



7.11

If the efficiency of a power plant goes up as the low temperature drops why not let the heat rejection go to a refrigerator at, say, $-10^{o}C$ instead of ambient $20^{o}C$?

The refrigerator must pump the heat up to $20^{o}C$ to reject it to the ambient. The refrigerator must then have a work input that will exactly offset the increased work output of the power plant, if they are both ideal. As we can not build ideal devices the actual refrigerator will require more work than the power plant will produce extra.

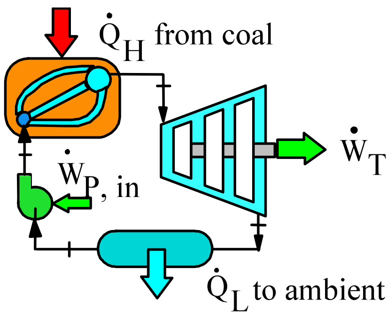
7.12

A coal-fired power plant operates with a high T of 600°C whereas a jet engine has about 1400 K. Does that mean we should replace all power plants with jet engines?

The thermal efficiency is limited by the Carnot heat engine efficiency.

That is, the low temperature is also important. Here the power plant has a much lower T in the condenser than the jet engine has in the exhaust flow so the jet engine does not necessarily have a higher efficiency than the power plant.

Gas-turbines are used in power plants where they can cover peak power demands needed for shorter time periods and their high temperature exhaust can be used to boil additional water for the steam cycle.



7.13

A heat transfer requires a temperature difference, see chapter 4, to push the $\dot{Q}$. What implications do that have for a real heat engine? A refrigerator?

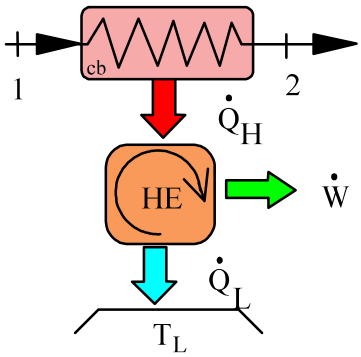
This means that there are temperature differences between the source of energy and the working substance so T_H is smaller than the source temperature. This lowers the maximum possible efficiency. As heat is rejected the working substance must have a higher temperature T_L than the ambient receiving the $\dot{Q}_L$, which lowers the efficiency further.

For a refrigerator the high temperature must be higher than the ambient to which the $\dot{Q}_H$ is moved. Likewise the low temperature must be lower than the cold space temperature in order to have heat transfer from the cold space to the cycle substance. So the net effect is the cycle temperature difference is larger than the reservoir temperature difference and thus the COP is lower than that estimated from the cold space and ambient temperatures.

7.14

Hot combustion gases (air) at 1500 K are used as heat source in a heat engine where the gas is cooled to 750 K and the ambient is at 300 K. This is not a constant T source. How does that affect the efficiency?

Solution:



If the efficiency is written as

$$\eta_{TH} = \dot{W}_{net} / \dot{Q}_H = 1 - \frac{T_L}{T_H}$$

then T_H is somewhere between 1500 K and 750 K and it is not a linear average.

After studying chapter 8 and 9 we can solve this problem and find the proper average high temperature based on properties at states 1 and 2.

Heat Engines and Refrigerators

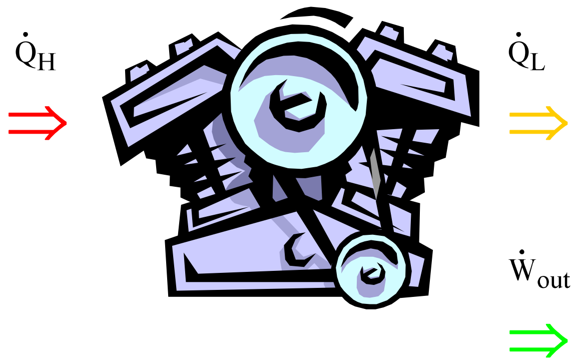
7.15

A gasoline engine produces 20 hp using 35 kW of heat transfer from burning fuel. What is its thermal efficiency and how much power is rejected to the ambient?

Conversion Table A.1: 20 hp = 20 × 0.7457 kW = 14.91 kW

Efficiency: $\eta_{TH} = \dot{W}_{out}/\dot{Q}_H = \frac{14.91}{35} = \mathbf{0.43}$

Energy equation: $\dot{Q}_L = \dot{Q}_H - \dot{W}_{out} = 35 - 14.91 = \mathbf{20.1\ kW}$



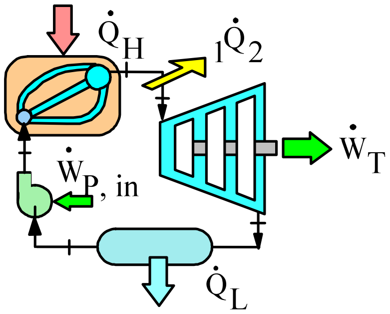
7.16

Calculate the thermal efficiency of the steam power plant cycle described in Example 6.9.

Solution:

From solution to Example 6.9,

$$w_{net} = w_t + w_p = 640.7 - 4$$
$$= 636.7 \text{ kJ/kg}$$
$$q_H = q_b = 2831 \text{ kJ/kg}$$
$$\eta_{TH} = w_{net}/q_H = \frac{636.7}{2831} = \mathbf{0.225}$$



Notice we cannot write $w_{net} = q_H - q_L$ as there is an extra heat transfer $_1\dot{Q}_2$ as a loss in the line. This needs to be accounted for in the overall energy equation.

7.17

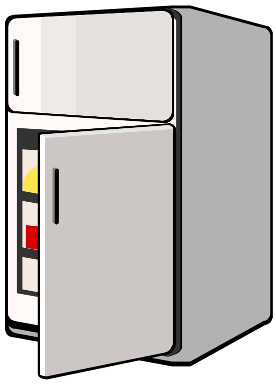
A refrigerator removes 1.5 kJ from the cold space using 1 kJ work input. How much energy goes into the kitchen and what is its coefficient of performance?

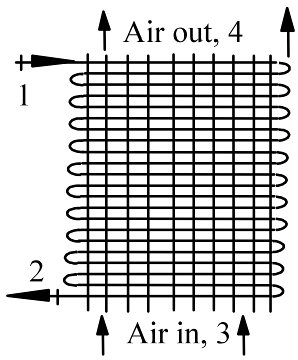
C.V. Refrigerator. The energy Q_H goes into the kitchen air.

Energy Eq.: $Q_H = W + Q_L = 1 + 1.5 = \mathbf{2.5\ kJ}$

COP: $\beta = \frac{Q_L}{W} = 1.5 / 1 = \mathbf{1.5}$

The back side of the refrigerator has a black grille that heats the kitchen air. Other models have that at the bottom with a fan to drive the air over it.





7.18

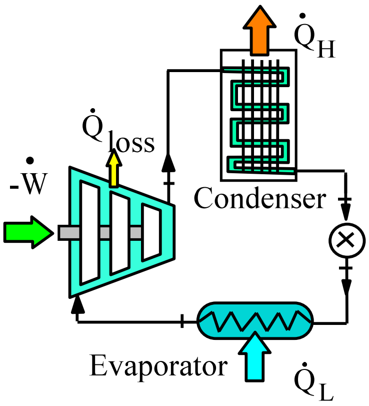
Calculate the coefficient of performance of the R-134a refrigerator given in Example 6.10.

Solution:

From the definition

$$\beta = \dot{Q}_L/\dot{W}_{IN} = \frac{14.54}{5} = 2.91$$

Notice we cannot write $\dot{W}_{IN} = \dot{Q}_H - \dot{Q}_L$ as there is a small $\dot{Q}$ in the compressor. This needs to be accounted for in the overall energy equation.



7.19

A coal fired power plant has an efficiency of 35% and produces net 500 MW of electricity. Coal releases 25 000 kJ/kg as it burns so how much coal is used per hour?

From the definition of the thermal efficiency and the energy release by the combustion called heating value HV we get

$$\dot{W} = \eta\ \dot{Q}_H = \eta \cdot \dot{m} \cdot HV$$

then

$$\dot{m} = \frac{\dot{W}}{\eta \times HV} = \frac{500 \text{ MW}}{0.35 \times 25000 \text{ kJ/kg}} = \frac{500 \times 1000 \text{ kJ/s}}{0.35 \times 25000 \text{ kJ/kg}}$$

$= 57.14$ kg/s = **205 714 kg/h**

7.20

Assume we have a refrigerator operating at steady state using 500 W of electric power with a COP of 2.5. What is the net effect on the kitchen air?

Take a C.V. around the whole kitchen. The only energy term that crosses the control surface is the work input $\dot{W}$ apart from energy exchanged with the kitchen surroundings. That is the kitchen is being heated with a rate of $\dot{W}$.

Remark: The two heat transfer rates are both internal to the kitchen. $\dot{Q}_H$ goes into the kitchen air and $\dot{Q}_L$ actually leaks from the kitchen into the refrigerated space, which is the reason we need to drive it out again.

7.21

A room is heated with a 1500 W electric heater. How much power can be saved if a heat pump with a COP of 2.0 is used instead?

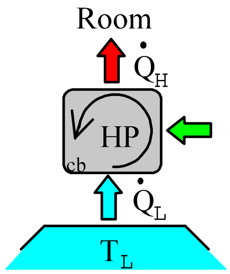
Assume the heat pump has to deliver 1500 W as the $\dot{Q}_H$.

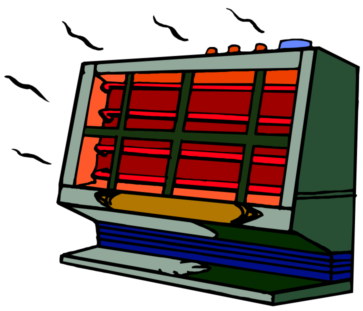
Heat pump: $\beta' = \dot{Q}_H/\dot{W}_{IN}$

$$\dot{W}_{IN} = \dot{Q}_H/\beta' = \frac{1500}{2} = 750 \text{ W}$$

So the heat pump requires an input of 750 W thus saving the difference

$$\dot{W}_{saved} = 1500 \text{ W} - 750 \text{ W} = \mathbf{750\ W}$$





7.22

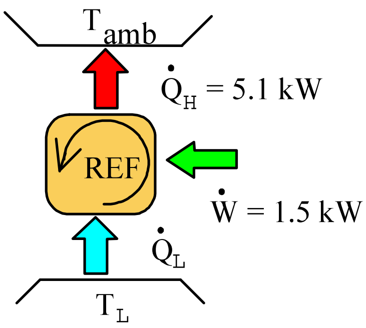
An air-conditioner discards 5.1 kW to the ambient with a power input of 1.5 kW. Find the rate of cooling and the coefficient of performance.

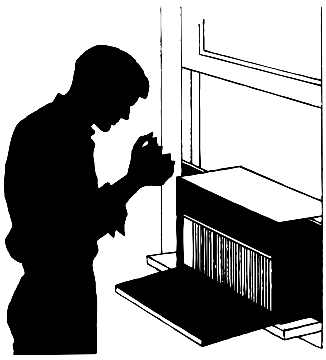
Solution:

In this case $\dot{Q}_H = 5.1$ kW goes to the ambient so

Energy Eq. : $\dot{Q}_L = \dot{Q}_H - \dot{W} = 5.1 - 1.5 = \mathbf{3.6\ kW}$

$$\beta_{REFRIG} = \frac{\dot{Q}_L}{\dot{W}} = \frac{3.6}{1.5} = \mathbf{2.4}$$





7.23

Calculate the thermal efficiency of the steam power plant cycle described in Problem 6.103.

From solution to Problem 6.103,

Turbine $A_5 = (\pi/4)(0.2)^2 = 0.03142 \text{ m}^2$

$$\mathbf{V}_5 = \dot{m}v_5/A_5 = 25 \times 0.06163 / 0.03142 = 49 \text{ m/s}$$

$$h_6 = 191.83 + 0.92 \times 2392.8 = 2393.2 \text{ kJ/kg}$$

$$w_T = 3404 - 2393.2 - (200^2 - 49^2)/(2 \times 1000) = 992 \text{ kJ/kg}$$

$$\dot{W}_T = \dot{m}w_T = 25 \times 992 = 24\,800 \text{ kW}$$

$$\dot{W}_{NET} = 24800 - 300 = 24\,500 \text{ kW}$$

From the solution to Problem 6.105

Economizer $A_7 = \pi D_7^2/4 = 0.004\,418 \text{ m}^2$, $v_7 = 0.001\,008 \text{ m}^3/\text{kg}$

$$\mathbf{V}_2 = \mathbf{V}_7 = \dot{m}v/A_7 = 25 \times 0.001008 / 0.004418 = 5.7 \text{ m/s},$$

$$\mathbf{V}_3 = (v_3/v_2)\mathbf{V}_2 = (0.001\,118 / 0.001\,008)\ 5.7 = 6.3 \text{ m/s} \approx \mathbf{V}_2$$

so kinetic energy change is unimportant

$$q_{ECON} = h_3 - h_2 = 744 - 194 = 550.0 \text{ kJ/kg}$$

$$\dot{Q}_{ECON} = \dot{m}q_{ECON} = 25\ (550.0) = 13\,750 \text{ kW}$$

Generator $A_4 = \pi D_4^2/4 = 0.031\,42 \text{ m}^2$, $v_4 = 0.060\,23 \text{ m}^3/\text{kg}$

$$\mathbf{V}_4 = \dot{m}v_4/A_4 = 25 \times 0.060\,23/0.031\,42 = 47.9 \text{ m/s}$$

$$q_{GEN} = 3426 - 744 + (47.9^2 - 6.3^2)/(2\times1000) = 2683 \text{ kJ/kg}$$

$$\dot{Q}_{GEN} = 25 \times (2683) = 67\,075 \text{ kW}$$

The total added heat transfer is

$$\dot{Q}_H = 13\,758 + 67\,075 = 80\,833 \text{ kW}$$

$$\Rightarrow\ \eta_{TH} = \dot{W}_{NET}/\dot{Q}_H = \frac{24500}{80833} = \mathbf{0.303}$$

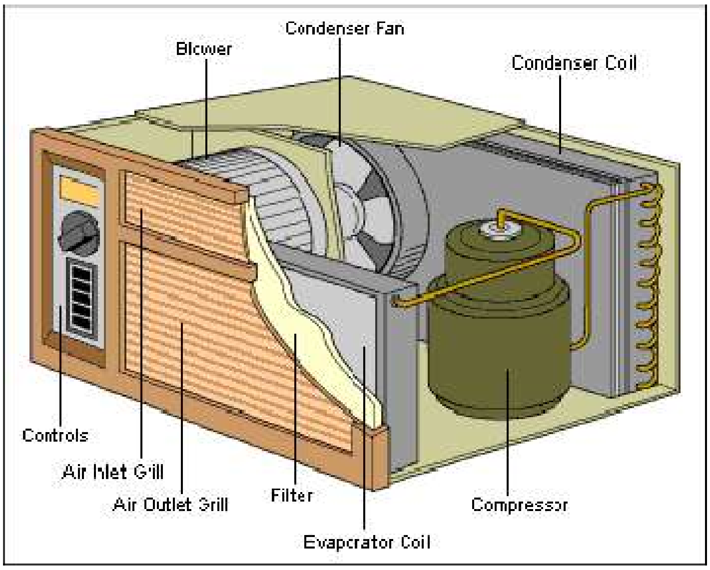
7.24

A window air-conditioner unit is placed on a laboratory bench and tested in cooling mode using 750 W of electric power with a COP of 1.75. What is the cooling power capacity and what is the net effect on the laboratory?

Definition of COP: $\beta = \dot{Q}_L / \dot{W}$

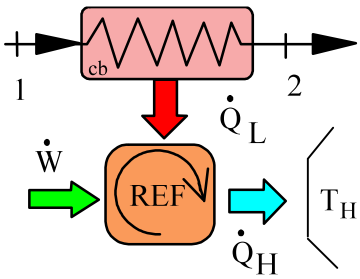
Cooling capacity: $\dot{Q}_L = \beta \dot{W} = 1.75 \times 750 =$ **1313 W**

For steady state operation the $\dot{Q}_L$ comes from the laboratory and $\dot{Q}_H$ goes to the laboratory giving a net to the lab of $\dot{W} = \dot{Q}_H - \dot{Q}_L = 750$ W, that is heating it.



7.25

A water cooler for drinking water should cool 25 L/h water from 18°C to 10°C using a small refrigeration unit with a COP of 2.5. Find the rate of cooling required and the power input to the unit.



The mass flow rate is

$$\dot{m} = \rho\dot{V} = \frac{25 \times 10^{-3}}{0.001002}\frac{1}{3600} \text{ kg/s} = 6.93 \text{ g/s}$$

Energy equation for heat exchanger

$$\dot{Q}_L = \dot{m}(h_1 - h_2) = \dot{m}\, C_P\, (T_1 - T_2)$$
$$= 6.93 \times 10^{-3} \times 4.18 \times (18 - 10) = 0.2318 \text{ kW}$$

$$\beta = \text{COP} = \dot{Q}_L / \dot{W} \quad \Rightarrow \quad \dot{W} = \dot{Q}_L / \beta = 0.2318 / 2.5 = \mathbf{0.093\ kW}$$

Comment: The unit does not operate continuously.

7.26

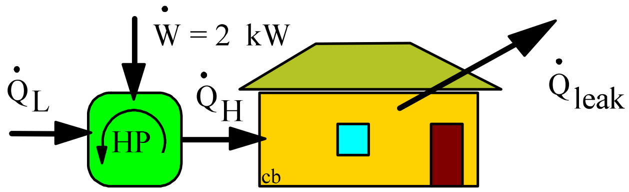
A farmer runs a heat pump with a 2 kW motor. It should keep a chicken hatchery at 30°C, which loses energy at a rate of 10 kW to the colder ambient T_{amb}. What is the minimum coefficient of performance that will be acceptable for the heat pump?

Solution:

Power input: $\dot{W} = 2$ kW

Energy Eq. for hatchery: $\dot{Q}_H = \dot{Q}_{Loss} = 10$ kW

Definition of COP: $\beta = COP = \frac{\dot{Q}_H}{\dot{W}} = \frac{10}{2} = \mathbf{5}$



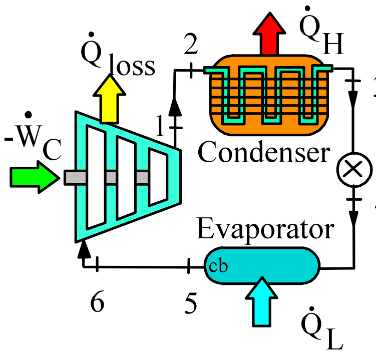
7.27

Calculate the coefficient of performance of the R-410a heat pump cycle described in Problem 6.108.

Solution:

From solution to Problem 6.108,

CV: Condenser



$\dot{Q}_{COND} = \dot{m}(h_3 - h_2)$

$= 0.05$ kg/s $(134 - 367)$ kJ/kg

$= -11.65$ kW

Then with the work as $-\dot{W}_{IN} = 5.0$ kW we have $\dot{Q}_H = -\dot{Q}_{COND}$

Heat pump: $\beta' = \dot{Q}_H/\dot{W}_{IN} = \frac{11.65}{5.0} =$ **2.33**

7.28

A power plant generates 150 MW of electrical power. It uses a supply of 1000 MW from a geothermal source and rejects energy to the atmosphere. Find the power to the air and how much air should be flowed to the cooling tower (kg/s) if its temperature cannot be increased more than 10°C.

Solution:

C.V. Total power plant.

Energy equation gives the amount of heat rejection to the atmosphere as

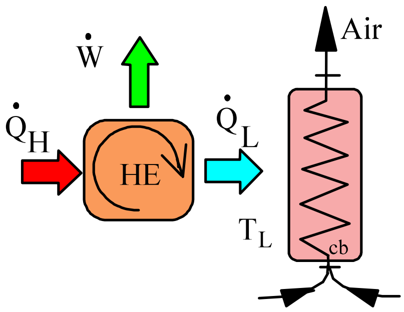
$$\dot{Q}_L = \dot{Q}_H - \dot{W} = 1000 - 150 = \mathbf{850\ MW}$$

The energy equation for the air flow that absorbs the energy is

$$\dot{Q}_L = \dot{m}_{air}\,\Delta h = \dot{m}_{air}\,C_p\,\Delta T$$

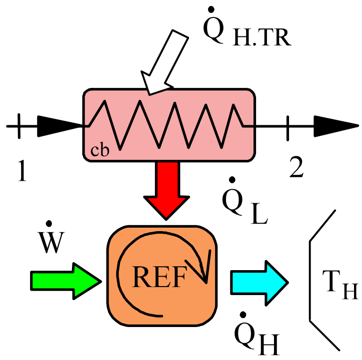
$$\dot{m}_{air} = \frac{\dot{Q}_L}{C_p \Delta T} = \frac{850 \times 1000}{1.004 \times 10} = \mathbf{84\ 661\ kg/s}$$

Probably too large to make, so some cooling by liquid water or evaporative cooling should be used.



7.29

A water cooler for drinking water should cool 25 L/h water from 18°C to 10°C while the water reservoir also gains 60 W from heat transfer. Assume a small refrigeration unit with a COP of 2.5 does the cooling. Find the total rate of cooling required and the power input to the unit.



The mass flow rate is

$$\dot{m} = \rho\dot{V} = \frac{25 \times 10^{-3}}{0.001002}\frac{1}{3600}\text{ kg/s} = 6.93\text{ g/s}$$

Energy equation for heat exchanger

$$\dot{Q}_L = \dot{m}(h_1 - h_2) + \dot{Q}_{H\,TR}$$

$$= \dot{m}\, C_P\, (T_1 - T_2) + \dot{Q}_{H\,TR}$$

$$= 6.93 \times 10^{-3} \times 4.18 \times (18 - 10)\text{ kW} + 60\text{ W}$$
$$= 291.8\text{ W}$$

$$\beta = \text{COP} = \dot{Q}_L / \dot{W} \quad \Rightarrow \quad \dot{W} = \dot{Q}_L / \beta = 291.8 / 2.5 = \mathbf{116.7\ W}$$

Comment: The unit does not operate continuously.

7.30

A car engine delivers 25 hp to the driveshaft with a thermal efficiency of 30%. The fuel has a heating value of 40 000 kJ/kg. Find the rate of fuel consumption and the combined power rejected through the radiator and exhaust.

Solution:

Heating value (HV): $\dot{Q}_H = \dot{m} \cdot HV$

From the definition of the thermal efficiency

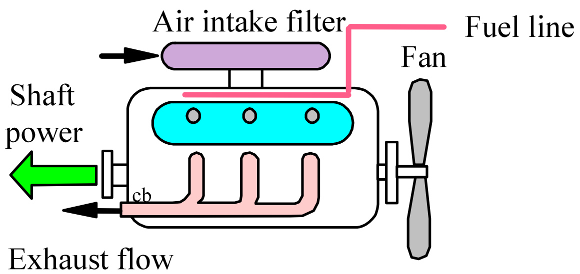
$$\dot{W} = \eta\, \dot{Q}_H = \eta \cdot \dot{m} \cdot HV$$

$$\dot{m} = \frac{\dot{W}}{\eta \cdot HV} = \frac{25 \times 0.7355}{0.3 \times 40\,000} = 0.00153 \text{ kg/s} = \mathbf{1.53\ g/s}$$

Conversion of power from hp to kW in Table A.1.

$$\dot{Q}_L = \dot{Q}_H - \dot{W} = (\dot{W}/\eta - \dot{W}) = (\frac{1}{\eta} - 1)\dot{W}$$

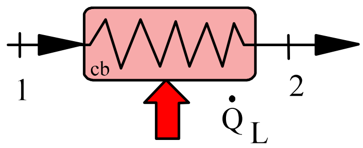
$$= (\frac{1}{0.3} - 1)\ 25 \times 0.7355 = \mathbf{42.9\ kW}$$



7.31

R-410a enters the evaporator (the cold heat exchanger) in an A/C unit at -20°C, x = 28% and leaves at -20°C, x = 1. The COP of the refrigerator is 1.5 and the mass flow rate is 0.003 kg/s. Find the net work input to the cycle.

Energy equation for heat exchanger



$$\dot{Q}_L = \dot{m}(h_2 - h_1) = \dot{m}[h_g - (h_f + x_1 h_{fg})]$$

$$= \dot{m}[h_{fg} - x_1 h_{fg}] = \dot{m}\,(1 - x_1)h_{fg}$$

$$= 0.003 \text{ kg/s} \times 0.72 \times 243.65 \text{ kJ/kg} = 0.5263 \text{ kW}$$

$$\beta = \text{COP} = \dot{Q}_L / \dot{W} \quad \Rightarrow \quad \dot{W} = \dot{Q}_L / \beta = 0.5263 / 1.5 = \mathbf{0.35\ kW}$$

7.32

For each of the cases below determine if the heat engine satisfies the first law (energy equation) and if it violates the second law.

a. $\dot{Q}_H = 6$ kW, $\dot{Q}_L = 4$ kW, $\dot{W} = 2$ kW

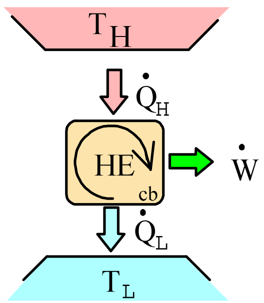
b. $\dot{Q}_H = 6$ kW, $\dot{Q}_L = 0$ kW, $\dot{W} = 6$ kW

c. $\dot{Q}_H = 6$ kW, $\dot{Q}_L = 2$ kW, $\dot{W} = 5$ kW

d. $\dot{Q}_H = 6$ kW, $\dot{Q}_L = 6$ kW, $\dot{W} = 0$ kW

Solution:

	1^{st}. law	2^{nd} law
a	Yes	Yes (possible)
b	Yes	No, impossible Kelvin - Planck
c	No	Yes, but energy not conserved
d	Yes	Yes (Irreversible $\dot{Q}$ over ΔT)



7.33

For each of the cases in problem 7.32 determine if a heat pump satisfies the first law (energy equation) and if it violates the second law.

a. $\dot{Q}_H = 6$ kW, $\dot{Q}_L = 4$ kW, $\dot{W} = 2$ kW

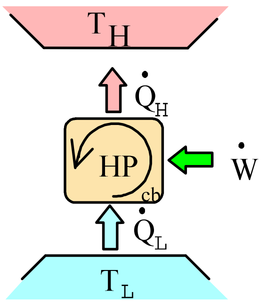
b. $\dot{Q}_H = 6$ kW, $\dot{Q}_L = 0$ kW, $\dot{W} = 6$ kW

c. $\dot{Q}_H = 6$ kW, $\dot{Q}_L = 2$ kW, $\dot{W} = 5$ kW

d. $\dot{Q}_H = 6$ kW, $\dot{Q}_L = 6$ kW, $\dot{W} = 0$ kW

Solution:

	1^{st}. law	2^{nd} law
a	Satisfied	Does not violate
b	Satisfied	Does not violate
c	Violated	Does not violate, but 1^{st} law
d	Satisfied	Does violate, Clausius



7.34

A large stationary diesel engine produces 15 MW with a thermal efficiency of 40%. The exhaust gas, which we assume is air, flows out at 800 K and the intake air is 290 K. How large a mass flow rate is that, assuming this is the only way we reject heat? Can the exhaust flow energy be used?

Heat engine: $\dot{Q}_H = \dot{W}_{out}/\eta_{TH} = \frac{15}{0.4} = 37.5$ MW

Energy equation: $\dot{Q}_L = \dot{Q}_H - \dot{W}_{out} = 37.5 – 15 = 22.5$ kW

Exhaust flow: $\dot{Q}_L = \dot{m}_{air}(h_{800} - h_{290})$

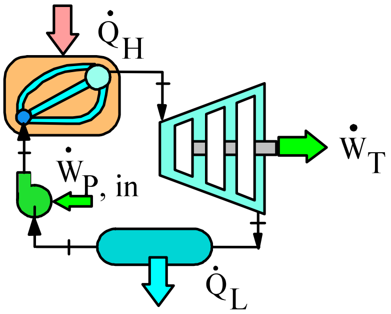
$$\dot{m}_{air} = \frac{\dot{Q}_L}{h_{800} - h_{290}} = \frac{22.5 \times 1000}{822.2 - 290.43} = \mathbf{42.3\ kg/s}$$

The flow of hot gases can be used to heat a building or it can be used to heat water in a steam power plant since that operates at lower temperatures.

7.35

In a steam power plant 1 MW is added in the boiler, 0.58 MW is taken out in the condenser and the pump work is 0.02 MW. Find the plant thermal efficiency. If everything could be reversed find the coefficient of performance as a refrigerator.

Solution:



CV. Total plant:

Energy Eq.:

$$\dot{Q}_H + \dot{W}_{P,in} = \dot{W}_T + \dot{Q}_L$$

$$\dot{W}_T = 1 + 0.02 - 0.58 = 0.44 \text{ MW}$$

$$\eta_{TH} = \frac{\dot{W}_T - \dot{W}_{P,in}}{\dot{Q}_H} = \frac{440 - 20}{1000} = \mathbf{0.42}$$

$$\beta = \frac{\dot{Q}_L}{\dot{W}_T - \dot{W}_{P,in}} = \frac{580}{440 - 20} = \mathbf{1.38}$$

7.36

Calculate the amount of work input a refrigerator needs to make ice cubes out of a tray of 0.25 kg liquid water at 10°C. Assume the refrigerator has $\beta = 3.5$ and a motor-compressor of 750 W. How much time does it take if this is the only cooling load?

C.V. Water in tray. We neglect tray mass.

Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process : $P = \text{constant} = P_o$

$${}_1W_2 = \int P\, dV = P_o m(v_2 - v_1)$$

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(h_2 - h_1)$$

Tbl. B.1.1 : $h_1 = 41.99$ kJ/kg, Tbl. B.1.5 : $h_2 = -\ 333.6$ kJ/kg

$${}_1Q_2 = 0.25(-333.4 - 41.99\ ) = -\ 93.848 \text{ kJ}$$

Consider now refrigerator

$$\beta = Q_L/W$$

$$W = Q_L/\beta = -\ {}_1Q_2/\ \beta = 93.848/3.5 = \mathbf{26.81\ kJ}$$

For the motor to transfer that amount of energy the time is found as

$$W = \int \dot{W}\, dt = \dot{W}\, \Delta t$$

$$\Delta t = W/\dot{W} = (26.81 \times 1000)/750 = \mathbf{35.75\ s}$$

Comment: We neglected a baseload of the refrigerator so not all the 750 W are available to make ice, also our coefficient of performance is very optimistic and finally the heat transfer is a transient process. All this means that it will take much more time to make ice-cubes.

Second Law and Processes

7.37

Prove that a cyclic device that violates the Kelvin–Planck statement of the second law also violates the Clausius statement of the second law.

Solution: Proof very similar to the proof in section 7.2.

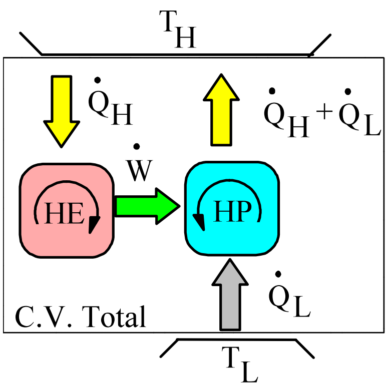
H.E. violating Kelvin receives Q_H from T_H and produces net $W = Q_H$.

This W input to H.P. receiving Q_L from T_L.

H.P. discharges $Q_H + Q_L$ to T_H.

Net Q to T_H is : $-Q_H + Q_H + Q_L = Q_L$.

H.E. + H.P. together transfers Q_L from T_L to T_H with no W thus violates Clausius.



7.38

Assume a cyclic machine that exchanges 6 kW with a 250°C reservoir and has

a. $\dot{Q}_L = 0$ kW, $\dot{W} = 6$ kW

b. $\dot{Q}_L = 6$ kW, $\dot{W} = 0$ kW

and $\dot{Q}_L$ is exchanged with a 30°C ambient. What can you say about the processes in the two cases a and b if the machine is a heat engine? Repeat the question for the case of a heat pump.

Solution:

Heat engine

a. Since $\dot{Q}_L = 0$ impossible Kelvin – Planck

b. Possible, irreversible, $\eta_{eng} = 0$

Heat pump

a. Possible, irreversible (like an electric heater)

b. Impossible, $\beta \rightarrow \infty$, Clausius

7.39

Discuss the factors that would make the power plant cycle described in Problem 6.103 an irreversible cycle.

Solution:

General discussion, but here are a few of the most significant factors.

1. Combustion process that generates the hot source of energy.
2. Heat transfer over finite temperature difference in boiler.
3. Flow resistance and friction in turbine results in less work out.
4. Flow friction and heat loss to/from ambient in all pipes.
5. Heat transfer over finite temperature difference in condenser.

7.40

Discuss the factors that would make the heat pump described in Problem 6.108 an irreversible cycle.

Solution:

General discussion but here are a few of the most significant factors.

1. Unwanted heat transfer in the compressor.
2. Pressure loss (back flow leak) in compressor
3. Heat transfer and pressure drop in line 1 => 2.
4. Pressure drop in all lines.
5. Throttle process 3 => 4.

7.41

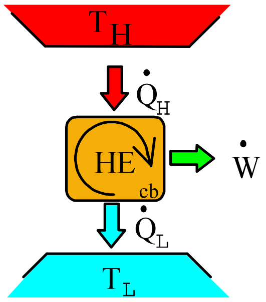
Consider the four cases of a heat engine in problem 7.32 and determine if any of those are perpetual machines of the first or second kind.

a. $\dot{Q}_H = 6$ kW, $\dot{Q}_L = 4$ kW, $\dot{W} = 2$ kW

b. $\dot{Q}_H = 6$ kW, $\dot{Q}_L = 0$ kW, $\dot{W} = 6$ kW

c. $\dot{Q}_H = 6$ kW, $\dot{Q}_L = 2$ kW, $\dot{W} = 5$ kW

d. $\dot{Q}_H = 6$ kW, $\dot{Q}_L = 6$ kW, $\dot{W} = 0$ kW



Solution:

	1st. law	2nd law
a	Yes	Yes (possible)
b	Yes	No, impossible Kelvin - Planck Perpetual machine second kind It violates the 2nd law converts all $\dot{Q}$ to $\dot{W}$
c	No	Yes, but energy not conserved Perpetual machine first kind It generates energy inside
d	Yes	Yes (Irreversible $\dot{Q}$ over ΔT)

7.42

Consider a heat engine and heat pump connected as shown in figure P7.42. Assume $T_{H1} = T_{H2} > T_{amb}$ and determine for each of the three cases if the setup satisfy the first law and/or violates the 2nd law.

	$\dot{Q}_{H1}$	$\dot{Q}_{L1}$	$\dot{W}_1$	$\dot{Q}_{H2}$	$\dot{Q}_{L2}$	$\dot{W}_2$
a	6	4	2	3	2	1
b	6	4	2	5	4	1
c	3	2	1	4	3	1

Solution:

	1st. law	2nd law
a	Yes	Yes (possible)
b	Yes	No, combine Kelvin - Planck
c	Yes	No, combination clausius

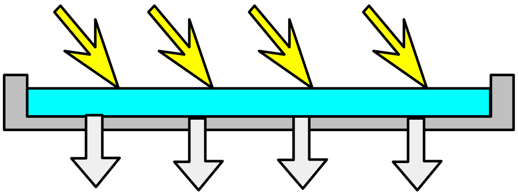
7.43

The water in a shallow pond heats up during the day and cools down during the night. Heat transfer by radiation, conduction and convection with the ambient thus cycles the water temperature. Is such a cyclic process reversible or irreversible?

Solution:

All the heat transfer takes place over a finite ΔT and thus all the heat transfer processes are irreversible.

Conduction and convection have ΔT in the water, which is internally irreversible and ΔT outside the water which is externally irreversible. The radiation is absorbed or given out at the water temperature thus internally (for absorption) and externally (for emission) irreversible.



Carnot Cycles and Absolute Temperature

7.44

Calculate the thermal efficiency of a Carnot cycle heat engine operating between reservoirs at 300°C and 45°C. Compare the result to that of Problem 7.16.

Solution:

$$\eta_{TH} = W_{net} / Q_H = 1 - \frac{T_L}{T_H} = 1 - \frac{45 + 273}{300 + 273} = \mathbf{0.445} \text{ (Carnot)}$$

$\eta_{7.16} = 0.225$ (efficiency about ½ of the Carnot)

7.45

A Carnot cycle heat engine has an efficiency of 40%. If the high temperature is raised 10% what is the new efficiency keeping the same low temperature?

Solution:

$$\eta_{TH} = W_{net} / Q_H = 1 - \frac{T_L}{T_H} = 0.4 \quad \Rightarrow \quad \frac{T_L}{T_H} = 0.6$$

so if T_H is raised 10% the new ratio becomes

$$\frac{T_L}{T_{H\,new}} = 0.6 / 1.1 = 0.5454 \quad \Rightarrow \quad \eta_{TH\,new} = 1 - 0.5454 = \mathbf{0.45}$$

7.46

Find the power output and the low T heat rejection rate for a Carnot cycle heat engine that receives 6 kW at 250°C and rejects heat at 30°C as in Problem 7.38.

Solution:

From the definition of the absolute temperature Eq. 7.8

$$\eta_{carnot} = 1 - \frac{T_L}{T_H} = 1 - \frac{303}{523} = 0.42$$

Definition of the heat engine efficiency gives the work as

$$\dot{W} = \eta\ \dot{Q}_H = 0.42 \times 6 = \mathbf{2.52\ kW}$$

Apply the energy equation

$$\dot{Q}_L = \dot{Q}_H - \dot{W} = 6 - 2.52 = \mathbf{3.48\ kW}$$

7.47

Consider the setup with two stacked (temperature wise) heat engines as in Fig. P7.4. Let $T_H = 900$ K, $T_M = 600$ K and $T_L = 300$ K. Find the two heat engine efficiencies and the combined overall efficiency assuming Carnot cycles.

The individual efficiencies

$$\eta_1 = 1 - \frac{T_M}{T_H} = 1 - \frac{600}{900} = \mathbf{0.333}$$

$$\eta_2 = 1 - \frac{T_L}{T_M} = 1 - \frac{300}{600} = \mathbf{0.5}$$

The overall efficiency

$$\eta_{TH} = \dot{W}_{net} / \dot{Q}_H = (\dot{W}_1 + \dot{W}_2) / \dot{Q}_H = \eta_1 + \dot{W}_2 / \dot{Q}_H$$

For the second heat engine and the energy Eq. for the first heat engine

$$\dot{W}_2 = \eta_2 \, \dot{Q}_M = \eta_2 \, (1 - \eta_1) \, \dot{Q}_H$$

so the final result is

$$\eta_{TH} = \eta_1 + \eta_2 \, (1 - \eta_1) = 0.333 + 0.5(1 - 0.333) = \mathbf{0.667}$$

Comment: It matches a single heat engine $\eta_{TH} = 1 - \frac{T_L}{T_H} = 1 - \frac{300}{900} = \frac{2}{3}$

7.48

At a few places where the air is very cold in the winter, like –30°C it is possible to find a temperature of 13°C down below ground. What efficiency will a heat engine have operating between these two thermal reservoirs?

Solution:

$$\eta_{TH} = 1 - \frac{T_L}{T_H}$$

The ground becomes the hot source and the atmosphere becomes the cold side of the heat engine

$$\eta_{TH} = 1 - \frac{273 - 30}{273 + 13} = 1 - \frac{243}{286} = \mathbf{0.15}$$

This is low because of the modest temperature difference.

7.49

Find the maximum coefficient of performance for the refrigerator in your kitchen, assuming it runs in a Carnot cycle.

Solution:

The refrigerator coefficient of performance is

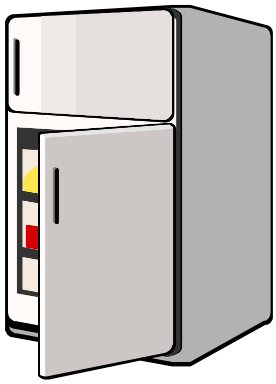
$$\beta = Q_L/W = Q_L/(Q_H - Q_L) = T_L/(T_H - T_L)$$

Assuming $T_L \sim 0°C$, $T_H \sim 35°C$,

$$\beta \le \frac{273.15}{35 - 0} = \mathbf{7.8}$$

Actual working fluid temperatures must be such that

$$T_L < T_{refrigerator} \quad \text{and} \quad T_H > T_{room}$$



A refrigerator does not operate in a Carnot cycle. The actual vapor compression cycle is examined in Chapter 11.

7.50

A refrigerator should remove 500 kJ from some food. Assume the refrigerator works in a Carnot cycle between –10°C and 45°C with a motor-compressor of 500 W. How much time does it take if this is the only cooling load?

Assume Carnot cycle refrigerator

$$\beta = \frac{\dot{Q}_L}{\dot{W}} = \dot{Q}_L / (\dot{Q}_H - \dot{Q}_L) \cong \frac{T_L}{T_H - T_L} = \frac{273 - 10}{45 - (-10)} = 4.785$$

This gives the relation between the low T heat transfer and the work as

$$\dot{Q}_L = \frac{Q}{t} = 4.785\ \dot{W}$$

$$t = \frac{Q}{\beta\ \dot{W}} = \frac{500 \times 1000}{4.785 \times 500} = \mathbf{209\ s}$$

7.51

A car engine burns 5 kg fuel (equivalent to addition of Q_H) at 1500 K and rejects energy to the radiator and the exhaust at an average temperature of 750 K. If the fuel provides 40 000 kJ/kg what is the maximum amount of work the engine can provide?

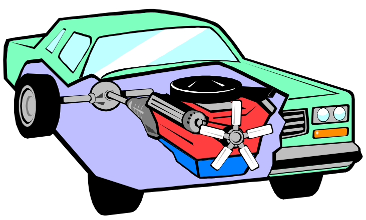
Solution:

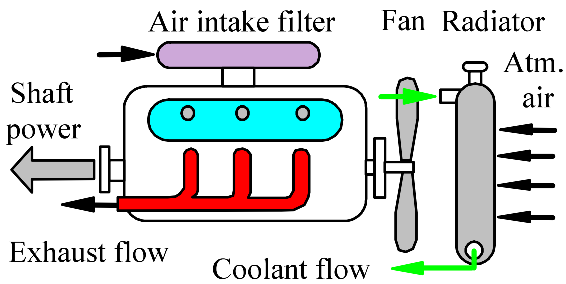
A heat engine $Q_H = m\ q_{fuel} = 5 \times 40\ 000 = 200\ 000$ kJ

Assume a Carnot efficiency (maximum theoretical work)

$$\eta = 1 - \frac{T_L}{T_H} = 1 - \frac{750}{1500} = 0.5$$

$W = \eta\ Q_H =$ **100 000 kJ**





7.52

A large heat pump should upgrade 5 MW of heat at 85°C to be delivered as heat at 150°C. What is the minimum amount of work (power) input that will drive this?

For the minimum work we assume a Carnot heat pump and $\dot{Q}_L = 5$ MW.

$$\beta_{HP} = \frac{\dot{Q}_H}{\dot{W}_{in}} = \frac{T_H}{T_H - T_L} = \frac{273.15 + 150}{150 - 85} = 6.51$$

$$\beta_{REF} = \beta_{HP} - 1 = \frac{\dot{Q}_L}{\dot{W}_{in}} = 5.51$$

Now we can solve for the work

$$\dot{W}_{in} = \dot{Q}_L/\beta_{REF} = 5/5.51 = \mathbf{0.907\ MW}$$

7.53

An air-conditioner provides 1 kg/s of air at 15°C cooled from outside atmospheric air at 35°C. Estimate the amount of power needed to operate the air-conditioner. Clearly state all assumptions made.

Solution:

Consider the cooling of air which needs a heat transfer as

$$\dot{Q}_{air} = \dot{m}\ \Delta h \cong \dot{m}\ C_p\Delta T = 1\ \text{kg/s} \times 1.004\ \text{kJ/kg K} \times 20\ \text{K} = 20\ \text{kW}$$

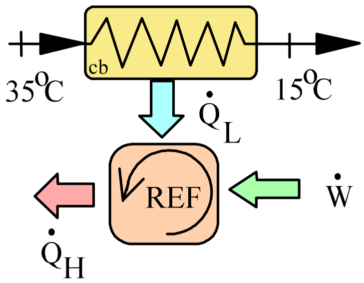
Assume Carnot cycle refrigerator

$$\beta = \frac{\dot{Q}_L}{\dot{W}} = \dot{Q}_L / (\dot{Q}_H - \dot{Q}_L) \cong \frac{T_L}{T_H - T_L} = \frac{273 + 15}{35 - 15} = 14.4$$

$$\dot{W} = \dot{Q}_L / \beta = \frac{20.0}{14.4} = \mathbf{1.39\ kW}$$

This estimate is the theoretical maximum performance. To do the required heat transfer $T_L \cong 5°C$ and $T_H = 45°C$ are more likely; secondly

$$\beta < \beta_{carnot}$$



7.54

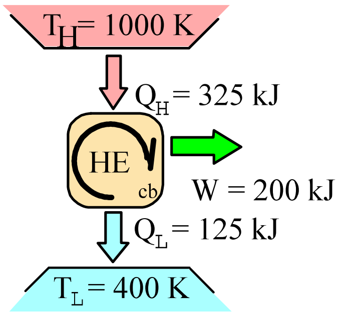
A cyclic machine, shown in Fig. P7.54, receives 325 kJ from a 1000 K energy reservoir. It rejects 125 kJ to a 400 K energy reservoir and the cycle produces 200 kJ of work as output. Is this cycle reversible, irreversible, or impossible?

Solution:

$$\eta_{Carnot} = 1 - \frac{T_L}{T_H} = 1 - \frac{400}{1000} = 0.6$$

$$\eta_{eng} = \frac{W}{Q_H} = \frac{200}{325} = 0.615 > \eta_{Carnot}$$

This is **impossible.**



7.55

A sales person selling refrigerators and deep freezers will guarantee a minimum coefficient of performance of 4.5 year round. How would you evaluate that? Are they all the same?

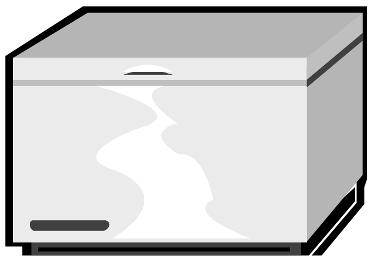
Solution:

Assume a high temperature of 35°C. If a freezer compartment is included $T_L \sim -20°C$ (deep freezer) and fluid temperature is then $T_L \sim -30°C$

$$\beta_{\text{deep freezer}} \le \frac{T_L}{T_H - T_L} = \frac{273.15 - 30}{35 - (-30)} = 3.74$$

A hot summer day may require a higher T_H to push Q_H out into the room, so even lower β.

Claim is possible for a refrigerator, but not for a deep freezer.



7.56

A temperature of about 0.01 K can be achieved by magnetic cooling. In this process a strong magnetic field is imposed on a paramagnetic salt, maintained at 1 K by transfer of energy to liquid helium boiling at low pressure. The salt is then thermally isolated from the helium, the magnetic field is removed, and the salt temperature drops. Assume that 1 mJ is removed at an average temperature of 0.1 K to the helium by a Carnot-cycle heat pump. Find the work input to the heat pump and the coefficient of performance with an ambient at 300 K.

Solution:

$$\beta = \dot{Q}_L/\dot{W}_{IN} = \frac{T_L}{T_H - T_L} = \frac{0.1}{299.9} = \mathbf{0.00033}$$

$$\dot{W}_{IN} = \frac{1\times10^{-3}}{0.00033} = \mathbf{3\ J}$$

Remark: This is an extremely large temperature difference for a heat pump.

7-57

The lowest temperature that has been achieved is about 1×10^{-6} K. To achieve this an additional stage of cooling is required beyond that described in the previous problem, namely nuclear cooling. This process is similar to magnetic cooling, but it involves the magnetic moment associated with the nucleus rather than that associated with certain ions in the paramagnetic salt. Suppose that 10 μJ is to be removed from a specimen at an average temperature of 10^{-5} K (ten micro-joules is about the potential energy loss of a pin dropping 3 mm). Find the work input to a Carnot heat pump and its coefficient of performance to do this assuming the ambient is at 300 K.

Solution:

$$Q_L = 10\ \mu\text{J} = 10\times10^{-6}\ \text{J} \quad \text{at} \quad T_L = 10^{-5}\ \text{K}$$

$$\Rightarrow Q_H = Q_L \times \frac{T_H}{T_L} = 10\times10^{-6} \times \frac{300}{10^{-5}} = 300\ \text{J}$$

$$W_{in} = Q_H - Q_L = 300 - 10\times10^{-6} \cong \mathbf{300\ J}$$

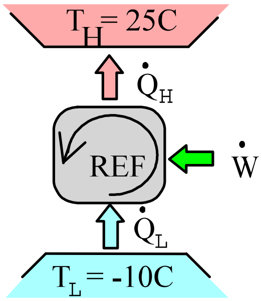
$$\beta = \frac{Q_L}{W_{in}} = \frac{10\times10^{-6}}{300} = \mathbf{3.33\times10^{-8}}$$

7.58

An inventor has developed a refrigeration unit that maintains the cold space at −10°C, while operating in a 25°C room. A coefficient of performance of 8.5 is claimed. How do you evaluate this?

Solution:

$$\beta_{Carnot} = \frac{Q_L}{W_{in}} = \frac{T_L}{T_H - T_L} = \frac{263.15}{25 - (-10)} = 7.52$$



$8.5 > \beta_{Carnot} \Rightarrow$ **impossible claim**

7.59

Calculate the amount of work input a refrigerator needs to make ice cubes out of a tray of 0.25 kg liquid water at 10°C. Assume the refrigerator works in a Carnot cycle between –8°C and 35°C with a motor-compressor of 750 W. How much time does it take if this is the only cooling load?

Solution:

C.V. Water in tray. We neglect tray mass.

Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process : $P = \text{constant} + P_o$

$${}_1W_2 = \int P\, dV = P_o m(v_2 - v_1)$$

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(h_2 - h_1)$$

Tbl. B.1.1 : $h_1 = 41.99$ kJ/kg, Tbl. B.1.5 : $h_2 = -\ 333.6$ kJ/kg

$${}_1Q_2 = 0.25(-333.4 - 41.99) = -\ 93.848 \text{ kJ}$$

Consider now refrigerator

$$\beta = \frac{Q_L}{W} = \frac{Q_L}{Q_H - Q_L} = \frac{T_L}{T_H - T_L} = \frac{273 - 8}{35 - (-8)} = 6.16$$

$$W = \frac{Q_L}{\beta} = -\frac{{}_1Q_2}{\beta} = \frac{93.848}{6.16} = 15.24 \text{ kJ}$$

For the motor to transfer that amount of energy the time is found as

$$W = \int \dot{W}\, dt = \dot{W}\, \Delta t$$

$$\Delta t = \frac{W}{\dot{W}} = \frac{15.24 \times 1000}{750} = \mathbf{20.3\ s}$$

Comment: We neglected a baseload of the refrigerator so not all the 750 W are available to make ice, also our coefficient of performance is very optimistic and finally the heat transfer is a transient process. All this means that it will take much more time to make ice-cubes.

7.60

A heat pump receives energy from a source at 80°C and delivers energy to a boiler that operates at 350 kPa. The boiler input is saturated liquid water and the exit is saturated vapor both at 350 kPa. The heat pump is driven by a 2.5 MW motor and has a COP that is 60% of a Carnot heat pump COP. What is the maximum mass flow rate of water the system can deliver?

$T_H = T_{sat} = 138.88^oC = 412$ K, $\quad h_{fg} = 2148.1$ kJ/kg

$$\beta_{HP\ Carnot} = \frac{\dot{Q}_H}{\dot{W}_{in}} = \frac{T_H}{T_H - T_L} = \frac{412}{138.88 - 80} = 7$$

$\beta_{HP\ ac} = 0.6 \times 7 = 4.2 = \dot{Q}_H/\dot{W}_{in}$

$\dot{Q}_H = 4.2\ \dot{W}_{in} = 4.2 \times 2.5\ \text{MW} = 10.5\ \text{MW} = \dot{m}\ h_{fg}$

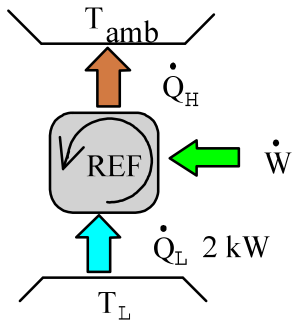
$\dot{m} = \dot{Q}_H / h_{fg} = 10\ 500\ \text{kW} / 2148.1\ \text{kJ/kg} =$ **4.89 kg/s**

7.61

A household freezer operates in a room at 20°C. Heat must be transferred from the cold space at a rate of 2 kW to maintain its temperature at –30°C. What is the theoretically smallest (power) motor required to operate this freezer?

Solution:

Assume a Carnot cycle between T_L = -30°C and T_H = 20°C:



$$\beta = \frac{\dot{Q}_L}{\dot{W}_{in}} = \frac{T_L}{T_H - T_L} = \frac{273.15 - 30}{20 - (-30)} = 4.86$$

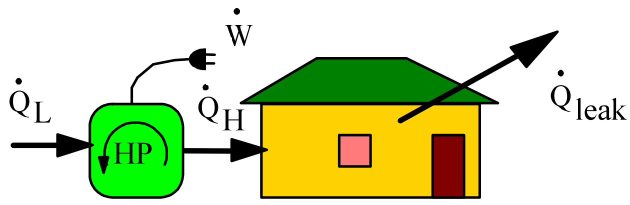
$$\dot{W}_{in} = \dot{Q}_L/\beta = 2/4.86 = \mathbf{0.41\ kW}$$

This is the theoretical minimum power input.
Any actual machine requires a larger input.

7.62

We propose to heat a house in the winter with a heat pump. The house is to be maintained at 20°C at all times. When the ambient temperature outside drops to −10°C, the rate at which heat is lost from the house is estimated to be 25 kW. What is the minimum electrical power required to drive the heat pump?

Solution:



Minimum power if we assume a Carnot cycle

$\dot{Q}_H = \dot{Q}_{leak} = 25$ kW

$$\beta' = \frac{\dot{Q}_H}{\dot{W}_{IN}} = \frac{T_H}{T_H - T_L} = \frac{293.2}{30} = 9.773 \quad \Rightarrow \quad \dot{W}_{IN} = \frac{25}{9.773} = \mathbf{2.56\ kW}$$

7.63

A certain solar-energy collector produces a maximum temperature of 100°C. The energy is used in a cyclic heat engine that operates in a 10°C environment. What is the maximum thermal efficiency? What is it, if the collector is redesigned to focus the incoming light to produce a maximum temperature of 300°C?

Solution:

For $T_H = 100°C = 373.2$ K & $T_L = 283.2$ K

$$\eta_{th\ max} = \frac{T_H - T_L}{T_H} = \frac{90}{373.2} = \mathbf{0.241}$$

For $T_H = 300°C = 573.2$ K & $T_L = 283.2$ K

$$\eta_{th\ max} = \frac{T_H - T_L}{T_H} = \frac{290}{573.2} = \mathbf{0.506}$$

7.64

Helium has the lowest normal boiling point of any of the elements at 4.2 K. At this temperature the enthalpy of evaporation is 83.3 kJ/kmol. A Carnot refrigeration cycle is analyzed for the production of 1 kmol of liquid helium at 4.2 K from saturated vapor at the same temperature. What is the work input to the refrigerator and the coefficient of performance for the cycle with an ambient at 300 K?

Solution:

For the Carnot cycle the ratio of the heat transfers is the ratio of temperatures

$$Q_L = n\,\bar{h}_{fg} = 1 \text{ kmol} \times 83.3 \text{ kJ/kmol} = 83.3 \text{ kJ}$$

$$Q_H = Q_L \times \frac{T_H}{T_L} = 83.3 \times \frac{300}{4.2} = 5950 \text{ kJ}$$

$$W_{IN} = Q_H - Q_L = 5950 - 83.3 = \mathbf{5886.7\ kJ}$$

$$\beta = \frac{Q_L}{W_{IN}} = \frac{83.3}{5886.7} = \mathbf{0.0142} \qquad [\ = \frac{T_L}{T_H - T_L}\ ]$$

7.65

A thermal storage is made with a rock (granite) bed of 2 m^3 which is heated to 400 K using solar energy. A heat engine receives a Q_H from the bed and rejects heat to the ambient at 290 K. The rock bed therefore cools down and as it reaches 290 K the process stops. Find the energy the rock bed can give out. What is the heat engine efficiency at the beginning of the process and what is it at the end of the process?

Solution:

Assume the whole setup is reversible and that the heat engine operates in a Carnot cycle. The total change in the energy of the rock bed is

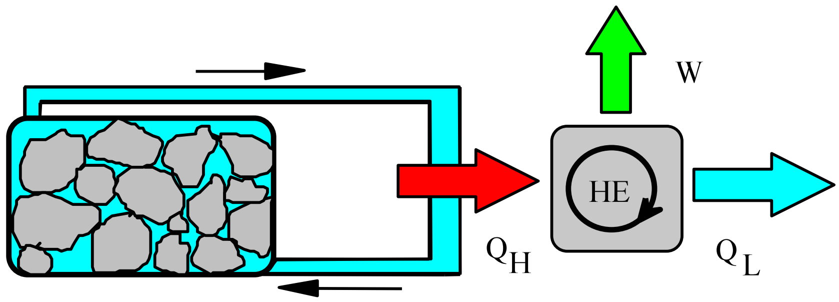
$u_2 - u_1 = q = C\ \Delta T = 0.89\ (400 - 290) = 97.9$ kJ/kg

$m = \rho V = 2750 \times 2 = 5500$ kg , $Q = mq = 5500 \times 97.9 =$ **538 450 kJ**

To get the efficiency use the CARNOT cycle result as

$\eta = 1 - T_o/T_H = 1 - 290/400 =$ **0.275** at the beginning of process

$\eta = 1 - T_o/T_H = 1 - 290/290 =$ **0.0** at the end of process



7.66

In a cryogenic experiment you need to keep a container at −125°C although it gains 100 W due to heat transfer. What is the smallest motor you would need for a heat pump absorbing heat from the container and rejecting heat to the room at 20°C?

Solution:

We do not know the actual device so find the work for a Carnot cycle

$$\beta_{REF} = \dot{Q}_L / \dot{W} = \frac{T_L}{T_H - T_L} = \frac{148.15}{20 - (-125)} = 1.022$$

=> $\dot{W} = \dot{Q}_L / \beta_{REF} = 100/1.022 = \mathbf{97.8\ W}$

7.67

It is proposed to build a 1000-MW electric power plant with steam as the working fluid. The condensers are to be cooled with river water (see Fig. P7.67). The maximum steam temperature is 550°C, and the pressure in the condensers will be 10 kPa. Estimate the temperature rise of the river downstream from the power plant.

Solution:

$$\dot{W}_{NET} = 10^6 \text{ kW}, \quad T_H = 550°\text{C} = 823.3 \text{ K}$$

$$P_{COND} = 10 \text{ kPa} \rightarrow T_L = T_G \text{ (P = 10 kPa)} = 45.8°\text{C} = 319 \text{ K}$$

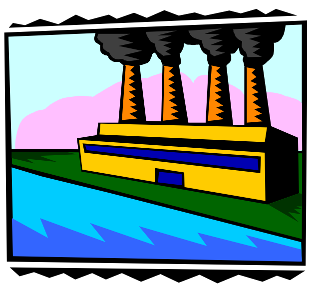
$$\eta_{TH\ CARNOT} = \frac{T_H - T_L}{T_H} = \frac{823.2 - 319}{823.2} = 0.6125$$

$$\Rightarrow \dot{Q}_{L\ MIN} = 10^6 \left(\frac{1 - 0.6125}{0.6125}\right) = 0.6327 \times 10^6 \text{ kW}$$

But $\dot{m}_{H_2O} = \dfrac{60 \times 8 \times 10/60}{0.001} = 80\ 000$ kg/s having an energy flow of

$$\dot{Q}_{L\ MIN} = \dot{m}_{H_2O}\, \Delta h = \dot{m}_{H_2O}\, C_{P\ LIQ\ H_2O}\, \Delta T_{H_2O\ MIN}$$

$$\Rightarrow \Delta T_{H_2O\ MIN} = \frac{\dot{Q}_{L\ MIN}}{\dot{m}_{H_2O} C_{P\ LIQ\ H_2O}} = \frac{0.6327 \times 10^6}{80000 \times 4.184} = \mathbf{1.9°C}$$



7.68

Repeat the previous problem using a more realistic thermal efficiency of 35%.

$$\dot{W}_{NET} = 10^6 \text{ kW} = \eta_{TH\ ac}\, \dot{Q}_H, \qquad \eta_{TH\ ac} = 0.35$$

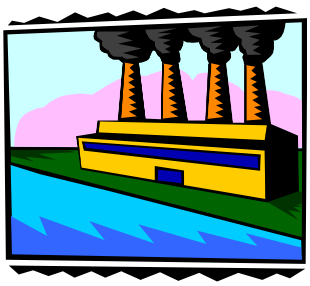
$$\Rightarrow \dot{Q}_L = \dot{Q}_H - \dot{W}_{NET} = \dot{W}_{NET}/\eta_{TH\ ac} - \dot{W}_{NET} = \dot{W}_{NET}(1/\eta_{TH\ ac} - 1)$$

$$= 10^6 \text{ kW} \left(\frac{1 - 0.35}{0.35}\right) = 1.857 \times 10^6 \text{ kW}$$

But $\dot{m}_{H_2O} = \dfrac{60 \times 8 \times 10/60}{0.001} = 80\ 000$ kg/s having an energy flow of

$$\dot{Q}_L = \dot{m}_{H_2O}\, \Delta h = \dot{m}_{H_2O}\, C_{P\ LIQ\ H_2O}\, \Delta T_{H_2O}$$

$$\Rightarrow \Delta T_{H_2O} = \frac{\dot{Q}_L}{\dot{m}_{H_2O} C_{P\ LIQ\ H_2O}} = \frac{1.857 \times 10^6}{80\ 000 \times 4.18} = \mathbf{5.6°C}$$



7.69

A steel bottle $V = 0.1\ m^3$ contains R-134a at 20°C, 200 kPa. It is placed in a deep freezer where it is cooled to -20°C. The deep freezer sits in a room with ambient temperature of 20°C and has an inside temperature of -20°C. Find the amount of energy the freezer must remove from the R-134a and the extra amount of work input to the freezer to do the process.

Solution:

C.V. R-134a out to the -20 °C space.

Energy equation: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process : V = Const $\Rightarrow v_2 = v_1 \Rightarrow {}_1W_2 = 0$

Table B.5.2: $v_1 = 0.11436\ m^3/kg$, $u_1 = 395.27\ kJ/kg$

$m = V/v_1 = 0.87443\ kg$

State 2: $v_2 = v_1 < v_g = 0.14649$ Table B.5.1 => 2 phase

$$\Rightarrow x_2 = \frac{v - v_f}{v_{fg}} = \frac{0.11436 - 0.000738}{0.14576} = 0.77957$$

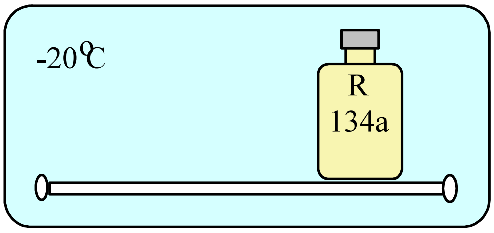
$$u_2 = 173.65 + 0.77957 \times 192.85 = 323.99\ kJ/kg$$

$${}_1Q_2 = m(u_2 - u_1) = \mathbf{-62.334\ kJ}$$

Consider the freezer and assume Carnot cycle

$$\beta = \frac{Q_L}{W} = \frac{Q_L}{Q_H - Q_L} = \frac{T_L}{T_H - T_L} = \frac{273 - 20}{20 - (-20)} = 6.33$$

$$W_{in} = Q_L / \beta = 62.334 / 6.33 = \mathbf{9.85\ kJ}$$

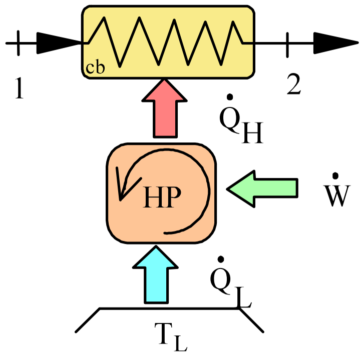


7.70

Sixty kilograms per hour of water runs through a heat exchanger, entering as saturated liquid at 200 kPa and leaving as saturated vapor. The heat is supplied by a Carnot heat pump operating from a low-temperature reservoir at 16°C. Find the rate of work into the heat pump.

Solution:

C.V. Heat exchanger



$\dot{m}_1 = \dot{m}_2$; $\dot{m}_1 h_1 + \dot{Q}_H = \dot{m}_1 h_2$

Table B.1.2: $h_1 = 504.7$ kJ/kg,

$h_2 = 2706.7$ kJ/kg

$T_H = T_{sat}(P) = 120.93 + 273.15$

$= 394.08$ K

$$\dot{Q}_H = \frac{60}{3600}(2706.7 - 504.7) = 36.7 \text{ kW}$$

Assume a Carnot heat pump.

$$\beta' = \dot{Q}_H/\dot{W} = T_H / (T_H - T_L) = 394.08 / 104.93 = 3.76$$

$$\dot{W} = \dot{Q}_H/\beta' = 36.7/3.76 = \mathbf{9.76\ kW}$$

7.71

A heat engine has a solar collector receiving 0.2 kW per square meter inside which a transfer media is heated to 450 K. The collected energy powers a heat engine which rejects heat at 40°C. If the heat engine should deliver 2.5 kW what is the minimum size (area) solar collector?

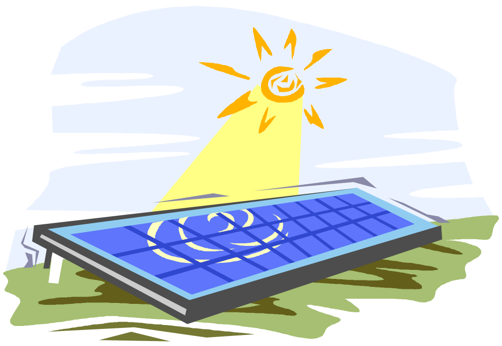
Solution:

$$T_H = 450 \text{ K} \qquad T_L = 40^oC = 313.15 \text{ K}$$

$$\eta_{HE} = 1 - \frac{T_L}{T_H} = 1 - \frac{313.15}{450} = 0.304$$

$$\dot{W} = \eta\, \dot{Q}_H \quad => \quad \dot{Q}_H = \frac{\dot{W}}{\eta} = \frac{2.5}{0.304} = 8.224 \text{ kW}$$

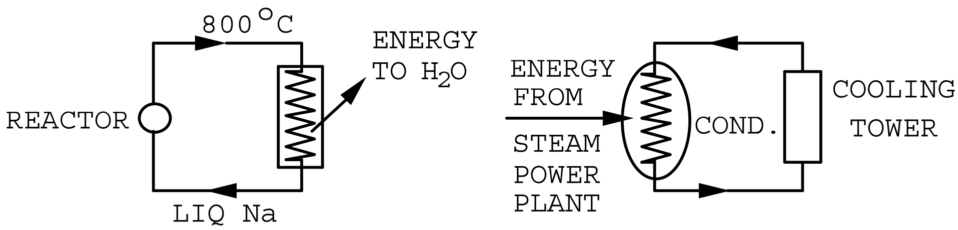
$$\dot{Q}_H = 0.2\, A \quad => \quad A = \frac{\dot{Q}_H}{0.2} = \mathbf{41\ m^2}$$



7.72

Liquid sodium leaves a nuclear reactor at 800°C and is used as the energy source in a steam power plant. The condenser cooling water comes from a cooling tower at 15°C. Determine the maximum thermal efficiency of the power plant. Is it misleading to use the temperatures given to calculate this value?

Solution:



T_H = 800°C = 1073.2 K, T_L = 15°C = 288.2 K

$$\eta_{TH\ MAX} = \frac{T_H - T_L}{T_H} = \frac{1073.2 - 288.2}{1073.2} = \mathbf{0.731}$$

It might be misleading to use 800°C as the value for T_H, since there is not a supply of energy available at a constant temperature of 800°C (liquid Na is cooled to a lower temperature in the heat exchanger).

⇒ The Na cannot be used to boil H_2O at 800°C.

Similarly, the H_2O leaves the cooling tower and enters the condenser at 15°C, and leaves the condenser at some higher temperature.

⇒ The water does not provide for condensing steam at a constant temperature of 15°C.

7.73

A power plant with a thermal efficiency of 40% is located on a river similar to Fig. P7.67. With a total river mass flow rate of 1×10^5 kg/s at 15°C find the maximum power production allowed if the river water should not be heated more than 1 degree.

The maximum heating allowed determines the maximum $\dot{Q}_L$ as

$$\dot{Q}_L = \dot{m}_{H_2O}\, \Delta h = \dot{m}_{H_2O}\, C_{P\ LIQ\ H_2O}\, \Delta T_{H_2O}$$

$$= 1 \times 10^5 \text{ kg/s} \times 4.18 \text{ kJ/kg-K} \times 1 \text{ K} = 418 \text{ MW}$$

$$= \dot{W}_{NET}(1/\eta_{TH\ ac} - 1)$$

$$\dot{W}_{NET} = \dot{Q}_L \,/\, (1/\eta_{TH\ ac} - 1) = \dot{Q}_L \frac{\eta_{TH\ ac}}{1 - \eta_{TH\ ac}}$$

$$= 418 \text{ MW} \times \frac{0.4}{1 - 0.4} = \mathbf{279\ MW}$$

7.74

A heat pump is driven by the work output of a heat engine as shown in figure P7.74. If we assume ideal devices find the ratio of the total power $\dot{Q}_{L1} + \dot{Q}_{H2}$ that heats the house to the power from the hot energy source $\dot{Q}_{H1}$ in terms of the temperatures.

$$\beta_{HP} = \dot{Q}_{H2}/\dot{W} = \dot{Q}_{H2}/(\dot{Q}_{H2} - \dot{Q}_{L2}) = \frac{T_{room}}{T_{room} - T_{amb}}$$

$$\dot{W} = \eta_{HE} \cdot \dot{Q}_{H1} = (1 - \frac{T_{room}}{T_H})\ \dot{Q}_{H1}$$

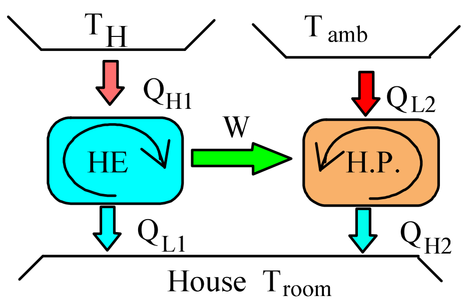
$$\dot{W} = \dot{Q}_{H2}/\beta_{HP} = \frac{T_{room}}{T_{room} - T_{amb}}\ \dot{Q}_{H2}$$

$$\dot{Q}_{L1} = \dot{Q}_{H1} - \dot{W} = [1 - 1 + \frac{T_{room}}{T_H}]\ \dot{Q}_{H1}$$

$$\frac{\dot{Q}_{H2} + \dot{Q}_{L1}}{\dot{Q}_{H1}} = 1 - 1 + \frac{T_{room}}{T_H} + \frac{1 - \frac{T_{room}}{T_H}}{\frac{T_{room} - T_{amb}}{T_{room}}} = \frac{T_{room}}{T_H} + \frac{T_{room} - T^2_{room}/T_H}{T_{room} - T_{amb}}$$

$$= T_{room}\ [\frac{1}{T_H} + \frac{1 - \frac{T_{room}}{T_H}}{T_{room} - T_{amb}}] = \frac{T_{room}}{T_H}\ [1 + \frac{T_H - T_{room}}{T_{room} - T_{amb}}]$$

$$= \frac{\mathbf{T_{room}}}{\mathbf{T_H}}\ \left[\frac{\mathbf{T_H - T_{amb}}}{\mathbf{T_{room} - T_{amb}}}\right]$$



7.75

A car engine with a thermal efficiency of 33% drives the air-conditioner unit (a refrigerator) besides powering the car and other auxiliary equipment. On a hot (35°C) summer day the A/C takes outside air in and cools it to 5°C sending it into a duct using 2 kW of power input and it is assumed to be half as good as a Carnot refrigeration unit. Find the rate of fuel (kW) being burned extra just to drive the A/C unit and its COP. Find the flow rate of cold air the A/C unit can provide.

$$\dot{W}_{extra} = \eta\, \dot{Q}_{H\ extra} \quad \Rightarrow \quad \dot{Q}_{H\ extra} = \dot{W}_{extra} / \eta = 2\ \text{kW} / 0.33 = \mathbf{6\ kW}$$

$$\beta = \frac{Q_L}{W_{IN}} = 0.5\ \beta_{Carnot} = 0.5\ \frac{T_L}{T_H - T_L} = 0.5\ \frac{5 + 273.15}{35 - 5} = 4.636$$

$$\dot{Q}_L = \beta\, \dot{W} = 4.636 \times 2\ \text{kW} = 9.272\ \text{kW} = \dot{m}_{air}\, C_{P\ air}\, \Delta T_{air}$$

$$\dot{m}_{air} = \dot{Q}_L / [C_{P\ air}\, \Delta T_{air}] = \frac{9.272\ \text{kW}}{1.004\ \text{kJ/kg-K} \times (35 - 5)\ \text{K}} = \mathbf{0.308\ kg/s}$$

7.76

Two different fuels can be used in a heat engine operating between the fuel burning temperature and a low temperature of 350 K. Fuel A burns at 2200 K delivering 30 000 kJ/kg and costs $1.50/kg. Fuel B burns at 1200 K, delivering 40 000 kJ/kg and costs $1.30/kg. Which fuel will you buy and why?

Solution:

Fuel A: $\eta_{TH,A} = 1 - \frac{T_L}{T_H} = 1 - \frac{350}{2200} = 0.84$

$W_A = \eta_{TH,A} \times Q_A = 0.84 \times 30\,000 = 25\,200$ kJ/kg

$W_A/\$_A = 25\,200/1.5 = 16\,800$ kJ/$

Fuel B: $\eta_{TH,B} = 1 - \frac{T_L}{T_H} = 1 - \frac{350}{1200} = 0.708$

$W_B = \eta_{TH,B} \times Q_B = 0.708 \times 40\,000 = 28\,320$ kJ/kg

$W_B/\$_B = 28\,320/1.3 = 21\,785$ kJ/$

Select fuel B for more work per dollar though it has a lower thermal efficiency.

7.77

A large heat pump should upgrade 5 MW of heat at 85°C to be delivered as heat at 150°C. Suppose the actual heat pump has a COP of 2.5 how much power is required to drive the unit. For the same COP how high a high temperature would a Carnot heat pump have assuming the same low T?

This is an actual COP for the heat pump as

$$\beta_{HP} = \text{COP} = \dot{Q}_H/\dot{W}_{in} = 2.5 \quad \Rightarrow \quad \dot{Q}_L/\dot{W}_{in} = 1.5$$

$$\dot{W}_{in} = \dot{Q}_L / 1.5 = 5 / 1.5 = \mathbf{3.333\ MW}$$

The Carnot heat pump has a COP given by the temperatures as

$$\beta_{HP} = \dot{Q}_H/\dot{W}_{in} = \frac{T_H}{T_H - T_L} = 2.5 \quad \Rightarrow \quad T_H = 2.5\, T_H - 2.5\, T_L$$

$$\Rightarrow \quad T_H = \frac{2.5}{1.5} T_L = \frac{5}{3} (85 + 273.15) = \mathbf{597\ K}$$

Finite ΔT Heat Transfer

7.78

The ocean near Havaii has 20^{o}C near the surface and 5^{o}C at some depth. A power plant based on this temperature difference is being planned. How large an efficiency could it have? If the two heat transfer terms (Q_H and Q_L) both require a 2 degree difference to operate what is the maximum efficiency then?

Solution:

$T_H = 20°C = 293.2\ K; \qquad T_L = 5°C = 278.2\ K$

$$\eta_{TH\ MAX} = \frac{T_H - T_L}{T_H} = \frac{293.2 - 278.2}{293.2} = \mathbf{0.051}$$

$$\eta_{TH\ mod} = \frac{T_{H'} - T_{L'}}{T_{H'}} = \frac{291.2 - 280.2}{291.2} = \mathbf{0.038}$$

This is a very low efficiency so it has to be done on a very large scale to be economically feasible and then it will have some environmental impact.

7.79

A refrigerator keeping 5°C inside is located in a 30°C room. It must have a high temperature ΔT above room temperature and a low temperature ΔT below the refrigerated space in the cycle to actually transfer the heat. For a ΔT of 0, 5 and 10°C respectively calculate the COP assuming a Carnot cycle.

Solution:

From the definition of COP and assuming Carnot cycle

$$\beta = \frac{Q_L}{W_{IN}} = \frac{T_L}{T_H - T_L} \quad \text{when T's are absolute temperatures}$$

	ΔT	T_H °C	T_H K	T_L °C	T_L K	β
a	0	30	303	5	278	11.1
b	5	35	308	0	273	7.8
c	10	40	313	-5	268	5.96

Notice how the COP drops significantly with the increase in ΔT.

7.80

A house is heated by a heat pump driven by an electric motor using the outside as the low-temperature reservoir. The house loses energy directly proportional to the temperature difference as $\dot{Q}_{loss} = K(T_H - T_L)$. Determine the minimum electric power to drive the heat pump as a function of the two temperatures.

Solution:

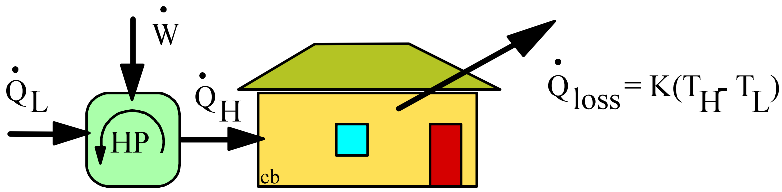
Heat pump COP: $\beta' = \dot{Q}_H/\dot{W}_{in} \le T_H/(T_H - T_L)$;

Heat loss must be added: $\dot{Q}_H = \dot{Q}_{loss} = K(T_H - T_L)$

Solve for required work and substitute in for β'

$$\dot{W}_{in} = \dot{Q}_H/\beta' \ \ge \ K(T_H - T_L) \times (T_H - T_L)/T_H$$

$$\dot{W}_{in} \ge K(T_H - T_L)^2/T_H$$



7.81

A house is heated by an electric heat pump using the outside as the low-temperature reservoir. For several different winter outdoor temperatures, estimate the percent savings in electricity if the house is kept at 20°C instead of 24°C. Assume that the house is losing energy to the outside as in Eq. 7.14.

Solution:

Heat Pump $\dot{Q}_{loss} \propto (T_H - T_L)$

$$\text{Max Perf.} \quad \frac{\dot{Q}_H}{\dot{W}_{IN}} = \frac{T_H}{T_H - T_L} = \frac{K(T_H - T_L)}{\dot{W}_{IN}}, \quad \dot{W}_{IN} = \frac{K(T_H - T_L)^2}{T_H}$$

A: T_{H_A} = 24°C = 297.2 K B: T_{H_B} = 20°C = 293.2 K

T_L,°C	$\dot{W}_{IN_A}/K$	$\dot{W}_{IN_B}/K$	% saving
-20	6.514	5.457	16.2 %
-10	3.890	3.070	21.1 %
0	1.938	1.364	29.6 %
10	0.659	0.341	48.3 %

7.82

A car engine operates with a thermal efficiency of 35%. Assume the air-conditioner has a coefficient of performance of $\beta = 3$ working as a refrigerator cooling the inside using engine shaft work to drive it. How much fuel energy should be spend extra to remove 1 kJ from the inside?

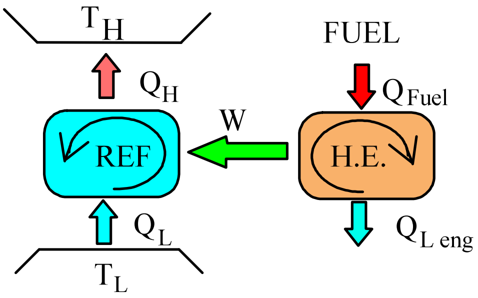
Solution:

Car engine: $W = \eta_{eng} Q_{fuel}$

Air conditioner: $\beta = \frac{Q_L}{W}$

$$W = \eta_{eng} Q_{fuel} = \frac{Q_L}{\beta}$$

$$Q_{fuel} = Q_L / (\eta_{eng} \beta) = \frac{1}{0.35 \times 3} = \mathbf{0.952\ kJ}$$



7.83

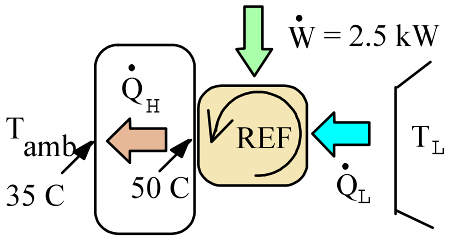
A refrigerator uses a power input of 2.5 kW to cool a 5°C space with the high temperature in the cycle as 50°C. The Q_H is pushed to the ambient air at 35°C in a heat exchanger where the transfer coefficient is 50 W/m^{2}K. Find the required minimum heat transfer area.

Solution:

$$\dot{W} = 2.5 \text{ kW} = \dot{Q}_H / \beta_{HP}$$

$$\dot{Q}_H = \dot{W} \times \beta_{HP} = 2.5 \times [323 / (50 - 5)] = 17.95 \text{ kW} = h\, A\, \Delta T$$

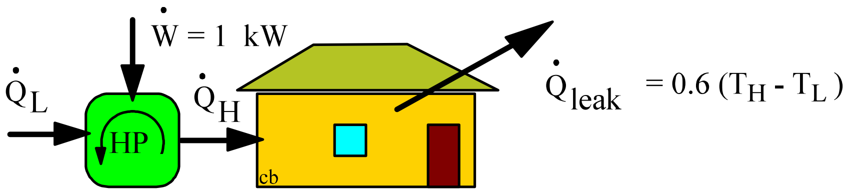
$$A = \frac{\dot{Q}_H}{h\, \Delta T} = \frac{17.95 \times 10^3}{50 \times 15} = \mathbf{23.9\ m^2}$$



7.84

A heat pump has a coefficient of performance that is 50% of the theoretical maximum. It maintains a house at 20°C, which leaks energy of 0.6 kW per degree temperature difference to the ambient. For a maximum of 1.0 kW power input find the minimum outside temperature for which the heat pump is a sufficient heat source.

Solution:



C.V. House. For constant 20°C the heat pump must provide $\dot{Q}_{leak} = 0.6\ \Delta T$

$$\dot{Q}_H = \dot{Q}_{leak} = 0.6\ (T_H - T_L) = \beta'\ \dot{W}$$

C.V. Heat pump. Definition of the coefficient of performance and the fact that the maximum is for a Carnot heat pump.

$$\beta' = \frac{\dot{Q}_H}{\dot{W}} = \frac{\dot{Q}_H}{\dot{Q}_H - \dot{Q}_L} = 0.5\ \beta'_{Carnot} = 0.5 \times \frac{T_H}{T_H - T_L}$$

Substitute into the first equation to get

$0.6\ (T_H - T_L) = [\ 0.5 \times T_H / (T_H - T_L)\ ]\ 1$ =>

$(T_H - T_L)^2 = (0.5 / 0.6)\ T_H \times 1 = 0.5 / 0.6 \times 293.15 = 244.29$

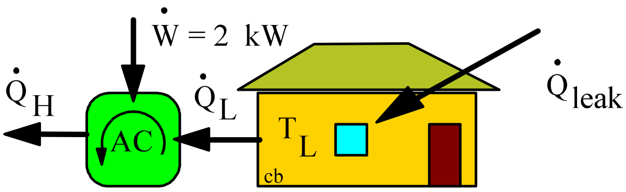
$T_H - T_L = 15.63$ => $T_L = 20 - 15.63 =$ **4.4 °C**

7.85

Consider a room at 20^{o}C that is cooled by an air conditioner with a COP of 3.2 using a power input of 2 kW and the outside is at 35^{o}C. What is the constant in the heat transfer Eq. 7.14 for the heat transfer from the outside into the room?

$$\dot{Q}_L = \beta_{AC}\dot{W} = 3.2 \times 2 \text{ kW} = 6.4 \text{ kW} = \dot{Q}_{\text{leak in}} = CA\ \Delta T$$

$$CA = \dot{Q}_L / \Delta T = \frac{6.4 \text{ kW}}{(35 - 20) \text{ K}} = \mathbf{0.427\ kW/K}$$



Here:

$T_L = T_{house} = 20^oC$

$T_H = T_{amb} = 35^oC$

7.86

A farmer runs a heat pump with a motor of 2 kW. It should keep a chicken hatchery at 30°C which loses energy at a rate of 0.5 kW per degree difference to the colder ambient. The heat pump has a coefficient of performance that is 50% of a Carnot heat pump. What is the minimum ambient temperature for which the heat pump is sufficient?

Solution:

C.V. Hatchery, steady state.

To have steady state at 30°C for the hatchery

Energy Eq.: $\dot{Q}_H = \dot{Q}_{Loss} = \beta_{AC}\dot{W}$

Process Eq.: $\dot{Q}_{Loss} = 0.5\ (T_H - T_{amb})$; $\qquad \beta_{AC} = ½\ \beta_{CARNOT}$

COP for the reference Carnot heat pump

$$\beta_{CARNOT} = \frac{\dot{Q}_H}{\dot{W}} = \frac{\dot{Q}_H}{\dot{Q}_H - \dot{Q}_L} = \frac{T_H}{T_H - T_L} = \frac{T_H}{T_H - T_{amb}}$$

Substitute the process equations and this β_{CARNOT} into the energy Eq.

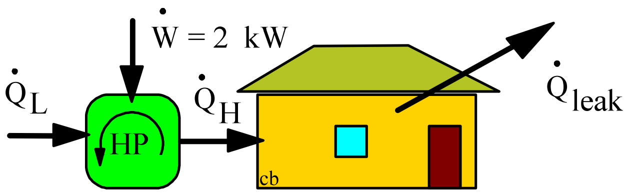
$$0.5\ (T_H - T_{amb}) = ½ \frac{T_H}{T_H - T_{amb}} \dot{W}$$

$$(T_H - T_{amb})^2 = ½\ T_H\dot{W}/0.5 = T_H\dot{W} = (273 + 30) \times 2 = 606\ K^2$$

$$T_H - T_{amb} = 24.62\ K$$

$$T_{amb} = 30 - 24.62 = \mathbf{5.38^oC}$$

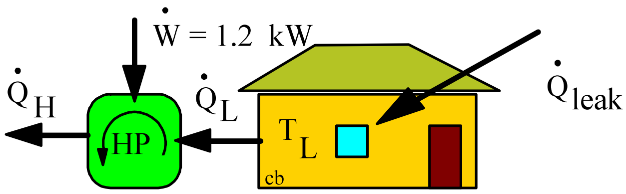
Comment: That of course is not a very low temperature and the size of the system is not adequate for most locations.



7.87

An air conditioner cools a house at T_L = 20°C with a maximum of 1.2 kW power input. The house gains 0.6 kW per degree temperature difference to the ambient and the refrigeration COP is $\beta = 0.6\ \beta_{Carnot}$. Find the maximum outside temperature, T_H, for which the air conditioner provides sufficient cooling.

Solution:



Here:

$T_L = T_{house}$

$T_H = T_{amb}$

In this setup the low temperature space is the house and the high temperature space is the ambient. The heat pump must remove the gain or leak heat transfer to keep it at a constant temperature.

$\dot{Q}_{leak} = 0.6\ (T_{amb} - T_{house}) = \dot{Q}_L$ which must be removed by the heat pump.

$$\beta = \dot{Q}_L / \dot{W} = 0.6\ \beta_{carnot} = 0.6\ T_{house} / (T_{amb} - T_{house})$$

Substitute in for $\dot{Q}_L$ and multiply with $(T_{amb} - T_{house})\dot{W}$:

$$0.6\ (T_{amb} - T_{house})^2 = 0.6\ T_{house}\ \dot{W}$$

Since T_{house} = 293.15 K and $\dot{W}$ = 1.2 kW it follows

$$(T_{amb} - T_{house})^2 = T_{house}\ \dot{W} = 293.15 \times 1.2 = 351.78\ K^2$$

Solving ⇒ $(T_{amb} - T_{house}) = 18.76$ ⇒ T_{amb} = **311.9 K = 38.8 °C**

7.88

A house is cooled by an electric heat pump using the outside as the high-temperature reservoir. For several different summer outdoor temperatures, estimate the percent savings in electricity if the house is kept at 25°C instead of 20°C. Assume that the house is gaining energy from the outside directly proportional to the temperature difference as in Eq. 7.14.

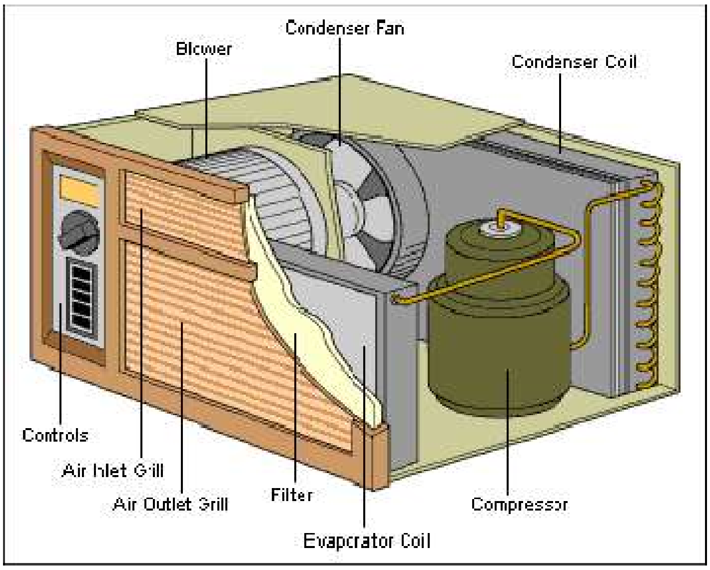
Solution:

Air-conditioner (Refrigerator) $\dot{Q}_{LEAK} \propto (T_H - T_L)$

$$\text{Max Perf.} \quad \frac{\dot{Q}_L}{\dot{W}_{IN}} = \frac{T_L}{T_H - T_L} = \frac{K(T_H - T_L)}{\dot{W}_{IN}}, \quad \dot{W}_{IN} = \frac{K(T_H - T_L)^2}{T_L}$$

A: $T_{L_A} = 20°C = 293.2$ K B: $T_{L_B} = 25°C = 298.2$ K

T_H,°C	$\dot{W}_{IN_A}/K$	$\dot{W}_{IN_B}/K$	% saving
45	2.132	1.341	37.1 %
40	1.364	0.755	44.6 %
35	0.767	0.335	56.3 %



7.89

A Carnot heat engine, shown in Fig. P7.89, receives energy from a reservoir at T_{res} through a heat exchanger where the heat transferred is proportional to the temperature difference as $\dot{Q}_H = K(T_{res} - T_H)$. It rejects heat at a given low temperature T_L. To design the heat engine for maximum work output show that the high temperature, T_H, in the cycle should be selected as $T_H = \sqrt{T_{res}T_L}$

Solution:

$$W = \eta_{TH} Q_H = \frac{T_H - T_L}{T_H} \times K(T_{res} - T_H) \; ; \quad \text{maximize } W(T_H) \Rightarrow \frac{\delta W}{\delta T_H} = 0$$

$$\frac{\delta W}{\delta T_H} = K(T_{res} - T_H) T_L T_H^{-2} - K(1 - T_L/T_H) = 0$$

$$\Rightarrow \mathbf{T_H = \sqrt{T_{res} T_L}}$$

7.90

Consider a Carnot cycle heat engine operating in outer space. Heat can be rejected from this engine only by thermal radiation, which is proportional to the radiator area and the fourth power of absolute temperature, $\dot{Q}_{rad} \sim KAT^4$. Show that for a given engine work output and given T_H, the radiator area will be minimum when the ratio $T_L/T_H = 3/4$.

Solution:

$$\dot{W}_{NET} = \dot{Q}_H \left(\frac{T_H - T_L}{T_H}\right) = \dot{Q}_L \left(\frac{T_H - T_L}{T_L}\right); \qquad \text{also} \quad \dot{Q}_L = KAT_L^4$$

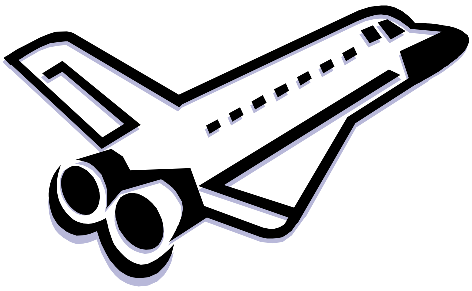
$$\frac{\dot{W}_{NET}}{KT_H^4} = \frac{AT_L^4}{T_H^4}\left(\frac{T_H}{T_L} - 1\right) = A\left[\left(\frac{T_L}{T_H}\right)^3 - \left(\frac{T_L}{T_H}\right)^4\right] = \text{const}$$

Differentiating,

$$dA\left[\left(\frac{T_L}{T_H}\right)^3 - \left(\frac{T_L}{T_H}\right)^4\right] + A\left[3\left(\frac{T_L}{T_H}\right)^2 - 4\left(\frac{T_L}{T_H}\right)^3\right] d\left(\frac{T_L}{T_H}\right) = 0$$

$$\frac{dA}{d(T_L/T_H)} = -A\left[3\left(\frac{T_L}{T_H}\right)^2 - 4\left(\frac{T_L}{T_H}\right)^3\right] \Big/ \left[\left(\frac{T_L}{T_H}\right)^3 - \left(\frac{T_L}{T_H}\right)^4\right] = 0$$

$$\frac{T_L}{T_H} = \frac{3}{4} \qquad \text{for min. A}$$



Check that it is minimum and not maximum with the 2nd derivative > 0.

7.91

On a cold (–10°C) winter day a heat pump provides 20 kW to heat a house maintained at 20°C and it has a COP_{HP} of 4. How much power does the heat pump require? The next day a winter storm brings the outside to -15°C, assuming the same COP and the same house heat transfer coefficient for the heat loss to the outside air. How much power does the heat pump require then?

If we look at the heat loss for the house we have

$$\dot{Q}_{loss} = 20 \text{ kW} = CA\,\Delta T \quad \Rightarrow \quad CA = \frac{20 \text{ kW}}{20 - (-10) \text{ K}} = 0.667 \text{ kW/K}$$

So now with the new outdoor temperature we get

$$\dot{Q}_{loss} = CA\,\Delta T = 0.667 \text{ kW/K} \times [20 - (-15)] \text{ K} = 23.3 \text{ kW}$$

$$\dot{Q}_{loss} = \dot{Q}_H = COP\ \dot{W} \quad \Rightarrow \quad \dot{W} = \dot{Q}_{loss}/COP = \frac{23.3 \text{ kW}}{4} = \mathbf{5.83\ kW}$$

Ideal Gas Carnot Cycles

7.92

Hydrogen gas is used in a Carnot cycle having an efficiency of 60% with a low temperature of 300 K. During the heat rejection the pressure changes from 90 kPa to 120 kPa. Find the high and low temperature heat transfer and the net cycle work per unit mass of hydrogen.

Solution:

As the efficiency is known, the high temperature is found as

$$\eta = 0.6 = 1 - \frac{T_L}{T_H} \quad => T_H = T_L /(1 - 0.6) = 750 \text{ K}$$

Now the volume ratio needed for the heat transfer, $T_3 = T_4 = T_L$, is

$$v_3 / v_4 = (RT_3 / P_3) / (RT_4 / P_4) = P_4 / P_3 = 120 / 90 = 1.333$$

so from Eq.7.9 we have with R = 4.1243 from Table A.5

$$q_L = RT_L \ln (v_3/v_4) = \mathbf{355.95 \text{ kJ/kg}}$$

Using the efficiency from Eq.7.4 then

$$q_H = q_L / (1 - 0.6) = \mathbf{889.9 \text{ kJ/kg}}$$

The net work equals the net heat transfer

$$w = q_H - q_L = \mathbf{533.9 \text{ kJ/kg}}$$

7.93

Carbon dioxide is used in an ideal gas refrigeration cycle, reverse of Fig. 7.24. Heat absorption is at 250 K and heat rejection is at 325 K where the pressure changes from 1200 kPa to 2400 kPa. Find the refrigeration COP and the specific heat transfer at the low temperature.

The analysis is the same as for the heat engine except the signs are opposite so the heat transfers move in the opposite direction.

$$\beta = \dot{Q}_L / \dot{W} = \beta_{carnot} = T_L / (T_H - T_L) = \frac{250}{325 - 250} = \mathbf{3.33}$$

$$q_H = RT_H \ln(v_2/v_1) = RT_H \ln\left(\frac{P_1}{P_2}\right) = 0.1889 \times 325 \ln\left(\frac{2400}{1200}\right) = 42.55$$

kJ/kg

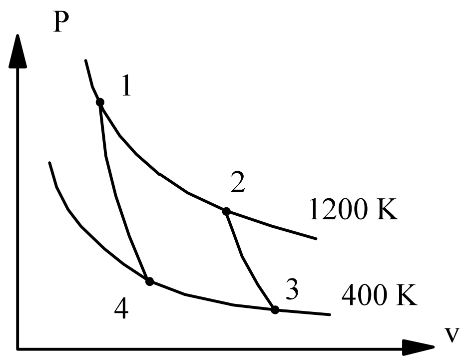
$$q_L = q_H\, T_L / T_H = 42.55 \times 250 / 325 = \mathbf{32.73\ kJ/kg}$$

7.94

An ideal gas Carnot cycle with air in a piston cylinder has a high temperature of 1200 K and a heat rejection at 400 K. During the heat addition the volume triples. Find the two specific heat transfers (q) in the cycle and the overall cycle efficiency.

Solution:

The P-v diagram of the cycle is shown to the right.
From the integration along the process curves done in the main text we have Eq.7.7



$q_H = R\ T_H \ln(v_2/v_1)$

$= 0.287 \times 1200 \ln(3)$

$=$ **378.4 kJ/kg**

Since it is a Carnot cycle the knowledge of the temperatures gives the cycle efficiency as

$$\eta_{TH} = 1 - \frac{T_L}{T_H} = 1 - \frac{400}{1200} = \mathbf{0.667}$$

from which we can get the other heat transfer from Eq.7.4

$$q_L = q_H\ T_L\ /\ T_H = 378.4\ \ 400\ /\ 1200 = \mathbf{126.1\ kJ/kg}$$

7.95

Air in a piston/cylinder goes through a Carnot cycle with the P-v diagram shown in Fig. 7.24. The high and low temperatures are 600 K and 300 K respectively. The heat added at the high temperature is 250 kJ/kg and the lowest pressure in the cycle is 75 kPa. Find the specific volume and pressure after heat rejection and the net work per unit mass.

Solution:

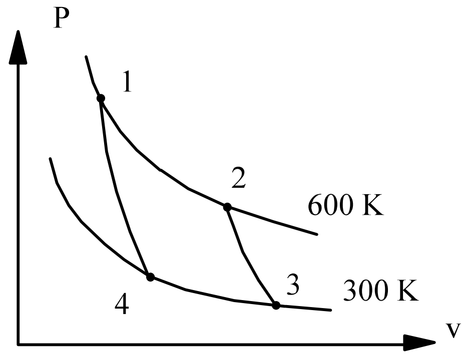
$q_H = 250$ kJ/kg , $T_H = 600$ K, $T_L = 300$ K, $P_3 = 75$ kPa

The states as shown in figure 7.21

1: 600 K , 2: 600 K, 3: 75 kPa, 300 K 4: 300 K

Since this is a Carnot cycle and we know the temperatures the efficiency is

$$\eta = 1 - \frac{T_L}{T_H} = 1 - \frac{300}{600} = 0.5$$



and the net work becomes

$$w_{NET} = \eta q_H = 0.5 \times 250$$
$$= \mathbf{125\ kJ/kg}$$

The heat rejected is

$$q_L = q_H - w_{NET} = 125 \text{ kJ/kg}$$

After heat rejection is state 4. From equation 7.9

$3 \rightarrow 4$ Eq.7.9 : $q_L = RT_L \ln (v_3/v_4)$

$v_3 = RT_3 / P_3 = 0.287 \times 300 / 75 = 1.148$ m^3/kg

$v_4 = v_3 \exp(-q_L/RT_L) = 1.148 \exp(-125/0.287 \times 300) =$ **0.2688 m^3/kg**

$P_4 = RT_4 / v_4 = 0.287 \times 300 / 0.2688 =$ **320 kPa**

Review Problems

7.96

At certain locations geothermal energy in undergound water is available and used as the energy source for a power plant. Consider a supply of saturated liquid water at 150°C. What is the maximum possible thermal efficiency of a cyclic heat engine using this source of energy with the ambient at 20°C? Would it be better to locate a source of saturated vapor at 150°C than use the saturated liquid at 150°C?

Solution:

$$T_{MAX} = 150°C = 423.2\ K = T_H\ ; \quad T_{Min} = 20°C = 293.2\ K = T_L$$

$$\eta_{TH\ MAX} = \frac{T_H - T_L}{T_H} = \frac{130}{423.2} = \mathbf{0.307}$$

Yes. Saturated vapor source at 150°C would remain at 150°C as it condenses to liquid, providing a large energy supply at that temperature.

7.97

A rigid insulated container has two rooms separated by a membrane. Room A contains 1 kg air at 200°C and room B has 1.5 kg air at 20°C, both rooms at 100 kPa. Consider two different cases

1) Heat transfer between A and B creates a final uniform T.
2) The membrane breaks and the air comes to a uniform state.

For both cases find the final temperature. Are the two processes reversible and different? Explain.

Solution:

C.V. Total A+B

1) Energy Eq.: $U_2 - U_1 = {}_1Q_2 - {}_1W_2 = 0 - \mathbf{0} = \mathbf{0}$

$$U_2 - U_1 = 0 = m_A (U_2 - U_1)_A + m_B (U_2 - U_1)_B$$

$$\cong m_A \mathbf{C_v} (T_2 - T_{A1}) + m_B \mathbf{C_v} (T_2 - T_{B1})$$

$$\Rightarrow T_2 = \frac{m_A}{m_A + m_B} T_{A1} + \frac{m_B}{m_A + m_B} T_{B1} = \frac{1}{2.5} \times 200 + \frac{1.5}{2.5} \times 20$$

$$= \mathbf{92^oC}$$

$$P_{A2} = P_{A1} \times T_2/ T_{A1} = 100 \times (273 + 92) /473 = 77.2 \text{ kPa}$$

$$P_{B2} = P_{B1} \times T_2/ T_{B1} = 100 \times (273 + 92) /293 = 124.6 \text{ kPa}$$

2) Same energy eq. Since ideal gas u(T) same $\mathbf{T_2 = 92^oC}$, but now also same P_2

$$P_2 = mRT_2 / V_1; \qquad V_1 = V_A + V_B$$

$$V_1 = m_{A1}RT_{A1}/ P_1 + m_{B1}RT_{B1}/ P_1$$

$$P_2 = (m_2RT_2 / (m_{A1}RT_{A1}/ P_1 + m_{B1}RT_{B1}/ P_1))$$

$$= P_1 (m_2T_2 / (m_{A1}T_{A1} + m_{B1}T_{B1})) = 100 \frac{2.5 \ (273 + 92)}{1 \times 473 + 1.5 \times 293}$$

$$= 100 \text{ kPa}$$

Both cases irreversible 1) Q over a finite ΔT and in 2) mixing of 2 different states (internal u redistribution)

(Case 2) is more irreversible as the final state in 1 could drive a turbine between the two different pressures until equal.

7.98

Consider the combination of the two heat engines as in Fig. P7.4. How should the intermediate temperature be selected so the two heat engines have the same efficiency assuming Carnot cycle heat engines.

Heat engine 1: $\eta_{TH\,1} = 1 - \frac{T_M}{T_H}$

Heat engine 2: $\eta_{TH\,2} = 1 - \frac{T_L}{T_M}$

$$\eta_{TH\,1} = \eta_{TH\,2} \;\Rightarrow\; 1 - \frac{T_M}{T_H} = 1 - \frac{T_L}{T_M} \;\Rightarrow\; \frac{T_M}{T_H} = \frac{T_L}{T_M}$$

$$\Rightarrow\; T_M = \sqrt{T_L T_H}$$

7.99

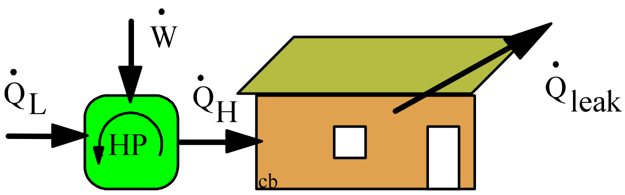
A house should be heated by a heat pump, $\beta' = 2.2$, and maintained at 20^oC at all times. It is estimated that it looses 0.8 kW per degree the ambient is lower than the inside. Assume an outside temperature of -10^oC and find the needed power to drive the heat pump?

Solution : Ambient $T_L = -10^oC$

Heat pump : $\beta' = \dot{Q}_H/\dot{W}$

House : $\dot{Q}_H = \dot{Q}_{leak} = 0.8 \, (T_H - T_L)$

$$\dot{W} = \dot{Q}_H/\beta' = \dot{Q}_{leak} / \beta' = 0.8 \, (T_H - T_L) / \beta'$$
$$= 0.8[20 - (-10)] /2.2 = \mathbf{10.91\ kW}$$



7.100

Consider a combination of a gas turbine power plant and a steam power plant as shown in Fig. P7.4. The gas turbine operates at higher temperatures (thus called a topping cycle) than the steam power plant (then called a bottom cycle). Assume both cycles have a thermal efficiency of 32%. What is the efficiency of the overall combination assuming Q_L in the gas turbine equals Q_H to the steam power plant?

Let the gas turbine be heat engine number 1 and the steam power plant the heat engine number 2. Then the overall efficiency

$$\eta_{TH} = \dot{W}_{net} / \dot{Q}_H = (\dot{W}_1 + \dot{W}_2) / \dot{Q}_H = \eta_1 + \dot{W}_2 / \dot{Q}_H$$

For the second heat engine and the energy Eq. for the first heat engine

$$\dot{W}_2 = \eta_2 \, \dot{Q}_M = \eta_2 \, (1 - \eta_1) \, \dot{Q}_H$$

so the final result is

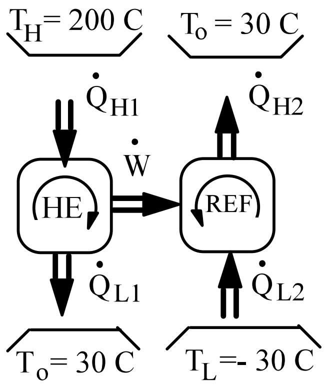
$$\eta_{TH} = \eta_1 + \eta_2 \, (1 - \eta_1) = 0.32 + 0.32(1 - 0.32) = \mathbf{0.538}$$

7.101

We wish to produce refrigeration at −30°C. A reservoir, shown in Fig. P7.101, is available at 200°C and the ambient temperature is 30°C. Thus, work can be done by a cyclic heat engine operating between the 200°C reservoir and the ambient. This work is used to drive the refrigerator. Determine the ratio of the heat transferred from the 200°C reservoir to the heat transferred from the −30°C reservoir, assuming all processes are reversible.

Solution:

Equate the work from the heat engine to the refrigerator.



$$W = Q_{H1} \left(\frac{T_H - T_0}{T_H} \right)$$

also

$$W = Q_{L2} \left(\frac{T_0 - T_L}{T_L} \right)$$

$$\frac{Q_{H1}}{Q_{L2}} = \left(\frac{T_o - T_L}{T_L} \right) \left(\frac{T_H}{T_H - T_o} \right) = \left(\frac{60}{243.2} \right) \left(\frac{473.2}{170} \right) = \mathbf{0.687}$$

7.102

A 4L jug of milk at 25°C is placed in your refrigerator where it is cooled down to 5°C. The high temperature in the Carnot refrigeration cycle is 45°C and the properties of milk are the same as for liquid water. Find the amount of energy that must be removed from the milk and the additional work needed to drive the refrigerator.

Solution:

C.V milk + out to the 5 °C refrigerator space

Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process : P = constant = 1 atm => ${}_1W_2 = Pm\,(v_2 - v_1)$

State 1: Table B.1.1, $v_1 \cong v_f = 0.001003\ m^3/kg$, $h_1 \cong h_f = 104.87$ kJ/kg

$m_2 = m_1 = V_1/v_1 = 0.004 / 0.001003 = 3.988$ kg

State 2: Table B.1.1, $h_2 \cong h_f = 20.98$ kJ/kg

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(u_2 - u_1) + Pm\,(v_2 - v_1) = m(h_2 - h_1)$$

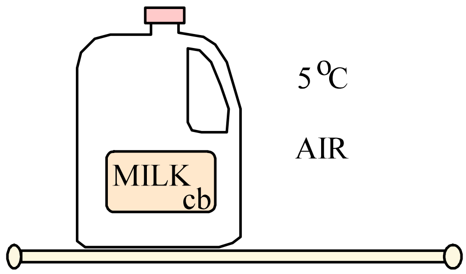
$${}_1Q_2 = 3.998\,(20.98 - 104.87) = -3.988 \times 83.89 = \mathbf{-\ 334.55\ kJ}$$

C.V. Refrigeration cycle $T_L = 5$ °C ; $T_H = 45$ °C, assume Carnot

Ideal : $\beta = Q_L / W = Q_L / (Q_H - Q_L) = T_L / (T_H - T_L)$

$= 278.15 / 40 = \mathbf{6.954}$

$W = Q_L / \beta = 334.55 / 6.954 = \mathbf{48.1\ kJ}$



Remark: If you calculate the work term ${}_1W_2$ you will find that it is very small, the volume does not change (liquid). The heat transfer could then have been done as $m(u_2 - u_1)$ without any change in the numbers.

7.103

An air-conditioner with a power input of 1.2 kW is working as a refrigerator (β = 3) or as a heat pump (β' = 4). It maintains an office at 20°C year round which exchanges 0.5 kW per degree temperature difference with the atmosphere. Find the maximum and minimum outside temperature for which this unit is sufficient.

Solution:

Analyze the unit in heat pump mode

Replacement heat transfer equals the loss: $\dot{Q} = 0.5\ (T_H - T_{amb})$

$$\dot{W} = \frac{\dot{Q}_H}{\beta'} = 0.5\ \frac{T_H - T_{amb}}{4}$$

$$T_H - T_{amb} = 4\ \frac{\dot{W}}{0.5} = 9.6\text{ K}$$

Heat pump mode: Minumum T_{amb} = 20 - 9.6 = **10.4 °C**

The unit as a refrigerator must cool with rate: $\dot{Q} = 0.5\ (T_{amb} - T_{house})$

$$\dot{W} = \frac{\dot{Q}_L}{\beta} = 0.5\ (T_{amb} - T_{house}) / 3$$

$$T_{amb} - T_{house} = 3\ \frac{\dot{W}}{0.5} = 7.2\text{ K}$$

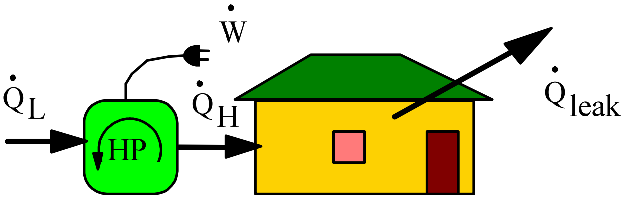
Refrigerator mode: Maximum T_{amb} = 20 + 7.2 = **27.2 °C**

7.104

Make some assumption about the heat transfer rates to solve problem 7.62 when the outdoor temperature is -20°C. Hint: look at the heat transfer given by Eq.7.14.

Solution:

Minimum power if we assume a Carnot cycle



We assume the heat transfer coefficient stays the same

$$\dot{Q}_H = \dot{Q}_{leak} = 25 \text{ kW} = CA\ \Delta T = CA\ [20 - (-10)] \Rightarrow \quad CA = \frac{5}{6} \text{ kW/K}$$

$$\dot{Q}_{leak\ new} = CA\ \Delta T = \frac{5}{6}\ [20 - (-20)] = 33.33 \text{ kW}$$

$$\beta' = \frac{\dot{Q}_H}{\dot{W}_{IN}} = \frac{T_H}{T_H - T_L} = \frac{293.15}{40} = 7.32875 \quad \Rightarrow \quad \dot{W}_{IN} = \frac{33.333}{7.32875} = \mathbf{4.55\ kW}$$

Comment. Leak heat transfer increases and COP is lower when T outside drops.

7.105

Air in a rigid 1 m^3 box is at 300 K, 200 kPa. It is heated to 600 K by heat transfer from a reversible heat pump that receives energy from the ambient at 300 K besides the work input. Use constant specific heat at 300 K. Since the coefficient of performance changes write $dQ = m_{air} C_v\, dT$ and find dW. Integrate dW with temperature to find the required heat pump work.

Solution:

$$\text{COP:} \qquad \beta' = \frac{Q_H}{W} = \frac{Q_H}{Q_H - Q_L} \cong \frac{T_H}{T_H - T_L}$$

$$m_{air} = P_1V_1 / RT_1 = 200 \times 1 / 0.287 \times 300 = 2.322 \text{ kg}$$

$$dQ_H = m_{air} C_v\, dT_H = \beta'\, dW \cong \frac{T_H}{T_H - T_L}\, dW$$

$$\Rightarrow \qquad dW = m_{air} C_v \left[\frac{T_H}{T_H - T_L}\right] dT_H$$

$${}_1W_2 = \int m_{air} C_v \left(1 - \frac{T_L}{T} \right) dT = m_{air} C_v \int \left(1 - \frac{T_L}{T} \right) dT$$

$$= m_{air} C_v \left[T_2 - T_1 - T_L \ln \frac{T_2}{T_1}\right]$$

$$= 2.322 \times 0.717 \left[600 - 300 - 300 \ln \frac{600}{300} \right] = \mathbf{153.1\ kJ}$$

7.106

A combination of a heat engine driving a heat pump (see Fig. P7.106) takes waste energy at 50°C as a source Q_{w1} to the heat engine rejecting heat at 30°C. The remainder Q_{w2} goes into the heat pump that delivers a Q_H at 150°C. If the total waste energy is 5 MW find the rate of energy delivered at the high temperature.

Solution:

Waste supply: $\dot{Q}_{w1} + \dot{Q}_{w2} = 5$ MW

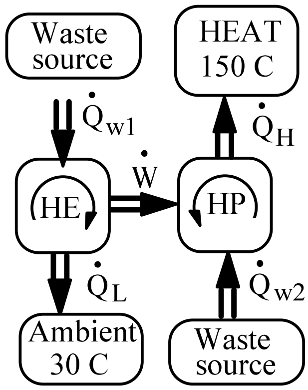
Heat Engine:

$$\dot{W} = \eta\, \dot{Q}_{w1} = (1 - T_{L1} / T_{H1})\, \dot{Q}_{w1}$$

Heat pump:

$$\dot{W} = \dot{Q}_H / \beta_{HP} = \dot{Q}_{W2} / \beta'$$

$$= \dot{Q}_{w2} / [T_{H1} / (T_H - T_{H1})]$$



Equate the two work terms:

$$(1 - T_{L1} / T_{H1})\, \dot{Q}_{w1} = \dot{Q}_{w2} \times (T_H - T_{H1}) / T_{H1}$$

Substitute $\dot{Q}_{w1} = 5 \text{ MW} - \dot{Q}_{w2}$

$$(1 - 303.15/323.15)(5 - \dot{Q}_{w2}) = \dot{Q}_{w2} \times (150 - 50) / 323.15$$

$$20 (5 - \dot{Q}_{w2}) = \dot{Q}_{w2} \times 100 \quad => \quad \dot{Q}_{w2} = 0.8333 \text{ MW}$$

$$\dot{Q}_{w1} = 5 - 0.8333 = 4.1667 \text{ MW}$$

$$\dot{W} = \eta\, \dot{Q}_{w1} = 0.06189 \times 4.1667 = 0.258 \text{ MW}$$

$\dot{Q}_H = \dot{Q}_{w2} + \dot{W} =$ **1.09 MW**

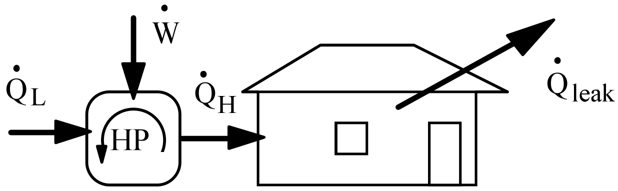
(For the heat pump $\beta' = 423.15 / 100 = 4.23$)

7.107

A heat pump heats a house in the winter and then reverses to cool it in the summer. The interior temperature should be 20°C in the winter and 25°C in the summer. Heat transfer through the walls and ceilings is estimated to be 2400 kJ per hour per degree temperature difference between the inside and outside.
a. If the winter outside temperature is 0°C, what is the minimum power required to drive the heat pump?
b.For the same power as in part (a), what is the maximum outside summer temperature for which the house can be maintained at 25°C?

Solution:

a) Winter:
House is T_H and ambient is at T_L

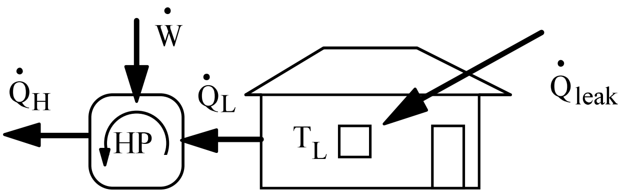


$T_H = 20°C = 293.2$ K , $T_L = 0°C = 273.2$ K and $\dot{Q}_H = 2400(20 - 0)$ kJ/h

$$\beta' = \dot{Q}_H/\dot{W}_{IN} = 2400\ (20 - 0)/\ \dot{W}_{IN} = \frac{T_H}{T_H - T_L} = \frac{293.2}{20}$$

$\Rightarrow \dot{W}_{IN} = 3275$ kJ/h = **0.91 kW** (For Carnot cycle)

b)



Summer:
$T_L = T_{house}$
$T_H = T_{amb}$

$T_L = 25°C = 298.2$ K, $\dot{W}_{IN} = 3275$ kJ/h and $\dot{Q}_L = 2400(T_H - 298.2)$ kJ/h

$$\beta = \frac{\dot{Q}_L}{\dot{W}_{IN}} = \frac{2400(T_H - 298.2)}{3275} = \frac{T_L}{T_H - T_L} = \frac{298.2}{T_H - 298.2}$$

$$\text{or,}\quad (T_H - 298.2)^2 = \frac{298.2 \times 3275}{2400} = 406.92$$

$T_H = 318.4$ K = **45.2°C**

7.108

A remote location without electricity operates a refrigerator with a bottle of propane feeding a burner to create hot gases. Sketch the setup in terms of cyclic devices and give a relation for the ratio of $\dot{Q}_L$ in the refrigerator to $\dot{Q}_{fuel}$ in the burner in terms of the various reservoir temperatures.

C.V.: Heat Eng.: $\dot{W}_{HE} = \eta_{HE}\dot{Q}_{fuel}$

C.V.: Refrigerator: $\dot{Q}_{L2} = \beta\ \dot{W}_{HE} = \beta\ \eta_{HE}\ \dot{Q}_{fuel}$

The ratio becomes

$$\dot{Q}_{L2} / \dot{Q}_{fuel} = \beta\ \eta_{HE}$$

$$= \frac{T_{L2}}{T_{H2} - T_{L2}} \left(1 - \frac{T_{L1}}{T_{H1}}\right) \qquad \text{If Carnot devices}$$

7.109

A furnace, shown in Fig. P7.109, can deliver heat, Q_{H1} at T_{H1} and it is proposed to use this to drive a heat engine with a rejection at T_{atm} instead of direct room heating. The heat engine drives a heat pump that delivers Q_{H2} at T_{room} using the atmosphere as the cold reservoir. Find the ratio Q_{H2}/Q_{H1} as a function of the temperatures. Is this a better set-up than direct room heating from the furnace?

Solution:

C.V.: Heat Eng.: $\dot{W}_{HE} = \eta\dot{Q}_{H1}$ where $\eta = 1 - T_{atm}/T_{H1}$

C.V.: Heat Pump: $\dot{W}_{HP} = \dot{Q}_{H2}/\beta'$ where $\beta' = T_{rm}/(T_{rm} - T_{atm})$

Work from heat engine goes into heat pump so we have

$$\dot{Q}_{H2} = \beta'\,\dot{W}_{HP} = \beta'\,\eta\,\dot{Q}_{H1}$$

and we may substitute T's for β', η. If furnace is used directly $\dot{Q}_{H2} = \dot{Q}_{H1}$, so if $\beta'\eta > 1$ this proposed setup is better. Is it? For $T_{H1} > T_{atm}$ formula shows that it is good for Carnot cycles. In actual devices it depends wether $\beta'\eta > 1$ is obtained.

7.110

Consider the rock bed thermal storage in Problem 7.65. Use the specific heat so you can write dQ_H in terms of dT_{rock} and find the expression for dW out of the heat engine. Integrate this expression over temperature and find the total heat engine work output.

Solution:

The rock provides the heat Q_H

$$dQ_H = -dU_{rock} = -mC\ \ dT_{rock}$$

$$dW = \eta dQ_H = -(\ 1 - \ T_o / T_{rock})\ mC\ dT_{rock}$$

$$m = \rho V = 2750 \times 2 = 5500 \text{ kg}$$

$$_1W_2 = \int -(\ 1 - T_o / T_{rock})\ mC\ dT_{rock} = -\ mC\ [T_2 - T_1 - T_o \ln \frac{T_2}{T_1}\ ]$$

$$= -\ 5500 \times 0.89\ [\ 290 - 400 - 290 \ln \frac{290}{400}\ ]\ = \mathbf{81\ 945\ kJ}$$

7.111

On a cold (-10°C) winter day a heat pump provides 20 kW to heat a house maintained at 20°C and it has a COP_{HP} of 4 using the maximum power available. The next day a winter storm brings the outside to -15°C, assuming the same COP and the house heat loss is to the outside air. How cold is the house then?

If we look at the heat loss for the house we have

$$\dot{Q}_{loss} = 20 \text{ kW} = CA\ \Delta T \quad \Rightarrow \quad CA = \frac{20 \text{ kW}}{20 - (-10) \text{ K}} = 0.667 \text{ kW/K}$$

$$\dot{Q}_{loss} = \dot{Q}_H = COP\ \dot{W} \quad \Rightarrow \quad \dot{W} = \dot{Q}_{loss}/COP = \frac{20 \text{ kW}}{4} = 5 \text{ kW}$$

With the same COP and the same power input we can get the same $\dot{Q}_H = 20$ kW.

$$\dot{Q}_{loss} = CA\ \Delta T = 0.667 \text{ kW/K} \times [T - (-15)] \text{ K} = 20 \text{ kW}$$

$$T = 20 \text{ kW} / 0.667 \text{ (kW/K)} - 15\ ^oC = (30 - 15)\ ^oC = \mathbf{15^oC}$$

Remark: Since $\dot{Q}_H = \dot{Q}_{loss} = CA\ \Delta T$ is the same the ΔT becomes the same so the house can be kept at 30°C above the ambient. In the real system the COP drops as the outdoor T drops unless the outside heat exchanger is buried under ground with a constant temperature independent upon the weather.

7.112

A Carnot heat engine operating between a high T_H and low T_L energy reservoirs has an efficiency given by the temperatures. Compare this to two combined heat engines one operating between T_H and an intermediate temperature T_M giving out work W_A and the other operating between T_M and T_L giving out W_B. The combination must have the same efficiency as the single heat engine so the heat transfer ratio $Q_H/Q_L = \psi(T_H,T_L) = [Q_H/Q_M]\ [Q_M/Q_L]$. The last two heat transfer ratios can be expressed by the same function $\psi()$ involving also the temperature T_M. Use this to show a condition the function $\psi()$ must satisfy.

The overall heat engine is a Carnot heat engine so

$$\dot{Q}_H / \dot{Q}_L = \frac{T_H}{T_L} = \psi(T_H,T_L)$$

The individual heat engines

$$\dot{Q}_H / \dot{Q}_M = \psi(T_H,T_M) \quad \text{and} \quad \dot{Q}_M / \dot{Q}_L = \psi(T_M,T_L)$$

Since an identity is

$$\dot{Q}_H / \dot{Q}_L = [\dot{Q}_H / \dot{Q}_M]\ \ [\dot{Q}_M / \dot{Q}_L] = \psi(T_H,T_L)$$

it follows that we have

$$\psi(T_H,T_L) = \psi(T_H,T_M) \times \ \psi(T_M,T_L)$$

Notice here that the product of the two functions must cancel the intermediate temperature T_M, this shows a condition the function $\psi()$ must satisfy. The Kelvin and Rankine temperature scales are determined by the choice of the function

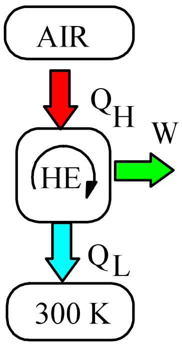
$$\psi(T_H,T_L) = \ T_H / T_L = \dot{Q}_H / \dot{Q}_L$$

satisfying the above restriction.

7.113

A 10-m^3 tank of air at 500 kPa, 600 K acts as the high-temperature reservoir for a Carnot heat engine that rejects heat at 300 K. A temperature difference of 25°C between the air tank and the Carnot cycle high temperature is needed to transfer the heat. The heat engine runs until the air temperature has dropped to 400 K and then stops. Assume constant specific heat capacities for air and find how much work is given out by the heat engine.

Solution:



$$T_H = T_{air} - 25°C, \quad T_L = 300 \text{ K}$$

$$m_{air} = \frac{P_1 V}{RT_1} = \frac{500 \times 10}{0.287 \times 600} = 29.04 \text{ kg}$$

$$dW = \eta dQ_H = \left(1 - \frac{T_L}{T_{air} - 25}\right) dQ_H$$

$$dQ_H = -m_{air} du = -m_{air} C_v dT_{air}$$

$$W = \int dW = -m_{air} C_v \int \left[1 - \frac{T_L}{T_a - 25}\right] dT_a$$

$$= -m_{air} C_v \left[T_{a2} - T_{a1} - T_L \ln \frac{T_{a2} - 25}{T_{a1} - 25}\right]$$

$$= -29.04 \times 0.717 \times \left[400 - 600 - 300 \ln \frac{375}{575}\right] = \mathbf{1494.3\ kJ}$$