Probability distributions of continues random variable

A Normal distribution as common case

Normal Probability Distribution

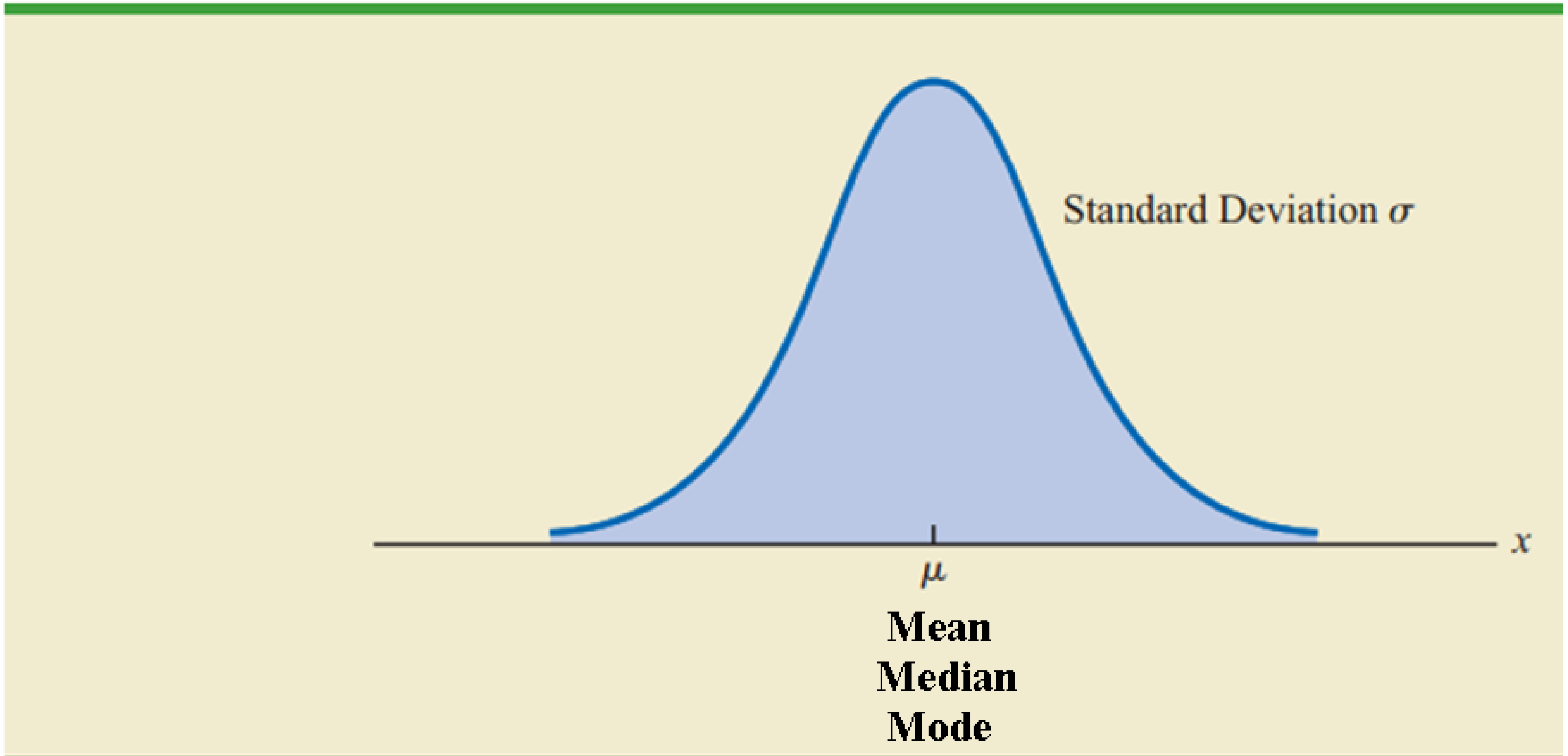
The most important probability distribution for describing a continuous random variable is the normal probability distribution.

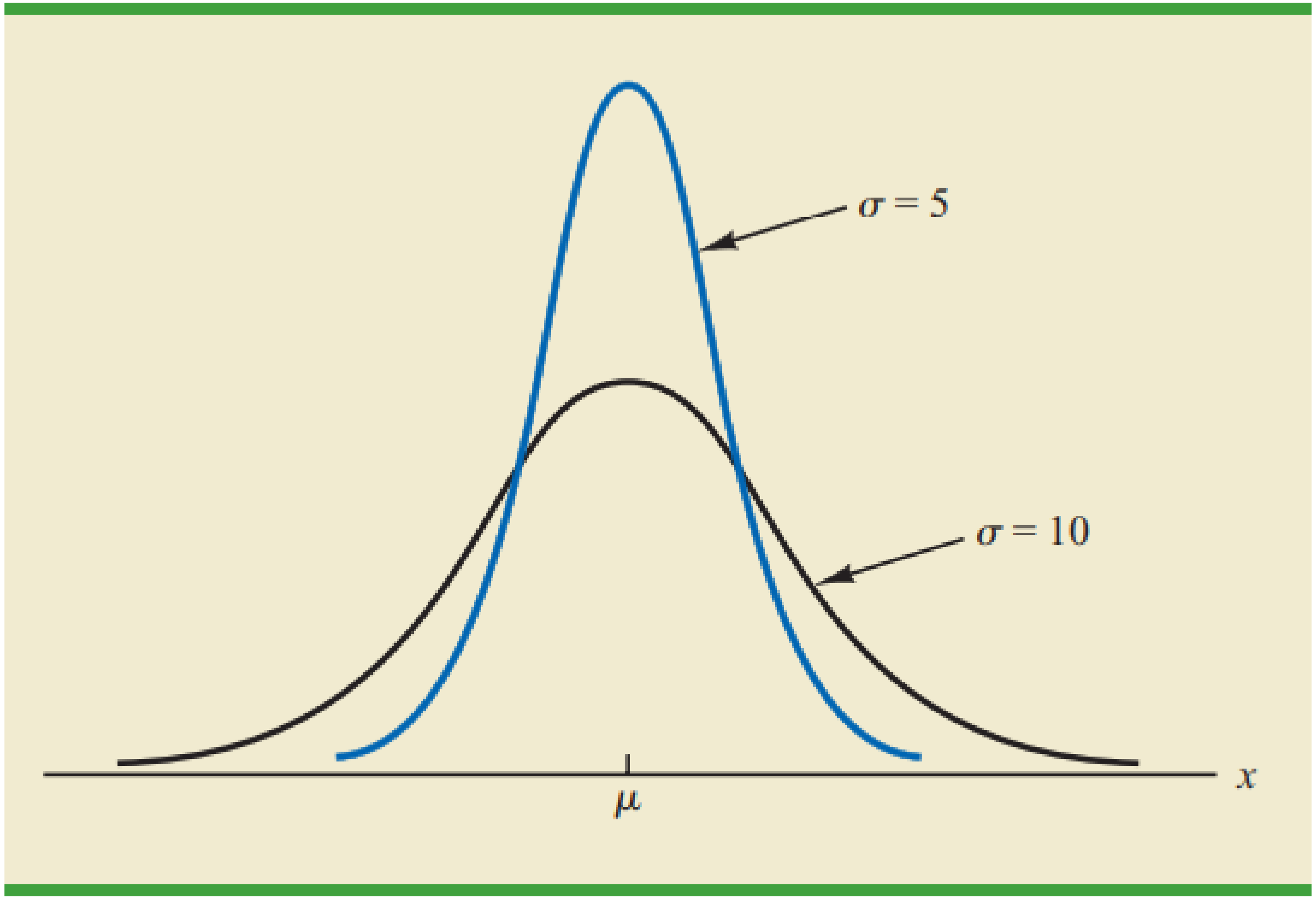
It is describes a lot of **continues** random variable.

The Normal distribution is based on two parameters: The mean ($\mu \in (-\infty, \infty)$) AND the variance ($\sigma^2 > 0$)

The normal distribution has the bell shaped curve and symmetric around mean (where mean=median=mode).

FIGURE 6.3 BELL-SHAPED CURVE FOR THE NORMAL DISTRIBUTION



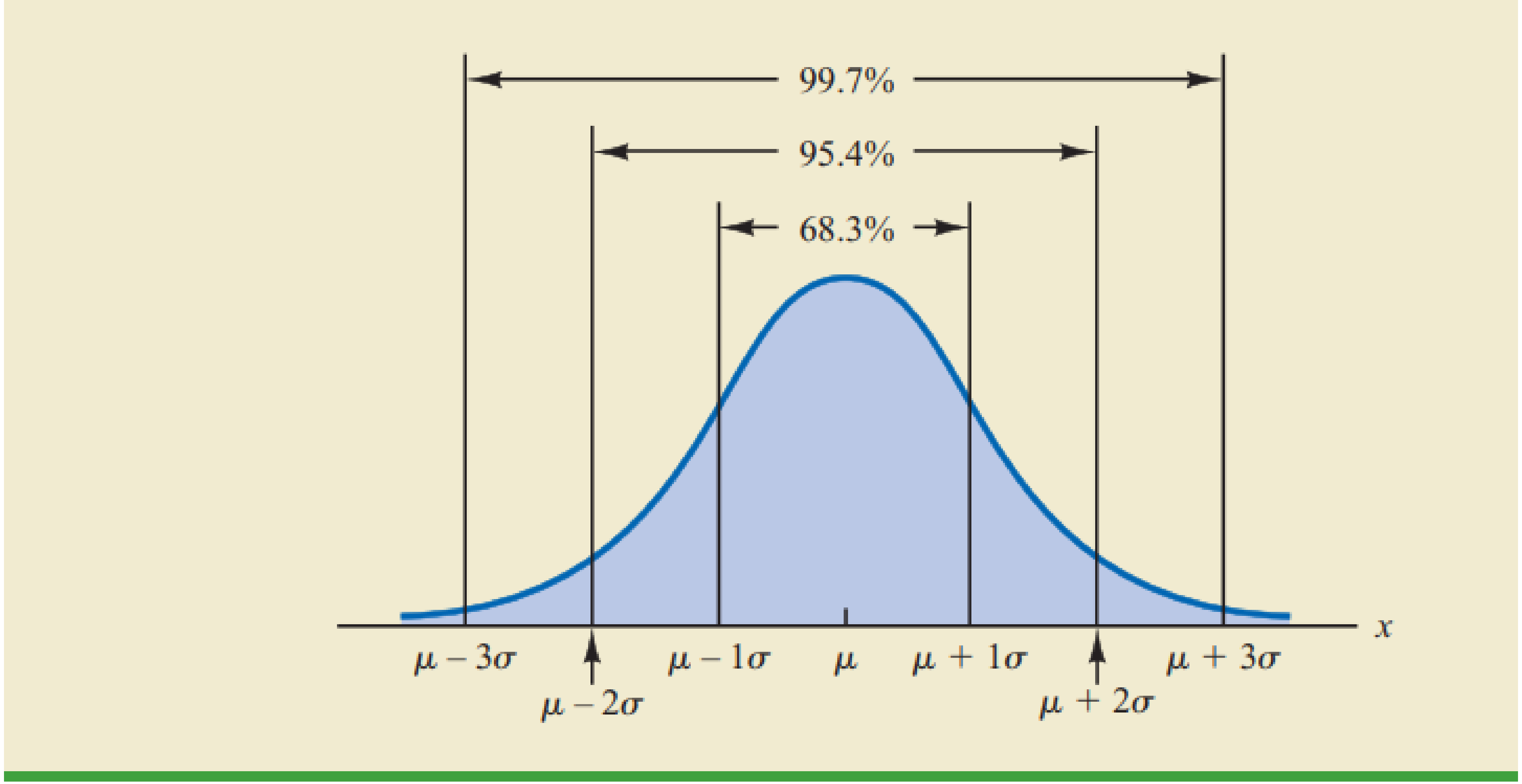


The Normal distribution shape is more spark when the variance (and S.D.) decrease.

Normal distribution properties:

1. The normal distributions is based on two parameters: the mean μ and the variance σ^2.
2. The highest point on the normal curve is at the mean, which is also the median and mode of the distribution.
3. The mean of the distribution can be any numerical value: negative, zero, or positive.
4. The normal distribution is symmetric, with the shape of the normal curve to the left of the mean a mirror image of the shape of the normal curve to the right of the mean. The tails of the normal curve extend to infinity in both directions and theoretically never touch the horizontal axis. Because it is symmetric, the normal distribution is not skewed; its skewness measure is zero.
5. The standard deviation determines how flat and wide the normal curve is. Larger values of the standard deviation result in wider, flatter curves, showing more variability in the data.
6. Probabilities for the normal random variable are given by areas under the normal curve. The total area under the curve for the normal distribution is (1). Because the distribution is symmetric, the area under the curve to the left of the mean is (0.50) and the area under the curve to the right of the mean is (0.50).
7. The percentage of values in some commonly used intervals are:
   a. 68.3% of the values of a normal random variable are within (+) or (-) one standard deviation of its mean.
   b. 95.4% of the values of a normal random variable are within (+) or (-) two standard deviations of its mean.
   c. 99.7% of the values of a normal random variable are within (+) or (-) three standard deviations of its mean

FIGURE 6.4 AREAS UNDER THE CURVE FOR ANY NORMAL DISTRIBUTION



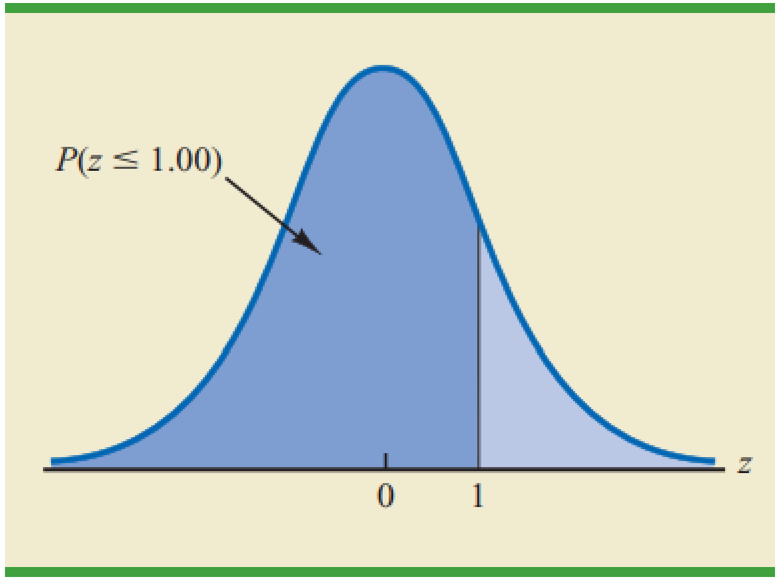
Standard Normal Probability Distribution

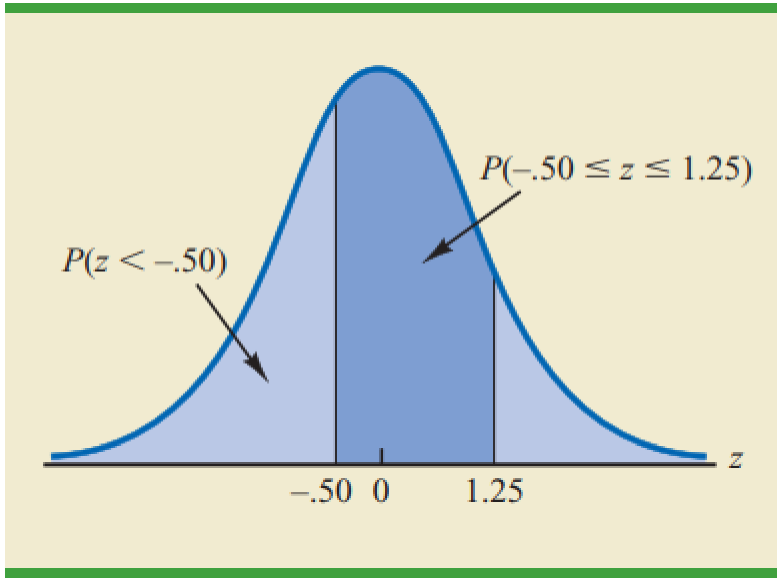
Is a normal distributions with mean $\mu = 0$ and variance $\sigma^2 = 1$.

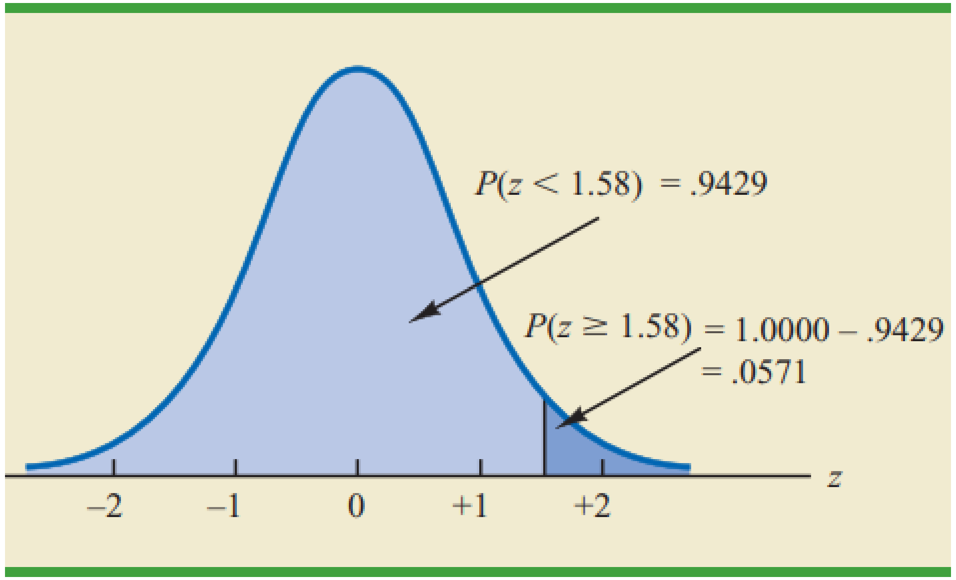
You can convert any normal distribution of (x) with mean μ and variance σ^2 to standard Normal distribution (z) by:

$$z = \frac{x - \mu}{\sigma}$$

Probabilities for the normal random variable are given by areas under the normal curve.
You can use standard Normal table (z).







Example:

Based on the standard Normal table (z), compute:

1. $P(Z < 0)$
2. $P(Z < 1.0)$
3. $P(Z < 1.65)$
4. $P(Z < 0.8)$
5. $P(Z < -1.28)$
6. $P(Z > 0)$
7. $P(Z > -1.34)$
8. $P(Z > 2.0)$
9. $P(-0.32 < Z < 0.5)$
10. $P(-1.28 < Z < 1.28)$
11. $P(-1.65 < Z < 1.65)$
12. $P(-1.96 < Z < 1.96)$
13. What is the value of Z that have 40% of chance (probability) less than Z, i.e. $P(Z < z) = 0.40$
14. What is the value of Z that have 80% of chance (probability) greater than Z, i.e. $P(Z > z) = 0.80$

Normal distribution application:

Example:

Assuming a population of 1000 students, if it mean of student marks is 70 and variance is 144. if the **student marks** is **normally distributed**.

a) Compute the probability that the student mark is greater than 79? $P(X > 79)$?

b) Compute the probability that the student mark is between 64 and 85?

c) How many students can there marks between 64 and 85?

d) How many students can there marks more than 85?

e) What is the mark that a 330 students will take less than it?

f) If you drawn a random sample from the population. if the sample size is 36. Compute the probability that the sample mean ($\bar{X}$) is greater than 75?

Example:

Assuming a population of 1000 student, if it mean of student marks is 70 and variance is 144. if the student marks is normally distributed.

$$\boldsymbol{N = 1000\,, \qquad \mu = 70\,, \qquad \sigma^2 = 144 \;\rightarrow\; \sigma = 12,}$$

Note:

$$\boldsymbol{Z = \frac{X-\mu}{\sigma}\,,}$$

a) Compute the probability that the student mark is greater than 79? $P(X > 79)$?

$$P(X > 75) = P\left(Z > \frac{\boldsymbol{X-\mu}}{\boldsymbol{\sigma}}\right) = P\left(Z > \frac{79-70}{12}\right) = P(Z > 0.75) = 0.2266$$

$$\boldsymbol{N = 1000}\,, \qquad \boldsymbol{\mu = 70}\,, \qquad \boldsymbol{\sigma^2 = 144 \rightarrow \sigma = 12},$$

Note:

$$\boldsymbol{Z = \frac{X-\mu}{\sigma}}\,,$$

b) Compute the probability that the student mark is between 64 and 85?

$P(64 < X < 85)$?

$$P(64 < X < 85) = P\left(\frac{X_1 - \mu}{\sigma} < Z < \frac{X_2 - \mu}{\sigma}\right)$$

$$= P\left(\frac{64-70}{12} < Z < \frac{85-70}{12}\right)$$

$$= P(-0.5 < Z < 1.25)$$

$$= T(1.25) - T(-0.5)$$

$$= 0.8944 - 0.3085 = 0.5859$$

$N = 1000$, $\mu = 70$, $\sigma^2 = 144 \rightarrow \sigma = 12$,

Note: $Z = \frac{X-\mu}{\sigma}$,

c) How many students can there marks between 64 and 85?

Note: Number of student is = Population size * Probability

$= N * P(64 < X < 85)$?

$$= 1000*0.5859$$

= "about 586 from 1000 " will take marks between than 64 and 85

d) How many students can there marks more than 85?

$= N * P(X > 85)$?

$$= 1000*(1-0.8944)=1000*0.1056$$

= "about 106 from 1000 " will take marks more than 85

$$N = 1000 , \quad \mu = 70 , \quad \sigma^2 = 144 \rightarrow \sigma = 12,$$

Note: $$Z = \frac{X-\mu}{\sigma} ,$$

e) What is the mark that a 330 students will take less than it?

$330 = 1000 * P(X < x_{330}) \rightarrow P(X < x_{330}) = 0.33?$

From Z table, find the z values that have probability (area) less than it= 0.33:

$$P(Z < -0.44) = 0.33 \quad , z = -0.44$$

$$Z = \frac{X - \mu}{\sigma}$$

$$-0.44 = \frac{x_{330} - \mu}{\sigma}$$

$$-0.44 = \frac{x_{330} - 70}{12}$$

$$x_{330} = 64.72$$

→ There are 330 students will take less than the mark 64.72

f) If you drawn a random sample from the population. if the sample size is 36. Compute the probability that the sample mean $(\bar{X})$ is greater than 75?

$$N = 1000, \quad \mu = 70, \quad \sigma^2 = 144, \quad n = 36, \quad P(\bar{X} > 75)\ ?$$

Note:

$$\boldsymbol{Z = \frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}}, \mu_{\bar{X}} = \mu, \quad \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}}$$

$$P(\bar{X} > 75) = P\left(Z > \frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}}\right) = P\left(Z > \frac{75 - 70}{\frac{12}{\sqrt{36}}}\right) = P(Z > 2.50) = 0.0062$$

g) If you drawn a random sample from the population. if the sample size is 25. Compute the probability that the sample mean ($\bar{X}$) is greater than 76?

$$N = 1000\,, \qquad \mu = 70\,, \qquad \sigma^2 = 144, \qquad n = 25\;\;, \qquad P(\bar{X} > 78)\,?$$

Note:

$$Z = \frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}}\;, \mu_{\bar{X}} = \mu\;, \quad \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$

$$P(\bar{X} > 75) = P\left(Z > \frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}}\right) = P\left(Z > \frac{76 - 70}{\frac{12}{\sqrt{25}}}\right) = P(Z > 0.42) =$$

$$= 1 - P(Z < 0.42) = 1 - 0.6628 = 0.3372$$