Principles of Statistics for Admin. (15060105)

Linear Correlation Coefficient

And

Simple Linear Regression

Independent Versus dependent variable:

X: independent (explanatory) variable

The independent variable is the cause. Its value is independent of other variables in your study.

Y: dependent (response) variable

The dependent variable effected by the independent variable.
Its value depends on changes in the independent variable.

Example:

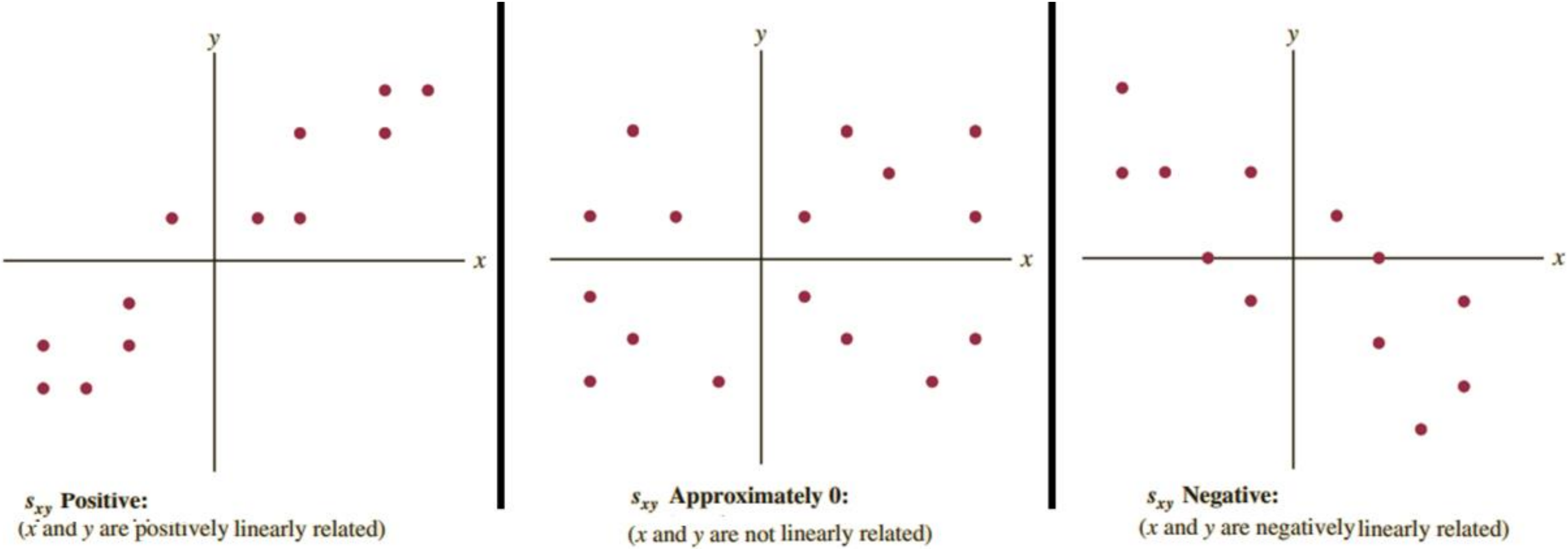
Determine independent and dependent variables:

An insurance company wants to predict sales from the amount of money they spend on advertising.

Correlation Coefficient

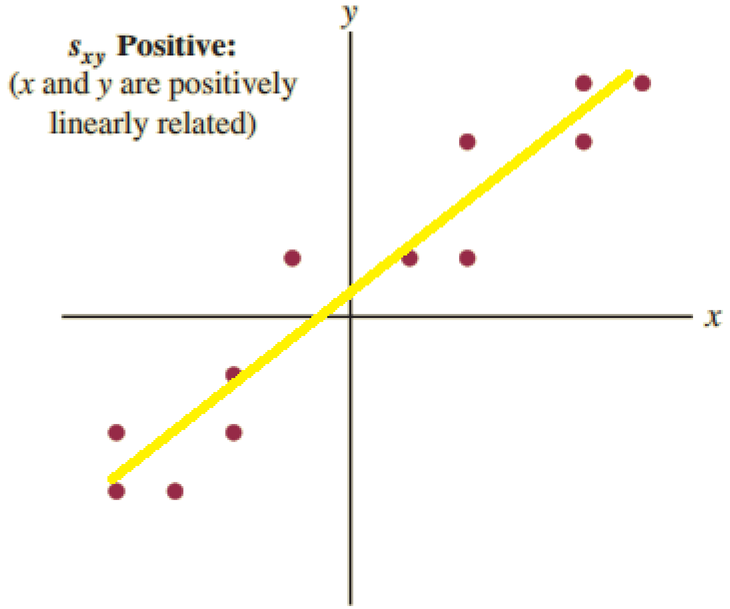
X: independent (explanatory) variable
Y: dependent (response) variable

The Scatter Plot



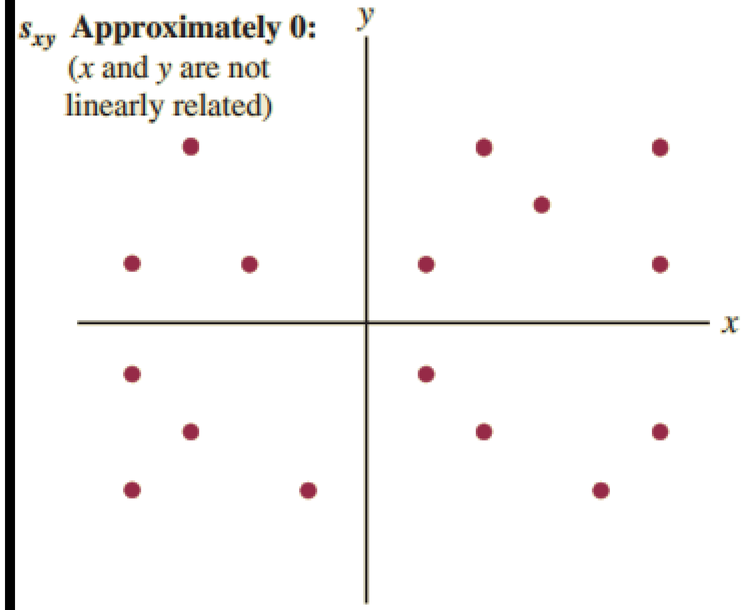
r : Pearson Correlation Coefficient $-1 \le r \le 1$

$$r = \frac{\sum (x_i - \bar{x})\ (y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2} \sqrt{\sum (y_i - \bar{y})^2}} \quad \textbf{OR} \quad r = \frac{\sum x_i y_i - n\bar{x} * \bar{y}}{\sqrt{\sum x_i^2 - n\bar{x}^2} \sqrt{\sum y_i^2 - n\bar{y}^2}}$$



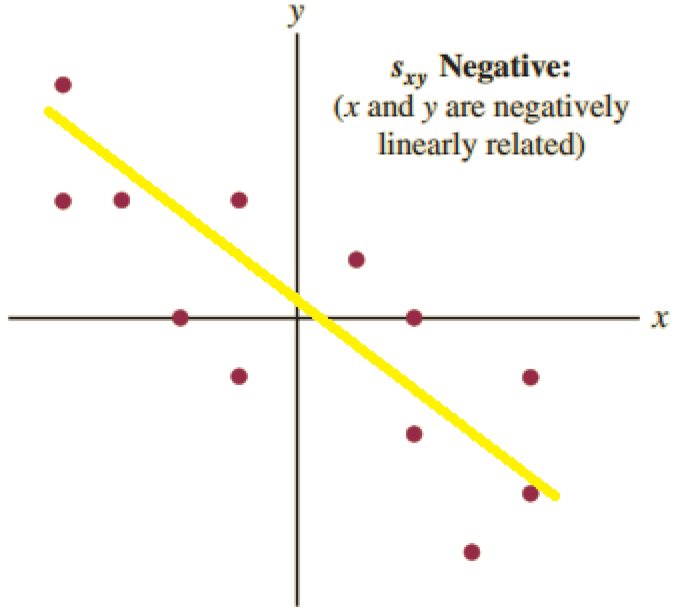
$0 < r \le 1$

r Positive



$r \approx 0$

r : around zero



$-1 \le r < 0$

r : Negative

r : Pearson Correlation Coefficient

r : <u>Pearson</u> Correlation Coefficient used to measure the <u>linear</u> relationship between two <u>quantitative</u> variable

The correlation coefficient (r) ranges from −1 to +1. $-1 \le r \le 1$

Values close to −1 or +1 indicate a strong linear relationship.

The closer the correlation is to zero, the weaker the relationship.

Negative

Positive

r = -1 **r = -0. 5** **r = 0** **r = 0.5** **r = +1**

Negative relationship

Positive relationship

r = -1 **r = -0. 5** **r = 0** **r = 0.5** **r = +1**

Here,

Some examples of the correlation coefficient measure values

r = 0.00 >>Means>> No Linear Relationship

r = - 1 >>>> Complete and negative linear relationship
r = - 0.98 >>>> Strong and negative linear relationship
r = - 0. 5 >>> Moderate and negative linear relationship
r = - 0.25 >>>> Weak and negative linear relationship

r = +1 >>>> Complete and positive linear relationship
r =+ 0.95 >>>> Strong and positive linear relationship
r =+ 0. 5 >>> Moderate and positive linear relationship
r =+ 0.34 >>>> Weak and positive linear relationship

<u>Note</u> about the correlation coefficient measure (r):

If r=0.00, this implies that there is no linear relationship between variables X & Y but there may be some other non-linear relationship.

Hence, r=0.00 doesn't necessarily imply that the two variables are independent.

So,
if X & Y are independent variables >> Then: r = 0.00

But
if r = 0.00, we can't said: X & Y are independent variables

r : Pearson Correlation Coefficient $-1 \le r \le 1$

$$r = \frac{\sum (x_i - \bar{x})\ (y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2} \sqrt{\sum (y_i - \bar{y})^2}} \quad \textbf{OR} \quad r = \frac{\sum x_i y_i - n\bar{x} * \bar{y}}{\sqrt{\sum x_i^2 - n\bar{x}^2} \sqrt{\sum y_i^2 - n\bar{y}^2}}$$

Example: Compute Pearson Correlation Coefficient

X	4	6	28	6
Y	5	8	14	9

Answer

	X	Y	(X-$\bar{x}$)	$(X-\bar{x})^2$	(Y-$\bar{y}$)	$(Y-\bar{y})^2$	(X-$\bar{x}$)(Y-$\bar{y}$)
	4	5	-7	49	-4	16	28
	6	8	-5	25	-1	1	5
	28	14	17	289	5	25	85
	6	9	-5	25	0	0	0
Total	**44**	**36**	**0**	**388**	**0**	**42**	**118**

$$r = \frac{\sum (x_i - \bar{x})\ (y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2} \sqrt{\sum (y_i - \bar{y})^2}}$$

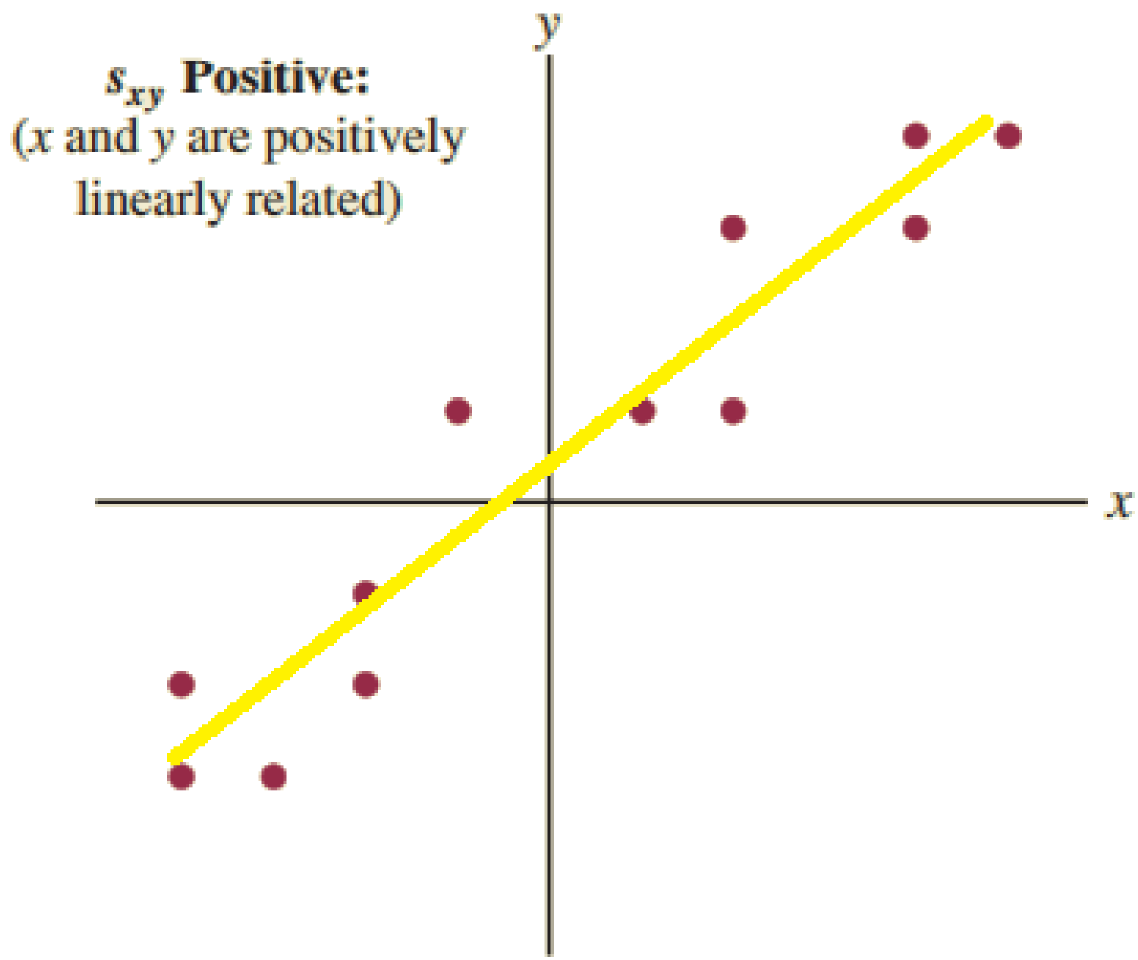
$$r = \frac{118}{\sqrt{388}\sqrt{42}} = 0.9243608$$

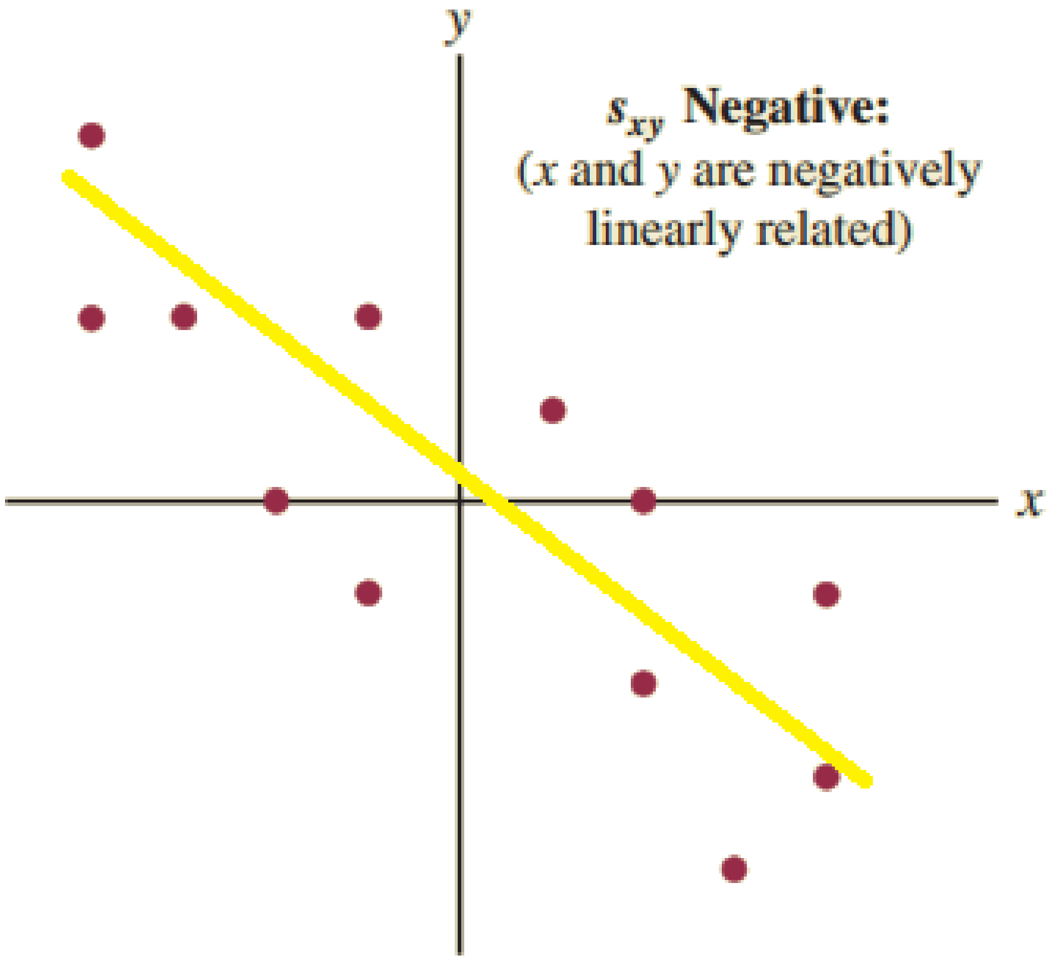
$$= 0.92$$

r = 0.92 >>>> Strong and positive linear relationship

Simple Linear Regression

$$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$$





Linear Regression

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x_i$$

Estimation Error (residual): $e = y - \hat{y}$

$\widehat{\boldsymbol{\beta}}_1$ **: Slope of the Regression Line**

$$\hat{\beta}_1 = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2}$$

OR $$\hat{\beta}_1 = \frac{\sum x_i y_i - n\bar{x} * \bar{y}}{\sum x_i^2 - n\bar{x}^2}$$

$\widehat{\boldsymbol{\beta}}_0$ **: intercept of the Regression Line**

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

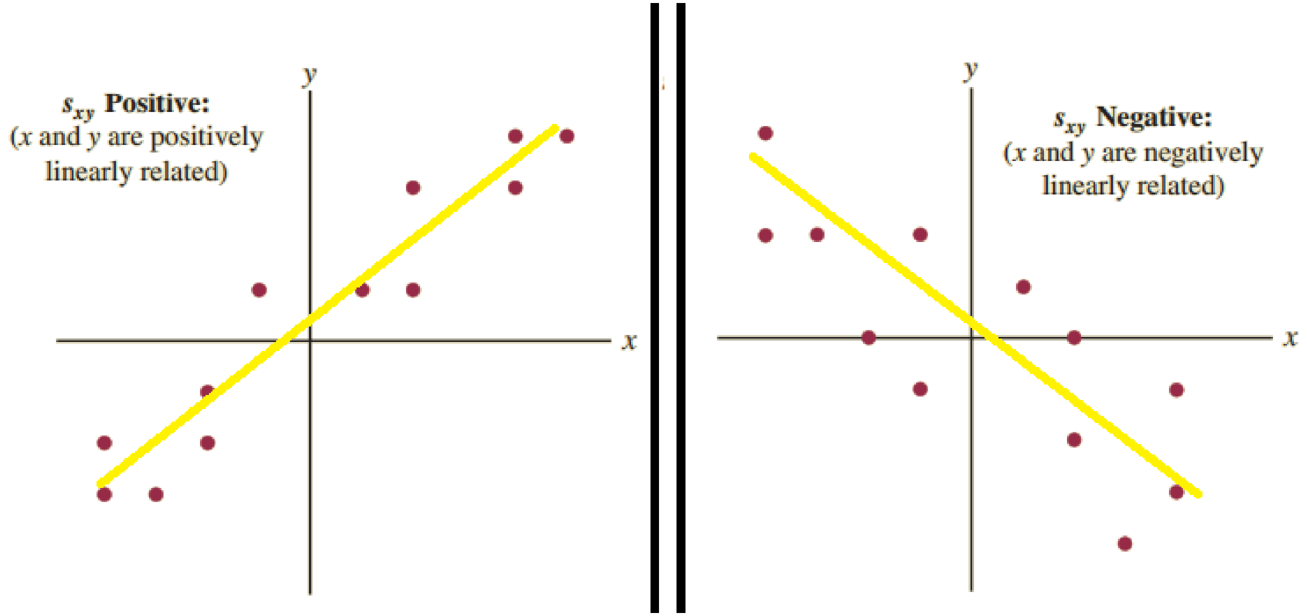
Note:

The sign of the linear regression Slope AND the sing of Pearson correlation coefficient are same

Exercise:

Based on the following two cases :

Determine when the error of estimation (residual) is a positive and when it is a negative.



Estimation Error (residual):

$$e = y - \hat{y}$$

Example:

For the following values

Find:

X	4	6	28	6
Y	5	8	14	9

a) Pearson correlation coefficient (r)

b) The relation type and strength

c) The linear regression equation $\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$

d) The regression estimate at x = 0

e) The regression estimate at x = 6

f) The regression estimate at x = 4

g) The error of regression estimate at x = 4

Answer

	X	Y	(X-$\bar{x}$)	$(X-\bar{x})^2$	(Y-$\bar{y}$)	(X-$\bar{x}$)(Y-$\bar{y}$)
	4	5	-7	49	-4	28
	6	8	-5	25	-1	5
	28	14	17	289	5	85
	6	9	-5	25	0	0
Total	**44**	**36**	**0**	**388**	**0**	**118**

$$\hat{\beta}_1 = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2} = \frac{118}{388} = 0.304$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x} = 9 - (0.308)(11) = 5.612$$

c) The linear regression equation is: $\hat{y}_i = 5.61 + 0.304 x_i$

Answer

a) **Pearson correlation coefficient (r = 0.92)**

b) **The relation type and strength : is strong and positive linear relationship**

c) **The linear regression equation** $\hat{y}_i = 5.61 + 0.304 x_i$

d) **The regression estimate at x = 0 is** $\hat{y} = 5.61 + 0.304(0) = 5.61$

e) **The regression estimate at x = 6 is** $\hat{y} = 5.61 + 0.304(6) = 7.434$

f) **The regression estimate at x = 4 is** $\hat{y} = 5.61 + 0.304(4) = 6.862$

g) **The error of regression estimate at x = 4 is** $e = y - \hat{y} = 5 - 6.862 = -1.826$

Example:

Based on the following data values

Find:

a) **Pearson correlation coefficient (r)**
b) **The relation type and strength**
c) **The linear regression equation**
d) **The regression estimate at x = 0**
e) **The regression estimate at x = 6**
f) **The regression estimate at x = 3**
g) **The error of regression estimate at x = 3, if y=4**

$$n=6, \bar{x}=9, \bar{y}=8, \sum x_i^2 = 930, \sum y_i^2 = 446, \sum x_i y_i = 582$$

X	4	3	7	6	28	6
Y	5	4	8	8	14	9

$$r = \frac{\sum x_i y_i - n\bar{x} * \bar{y}}{\sqrt{\sum x_i^2 - n\bar{x}^2}\sqrt{\sum y_i^2 - n\bar{y}^2}} = \frac{582-(6)(9)(8)}{\sqrt{930-(6)(9^2)}\sqrt{446-(6)(8^2)}} = 0.904$$

$$\hat{\beta}_1 = \frac{\sum x_i y_i - n\bar{x} * \bar{y}}{\sum x_i^2 - n\bar{x}^2} = \frac{582-(6)(9)(8)}{930-(6)(9^2)} = \frac{150}{444} = 0.338$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x} = 8-(0.338)(9) = 4.96$$

$$\hat{y}_i = 4.96 + 0.338 x_i$$

Answer:

a) **Pearson correlation coefficient (r = 0.904)**

b) **The relation type and strength : is strong and positive linear relationship**

c) **The linear regression equation** $\hat{y}_i = 4.96 + 0.338x_i$

d) **The regression estimate at x = 0** $\hat{y}_i = 4.96$

e) **The regression estimate at x = 6** $\hat{y}_i = 4.96 + 0.338(6) = 6.99$

f) **The regression estimate at x = 3** $\hat{y}_i = 4.96 + 0.338(3) = 5.974$

g) **The error of regression estimate at x = 3 , if y=4** $e = y - \hat{y} = 4 - 5.974 = -1.974$

The coefficient of determination (R^2):

is the square of the Pearson correlation (r) it ranges

$$0 \leq R^2 \leq 1$$

The coefficient of determination (R^2):

is a measurement used to explain how much variability of dependent variable Y can be caused by its relationship to independent variable X.

Ex. If the Pearson correlation (r=0.9) then $R^2 = (0.9)^2 = 0.81 = 81\%$

this means that:

The independent variable is explained about 81% of the variation of the dependent variable Y.

And 19% of the dependent variable variation comes from another factor(s).

Ex. Assuming the regression equation is $\hat{y}_i = 0.2 - 1.5x_i$,

If the coefficient of determination is (R^2=0.64). Then Pearson correlation (r) is:

a) 0.8 b) -0.8 c) 0.64 d) -0.64 e) 0.2 f) -0.2

Example:

Based on the following data values

X	20	8	12	14	24
Y	2	5	7	8	2

Find:

a) **Pearson correlation coefficient (r)**
b) **The relation type and strength**
c) **The linear regression equation**
d) **What is the slope of the linear regression line**
e) **The regression estimate at x = 0**
f) **What is the value of Y that the linear regression line intercept Y-axis**
g) **What is the value of X that the linear regression line intercept X-axis**
h) **The regression estimate at x = 8**
i) **The error of regression estimate at x =8,**
j) **What is the percent of X variable interpret the variation of variable Y (How much the coefficient of determination)?**