Principles of Statistics for Admin. (15060105)

Chapter 9: Hypothesis Tests

In hypothesis testing we begin by making a tentative assumption about a population parameter. This tentative assumption is called the **null hypothesis** and is denoted by H_0 . We then define another hypothesis, called the alternative hypothesis, which is the opposite of what is

The alternative hypothesis is denoted by H_{α} or H_{1} .

The hypothesis testing procedure uses data from a sample to test the two competing statements indicated by H_0 and H_a .

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Note: The null hypothesis H_0 includes (=):
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stated in the null hypothesis.

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Ex: H_0: \mu = 5 Versus H_a: \mu \neq 5 This H_a is called: Two tailed hypothesis
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Ex:
$$H_0$$
: $\mu \le 5$ Versus H_a : $\mu > 5$ This H_a is called: One tailed hypothesis

Ex: H_0 : $\mu \ge 5$ Versus H_a : $\mu < 5$ This H_a is called: One tailed hypothesis

Note:

The null hypothesis H_0 is a negative sentence that formulated to denial any of the relationship, effect, or differences:

Ex. The null hypothesis H_0 is formulated to say that: there is no relationship between variables, or no differences between groups, or no effect of a variable or a method

Example:

 H_0 : There is No Linear relationship between X & Y (r = 0).

Versus H_a : There is a Linear relationship between X & Y $(r \neq 0)$.

Example:

 H_0 : There is No effect of gender variable on the student marks $(\mu_{male} = \mu_{female})$.

Versus H_a : There is an effect of gender variable on the student marks $(\mu_{male} \neq \mu_{female})$.

Note:

The null hypothesis H_0 usually assumed the population data distributed is from a specific distribution.

Example:

 H_0 : The student marks is normally distributed. ($X \sim Normal(\mu, \sigma^2)$).

Versus H_a : The student marks is **NOT** normally distributed. (X is $Not \sim Normal(\mu, \sigma^2)$).

The Alternative Hypothesis as a Research Hypothesis

Many applications of hypothesis testing involve an attempt to gather evidence in support of a research hypothesis.

in these situations, it is often best to begin with the alternative hypothesis and make it the conclusion that the researcher hopes to support. consider a particular automobile that currently attains a fuel efficiency of 24 miles per gallon in city driving.

a product research group has developed a new fuel injection system designed to increase the milesper-gallon rating. the group will run controlled tests with the new fuel injection system looking for statistical support for the conclusion that the new fuel injection system provides more miles per gallon than the current system.

Several new fuel injection units will be manufactured, installed in test automobiles, and subjected to research-controlled driving conditions. the sample mean miles per gallon for these automobiles will be computed and used in a hypothesis test to determine if it can be concluded that the new system provides more than 24 miles per gallon. In terms of the population mean miles per gallon μ , the research hypothesis $\mu > 24$ becomes the alternative hypothesis H_a . Since the current system provides an average or mean of 24 miles per gallon, we will make the tentative assumption that the new system is not any better than the current system and choose $\mu \leq 24$ as the null hypothesis H_0 . The null and alternative hypotheses are:

 $H_0: \mu \leq 24 \quad Vs \quad H_a: \mu > 24$

Write the null and alternative hypotheses in the following cases:

- a) A researcher needs to test if the student marks mean is more than 75: H_0 : $\mu \le 75$ Vs H_a : $\mu > 75$ Here, we write H_a first, and then H_0 is the opposite.
- b) A researcher needs to test if the student marks mean equals to 75: H_0 : $\mu = 75$ Vs H_a : $\mu \neq 75$ Here, we write H_0 first, and then H_a is the opposite.
- c) A researcher needs to test if the student marks mean is less than 75: H_0 : $\mu \ge 75$ Vs H_a : $\mu < 75$ Here, we write H_a first, and then H_0 is the opposite.
- d) A company wants to test if there product can be used after one year (product life>1): H_0 : $\mu \le 1$ Vs H_a : $\mu > 1$ Here, we write H_a first, and then H_0 is the opposite.
- e) A company wants to test if there batteries have a mean life of 112 hours: H_0 : $\mu = 112$ Vs H_a : $\mu \neq 112$

Summary of Forms for Null and Alternative Hypotheses

Assuming μ_0 be the hypothesized value

$$H_0: \mu \ge \mu_0$$
 $H_0: \mu \le \mu_0$ $H_0: \mu = \mu_0$
 $H_a: \mu < \mu_0$ $H_a: \mu > \mu_0$ $H_a: \mu \ne \mu_0$

OR:

OR:

 H_0 : $\mu = \mu_0$ H_0 : $\mu = \mu_0$ Vs. H_1 : $\mu < \mu_0$ Vs. H_1 : $\mu > \mu_0$

 H_0 ملاحظة: اشارة المساواة تكون دائما لصالح الفرضية الصفرية

Example: Here μ_0 =600 be the hypothesized value

$$H_0$$
: $\mu \ge 600$ H_0 : $\mu \le 600$ H_0 : $\mu = 600$ H_a : $\mu < 600$ H_a : $\mu \ge 600$ H_a : $\mu \ne 600$

The Null Hypothesis as an Assumption to be Challenged

Of course, not all hypothesis tests involve research hypotheses.

In the following discussion we consider applications of hypothesis testing where we begin with a belief or an assumption that a statement about the value of a population parameter is true. We will then use a hypothesis test to challenge the assumption and determine if there is statistical evidence to conclude that the assumption is incorrect. In these situations, it is helpful to develop the **null** hypothesis first. The null hypothesis H_0 expresses the belief or assumption about the value of the population parameter. The alternative hypothesis H_0 is that the belief or assumption is incorrect.

As an example, consider the situation of a manufacturer of soft drink products. The label on a soft drink bottle states that it contains 67.6 fluid ounces. we consider the label correct provided the population mean filling weight for the bottles is at least 67.6 fluid ounces. Without any reason to believe otherwise, we would give the manufacturer the benefit of the doubt and assume that the statement provided on the label is correct. Thus, in a hypothesis test about the population mean fluid weight per bottle, we would begin with the assumption that the label is correct and state the **null hypothesis** as $\mu \geq 67.6$.

The challenge to this assumption would imply that the label is incorrect and the bottles are being underfilled. This challenge would be stated as the **alternative hypothesis** μ < 67.6.

Thus, the null and alternative hypotheses are:

 $H_0: \mu \geq 67.6 \quad Vs. \quad H_1: \mu < 67.6$

Type I and Type II Errors

The <u>null</u> and <u>alternative</u> hypotheses are competing statements about the population. Either the null hypothesis H_0 is true or the alternative hypothesis H_a is true, but **NOT BOTH**.

Ideally the hypothesis testing procedure should lead to [correct conclusions]

>> the acceptance of H_0 when H_0 is true (H_a is false).

and

>> the rejection of H_0 when H_a is true (H_0 is false).

Unfortunately, the correct conclusions are not always possible, because hypothesis tests are based on sample information, we must allow for the possibility of errors.

		Population Condition	
		H_0 True	H _a True
Conclusion	Accept H_0	Correct Conclusion	Type II Error
	Reject H ₀	Type I Error	Correct Conclusion

		Population Condition	
		H ₀ True	H _a True
Conclusion	Accept H ₀	Correct Conclusion	Type II Error
	Reject H ₀	Type I Error	Correct Conclusion

Note: The hypothesis: $H_0 \& H_a$ are two competing statements.

If one is true; the another is false. If one is false; the another is true.

The **first row** of this table shows what can happen if the conclusion is to **accept** H_0 . If H_0 is true, this conclusion is correct.

However, if H_a is true, we make a Type II error; that is, we accept H_0 when it is false.

The **second row** of this table shows what can happen if the conclusion is to **reject** H_0 . If H_0 is true, we make a Type I error; that is, we reject H_0 when it is true. However, if H_a is true, rejecting H_0 is correct.

The probability of making a type I error when the null hypothesis is true as an equality is called the level of significance (α) .

Where

Type I error; when we reject H_0 when it is true.

The level of significance is the <u>probability</u> of making a type I error when the null hypothesis is true as an equality.

The common choices for (α) are .05 and .10

The probability of making a type II error is called (β) .

Where:

Type II error; when we Accept H_0 when it is false.

Type II error; when we [Don't Reject] H_0 when it is false.

The probability of making a type II error is called (β) .

The **power** of the test: is the probability of correctly rejecting H_0 when it is false. [power = $1 - \beta$]

For any particular value of μ , The Power is $(1 - \beta)$; that is, the probability of correctly rejecting the null hypothesis is 1 minus the probability of making a type II error.

Such a graph is called a power curve. note that the power curve extends over the values of μ for which the null hypothesis is false.

The height of the power curve at any value of μ indicates the probability of correctly rejecting H_0 when H_0 is false.

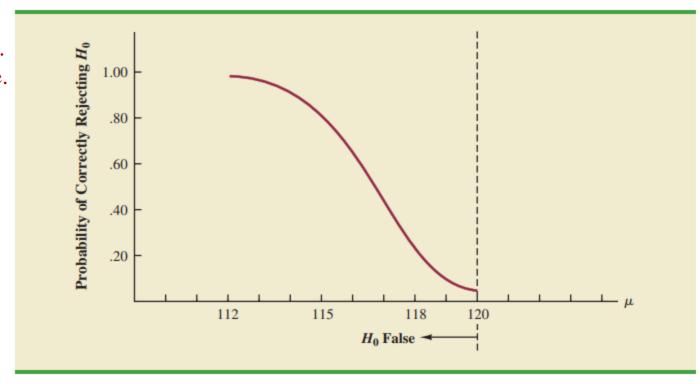
Value of μ	$z = \frac{116.71 - \mu}{12/\sqrt{36}}$	Probability of a Type II Error (β)	Power (1 – β)
112	2.36	.0091	.9909
114	1.36	.0869	.9131
115	.86	.1949	.8051
116.71	.00	.5000	.5000
117	15	.5596	.4404
118	65	.7422	.2578
119.999	-1.645	.9500	.0500

The probability of making a type II error is called (β) .

Where:

Type II error; when we Accept H_0 when it is false. **Type II error**; when we [Don't Reject] H_0 when it is false.

POWER CURVE FOR THE LOT-ACCEPTANCE HYPOTHESIS TEST



In practice, the person responsible for the hypothesis test specifies the level of significance. By selecting (α) , that person is controlling the probability of making a type I error.

If the cost of making a type I error is high, small values of (α) are preferred. if the cost of making a type I error is not too high, larger values of (α) are typically used.

Applications of hypothesis testing that only control for the type I error are called **significance tests**. Many applications of hypothesis testing are of this type.

Although most applications of hypothesis testing control for the probability of making a type I error, they do not always control for the probability of making a type II error. Hence, if we decide to accept H_0 , we cannot determine how confident we can be with that decision.

Because of the uncertainty associated with making a type II error when conducting significance tests, statisticians usually recommend that we use the statement "do not reject H_0 " instead of "accept H_0 ." Using the statement "do not reject H_0 " carries the recommendation to with hold both judgment and action. In effect, by not directly accepting H_0 , the statistician avoids the risk of making a type II error. Whenever the probability of making a type II error has not been determined and controlled, we will not make the statement "accept H_0 ."

In such cases, only **two conclusions** are **possible**: **Do not reject** H_0 or **reject** H_0 . Although controlling for a type II error in hypothesis testing is not common, it can be done.

In summary, the following step-by-step procedure can be used to compute the probability of making a type II error in hypothesis tests about a population mean (μ) :

- 1. formulate the null and alternative hypotheses.
- 2. use the level of significance (α) and the critical value approach to determine the critical value and the rejection rule for the test.
- 3. use the rejection rule to solve for the value of the sample mean corresponding to the critical value of the test statistic.
- 4. use the results from step 3 to state the values of the sample mean that lead to the acceptance of H_0 . these values define the acceptance region for the test.
- 5. use the sampling distribution of \bar{x} for a value of μ satisfying the alternative hypothesis, and the acceptance region from step 4, to compute the probability that the sample mean will be in the acceptance region.

This probability is the probability of making a type II error at the chosen value of μ .

In summary, the following step-by-step to test any hypothesis about the population mean (μ) :

- 1) Write the hypothesis: H_0 & H_a
- 2) Compute the test statistic
- 3) Draw Acceptance-Rejection regions of H_0 [This is based on H_a]
- 4) Make your conclusion:

[Reject H_0] OR [Accept $H_0 = Don't$ Reject H_0] we Reject H_0 if **test statistic** \in **Rejection regions**

Acceptance-Rejection regions of H_0

(One Tail Test)

Sampling distribution

of \overline{x}

This is based on H_a :

If H_a : $\mu < \mu_0$

The Rejection region is on the left

Lower Tail Test Left Tail Test (One Tail Test)

Accept H0Reject H0 μ_0 $-Z_{\alpha}$

The critical value is $-Z_{\alpha}$

Acceptance-Rejection regions of H_0

(One Tail Test)

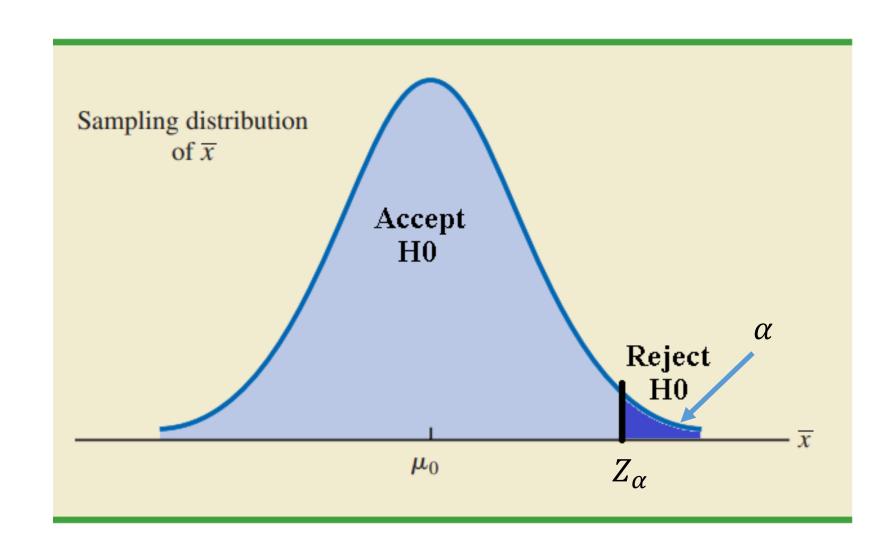
This is based on H_a :

If H_a : $\mu > \mu_0$

The Rejection region is on the Right

Upper Tail Test Right Tail Test (One Tail Test)

The critical value is Z_{α}



Acceptance-Rejection regions of H_0 (Two Tail Test)

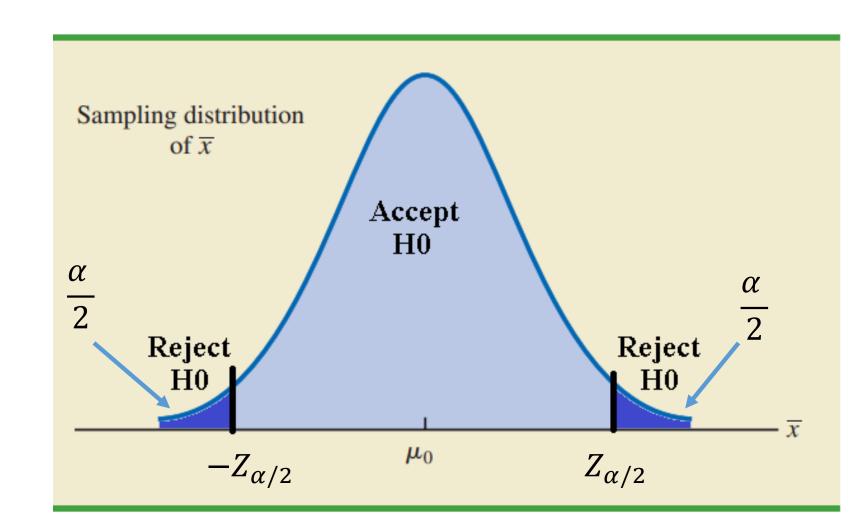
This is based on H_a :

If H_a : $\mu \neq \mu_0$ (Two Tail Test)

Then May be $\mu < \mu_0$

OR

 $\mu > \mu_0$



TEST STATISTIC FOR HYPOTHESIS TESTS ABOUT A POPULATION MEAN:

 σ KNOWN

$$z = \frac{\overline{x} - \mu_0}{\sigma / \sqrt{n}} \tag{9.1}$$

If (σ) is unknown, we replace it value by (S) when the sample size is large

TEST STATISTIC FOR HYPOTHESIS TESTS ABOUT A POPULATION MEAN:

 σ UNKNOWN

t distribution with (n-1) degrees of freedom

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \tag{9.2}$$

TEST STATISTIC FOR HYPOTHESIS TESTS ABOUT A POPULATION

$$z = \frac{\bar{p} - p_0}{\sqrt{\frac{p_0(1 - p_0)}{n}}}$$

(9.4)

p-VALUE

A p-value is a probability that provides a measure of the evidence against the null hypothesis provided by the sample. Smaller p-values indicate more evidence against H_0 .

STEPS OF HYPOTHESIS TESTING

- **Step 1.** Develop the null and alternative hypotheses.
- **Step 2.** Specify the level of significance.
- Step 3. Collect the sample data and compute the value of the test statistic.

p-Value Approach

- **Step 4.** Use the value of the test statistic to compute the *p*-value.
- **Step 5.** Reject H_0 if the *p*-value $\leq \alpha$.
- **Step 6.** Interpret the statistical conclusion in the context of the application.

Critical Value Approach

- Step 4. Use the level of significance to determine the critical value and the rejection rule.
- **Step 5.** Use the value of the test statistic and the rejection rule to determine whether to reject H_0 .
- **Step 6.** Interpret the statistical conclusion in the context of the application.

REJECTION RULE USING p-VALUE

Reject H_0 if p-value $\leq \alpha$

Relation between (Two Tail Test) & Confidence intervals:

If the hypothesized value falls outside the interval, we reject the null hypothesis.

The population variance is known. (use Z-tables)

$$\overline{x} + \text{margin of error}$$
: $\overline{x} + Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$, $Z_{\frac{\alpha}{2}}$ value providing an area of $\alpha/2$ in the upper tail of the Z distribution

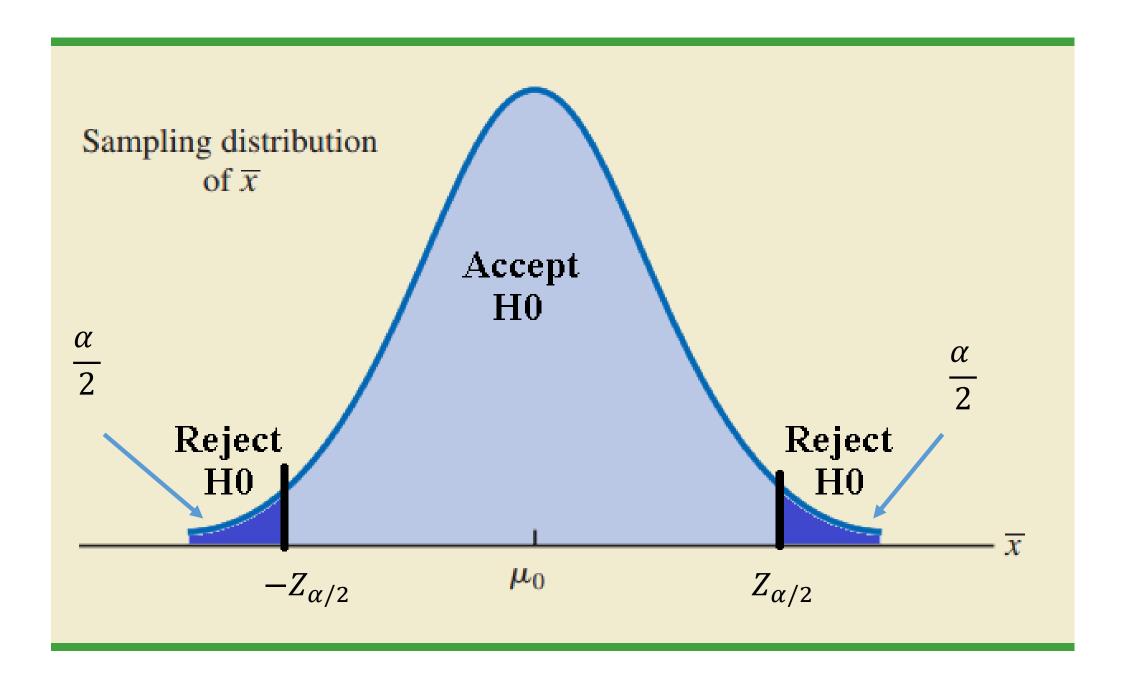
If (σ) is unknown, we replace it value by (S) when the sample size is large

The population variance is Unknown. (use t-tables)

$$\overline{x} + \text{margin of error}$$
: $\overline{x} + t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$, $t_{\frac{\alpha}{2}}$ value providing an area of $\alpha/2$ in the upper tail of the t distribution with $(n-1)$ degrees of freedom

* Confidence intervals for population proportion (p). (using Z-tables)

$$\overline{p} \mp \text{margin of error}: \quad \overline{p} \mp Z_{\frac{\alpha}{2}} \sqrt{\frac{\overline{p}(1-\overline{p})}{n}}$$



Note:

The most Z critical values for Confidence Intervals & Hypothesis Test

(Two tailed test):

from Normal Table

Assuming 95% confidence interval (C.I.) >>>>
$$Z_{\frac{0.05}{2}} = 1.96$$
 , $\alpha = 0.05$

Assuming 90% confidence interval (C.I.) >>>>
$$Z_{\frac{0.10}{2}}=1.645$$
 , $\alpha=0.10$

Assuming 85% confidence interval (C.I.) >>>>
$$Z_{\frac{0.15}{2}} = 1.44$$
 , $\alpha = 0.15$

Assuming 80% confidence interval (C.I.) >>>>
$$Z_{\frac{0.20}{2}} = 1.28$$
 , $\alpha = 0.20$

In summary, the following step-by-step to test any hypothesis about the population mean (μ) :

- 1) Write the hypothesis: H_0 & H_a
- 2) Compute the test statistic
- 3) Draw Acceptance-Rejection regions of H_0 [This is based on H_a]
- 4) Make your conclusion:

[Reject H_0] OR [Accept $H_0 = Don't$ Reject H_0] we Reject H_0 if **test statistic** \in **Rejection regions**

Make your conclusion:

[Reject H_0] OR [Accept $H_0 = Don't$ Reject H_0]

3 ways to make your conclusion:

we Reject H_0 if test statistic \in Rejection regions

OR

we Reject H_0 if the hypothesized value falls outside the confidence interval

OR

we Reject H_0 if $P_{value} \leq \alpha$

What is P_value: = a Probability of rejection using the test statistics calculated under H_1

p-VALUE

A p-value is a probability that provides a measure of the evidence against the null hypothesis provided by the sample. Smaller p-values indicate more evidence against H_0 .

How the P_Value calculated?

If H_1 is right tailed test:

$$P_{value} = P\{Z > Test Statistic\}$$

If H_1 is left tailed test:

$$P_{value} = P\{Z < Test Statistic\}$$

If H_1 is Two tailed test:

$$P_{value} = 2 * P\{Z > |Test Statistic|\}$$

Consider the following hypothesis test H_0 : $\mu = 50$ Vs. H_1 : $\mu \neq 50$. A sample of 64 provided a sample mean of 51.44. If the population standard deviation is 8.

- 1) What is the p-value of the test.
- 2) What is your decision about this hypothesis based on $\alpha = 0.05$
- 3) What is your decision about this hypothesis based on the p_value

Solve

1)

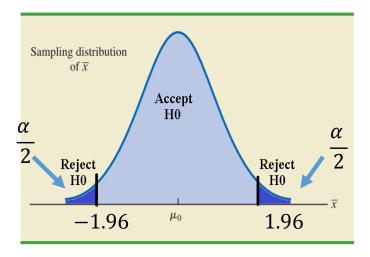
The hypothesis: H_0 : $\mu = 50$ $Vs. H_1$: $\mu \neq 50$

The Test Statistic: $z = \frac{51.44-50}{8/\sqrt{64}} = \frac{1.44}{1} = 1.44$

$$P(Z > 1.44) = 0.0749$$
 & $P(Z < -1.44) = 0.0749$

This is two tailed test Then $P_{value} = P(Z > 1.44) + P(Z < -1.44) = 0.1498 = 0.15$

Draw Acceptance-Rejection region



2) Make the conclusion:

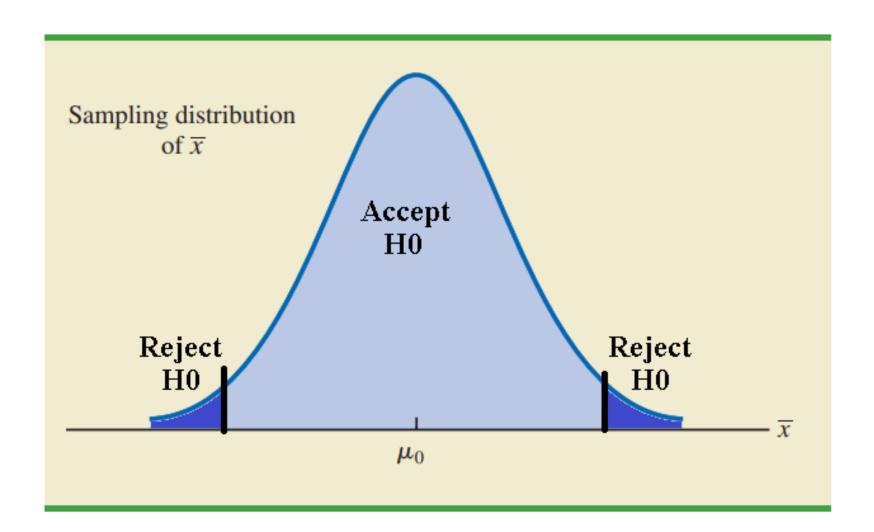
the test statistics falls in the acceptance region Then, Don't Reject H_0

3) Make the conclusion:

$$P_{value} = 0.15 > \alpha = 0.05 \rightarrow \text{Then, Don't Reject } H_0$$

TEST STATISTIC FOR HYPOTHESIS TESTS ABOUT A POPULATION MEAN: σ KNOWN

$$z = \frac{\overline{x} - \mu_0}{\sigma / \sqrt{n}} \tag{9.1}$$



Consider the following hypothesis test H_0 : $\mu = 50$ Vs. H_1 : $\mu > 50$. A sample of 64 provided a sample mean of 51.44. If the population standard deviation is 8.

- 1) What is the p-value of the test.
- What is your decision about this hypothesis based on $\alpha = 0.10$
- What is your decision about this hypothesis based on the p_value

Solve

1)

The hypothesis:
$$H_0$$
: $\mu = 50$ $Vs. H_1$: $\mu > 50$,

 σ KNOWN

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} \tag{9.1}$$

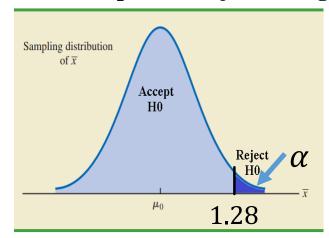
TEST STATISTIC FOR HYPOTHESIS TESTS ABOUT A POPULATION MEAN:

The Test Statistic:
$$z = \frac{51.44-50}{8/\sqrt{64}} = \frac{1.44}{1} = 1.44$$

$$P(Z > 1.44) = 0.0749$$

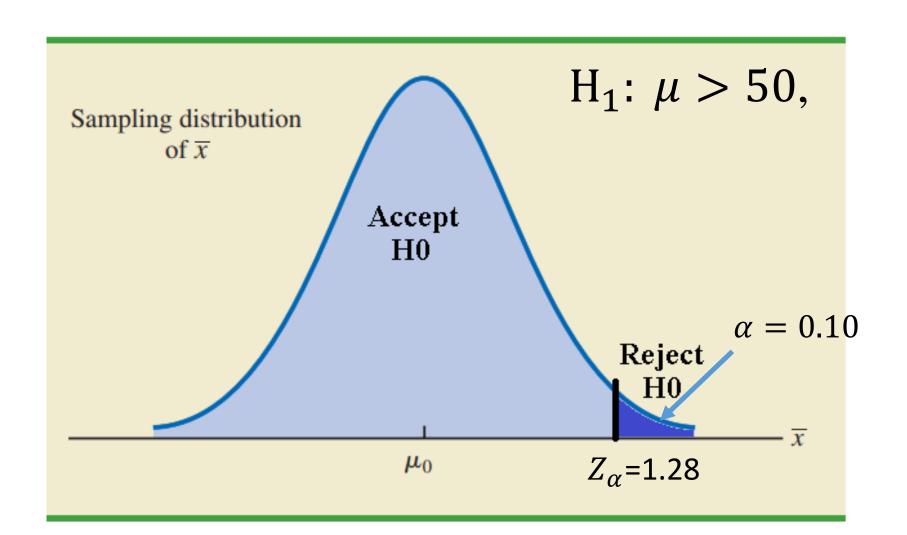
This is One tailed test Then $P_{value} = P(Z > 1.44) = 0.0749$

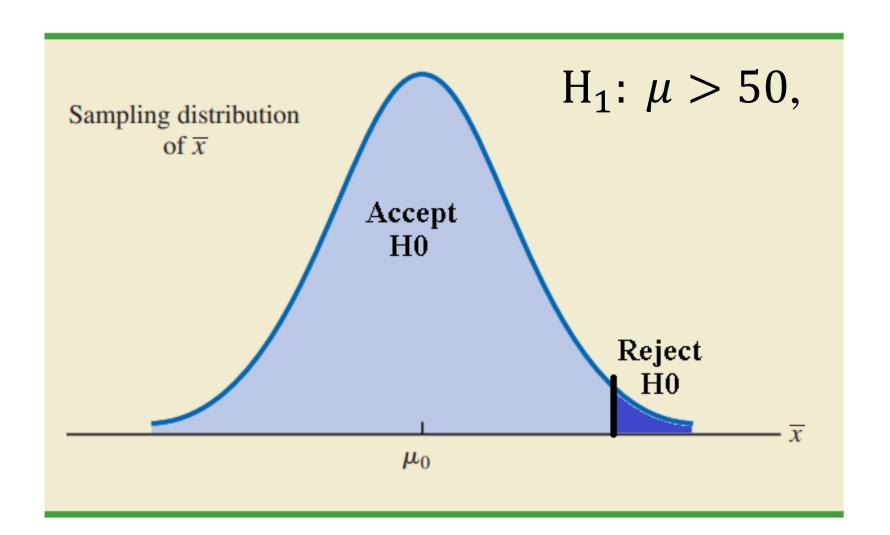
Draw Acceptance-Rejection region



2) Make the conclusion: the test statistics falls in the Rejection region Then, Reject H_0

3) Make the conclusion: $P_{value} = 0.0749 < \alpha = 0.10 \rightarrow \text{Then, Reject } H_0$





Assuming that a random sample of 240 students from PTUK university has been given a qualified exam, and 192 of these students had success in the exam by levels over than 70. Test if the population proportion of students in PTUK who passed the qualifying exam with graded over than 70 is more than 0.75 based on $\alpha = 0.05$.

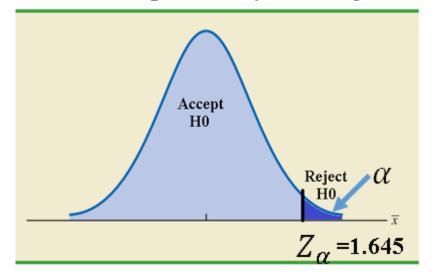
Solve

The hypothesis:
$$H_0: p \le 0.75$$
 Vs. $H_a: p > 0.75$, In our question: $\overline{p} = \frac{192}{240} = 0.8$

The Test Statistic:
$$z = \frac{0.8-0.75}{\sqrt{\frac{0.75(1-0.75)}{240}}} = \frac{0.05}{\sqrt{0.00078125}} = 1.7889$$

TEST STATISTIC FOR HYPOTHESIS TESTS ABOUT A POPULATION
PROPORTION
$$z = \frac{\bar{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$
 (9.4)

Draw Acceptance-Rejection region



Make the conclusion:

Reject H_0

Consider the following hypothesis test H_0 : $\mu = 70$ Vs. H_1 : $\mu \neq 70$. A sample of 25 provided a sample mean of 73. If the sample standard deviation is 8.

What is your decision about this hypothesis based on $\alpha = 0.05$

TEST STATISTIC FOR HYPOTHESIS TESTS ABOUT A POPULATION MEAN: σ UNKNOWN

t distribution with (n-1) degrees of freedom

$$=\frac{\bar{x}-\mu_0}{s/\sqrt{n}}\tag{9.2}$$

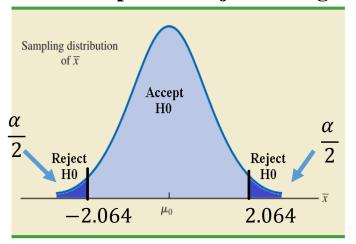
Solve

1)

The hypothesis: H_0 : $\mu = 70$ $Vs. H_1$: $\mu \neq 70$,

The Test Statistic: $t = \frac{73-70}{8/\sqrt{25}} = 1.875$ it is t distributed with 24 degrees of freedom

Draw Acceptance-Rejection region

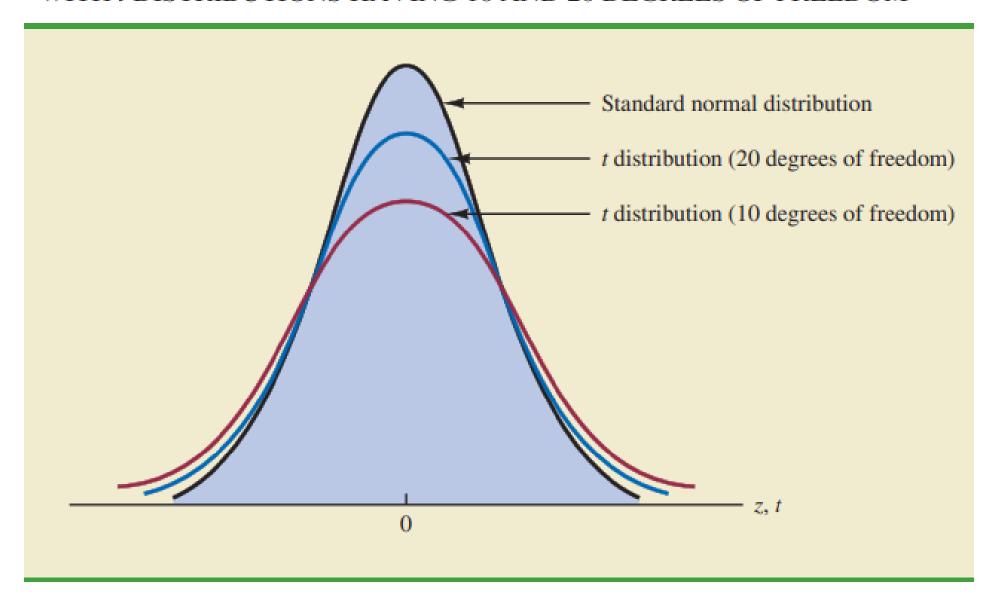


Make the conclusion:

The test statistics falls in the Acceptance region

Then, Accept H_0

COMPARISON OF THE STANDARD NORMAL DISTRIBUTION WITH t DISTRIBUTIONS HAVING 10 AND 20 DEGREES OF FREEDOM



If a p-value for your hypothesis testing about the population mean is 0.071, and the level of significance used was 5%, then the conclusion would be to:

1) Reject the null hypothesis H_o OR 2) Don't Reject the null hypothesis H_o OR 3) Not enough information to make conclusion

Example:

Consider a hypothesis H_0 : $\mu = 30$ Vs. H_a : $\mu \neq 30$, if the test critical value is 1.645, and the test statistic value (z=0.874) then

- 1) The null hypothesis H_0 should be rejected
- 2) The null hypothesis H_0 should not be rejected
- 3) Not enough information is given to make conclusion
- 4) The alternative hypothesis H_a must be accepted.

Example:

A survey was conducted to get an estimate of the proportion of shisha smokers among the school students. A government report says that 8% of them are shisha smokers.

The school manager doubts the report result and thinks that the actual proportion is much less than this proportion.

Choose the correct choice of null and alternative hypothesis the school manager wants to test:

- 1) H_0 : p = 0.08 versus H_a : $p \le 0.08$
- 2) H_0 : p = 0.08 versus H_a : p > 0.08.
- 3) H_0 : p = 0.08 versus H_a : p < 0.08.
- 4) H_0 : p = 0.08 versus H_a : $p \neq 0.08$

A hypothesis test is done in which the alternative hypothesis is that more than 8% of a population is left-handed. The p-value for the test is calculated to be (0.18).

Which statement is correct? we can:

- 1) conclude that more than 8% of the population is left-handed.
- :2) conclude that more than 18% of the population is left-handed
- 3) conclude that exactly 18% of the population is left-handed
- 4) Not conclude that more than 8% of the population is left-handed

Example:

The test statistic for a population mean in a One-tailed test is z = -1.18, then the p-value is

1) 0.119

2) 0.05

3) 0.118

4) 0.238

Example:

The test statistic for a population mean in a Two-tailed test is z = -1.18, then the p-value is

1) 0.119

2) 0.05

3) 0.118

4) 0.238

Example:

Suppose 90% confidence interval for population mean (μ) is (42.3,56.7), If the test is H₀: $\mu = 50$ Vs. H_a: $\mu \neq 50$ at $\alpha = 10\%$, then, the p_value of this test is : 1) more than 0.10 2) less than 0.10 3) more than 0.90 3) equals to 0.05

Example:

Assuming the power of a test is 0.85. The probability of [Don't Reject H_0 when it is false] is: 1) 0.85 **2) 0.15** 3) 0.075 4)0.05.