4 Work and Heat

In this chapter we consider work and heat. It is essential for the student of thermodynamics to understand clearly the definitions of both work and heat, because the correct analysis of many thermodynamic problems depends on distinguishing between them.

Work and heat are energy in transfer from one system to another and thus play a crucial role in most thermodynamic systems or devices. To analyze such systems, we need to model heat and work as functions of properties and parameters characteristic of the system or the way it functions. An understanding of the physics involved allows us to construct a model for heat and work and use the result in our analysis of energy transfers and changes, which we will do with the first law of thermodynamics in Chapter 5.

To facilitate understanding of the basic concepts, we present a number of physical arrangements that will enable us to express the work done from changes in the system during a process. We also examine work that is the result of a given process without describing in detail how the process physically can be made to occur. This is done because such a description is too complex and involves concepts that have not been covered so far, but at least we can examine the result of the process.

Heat transfer in different situations is a subject that usually is studied separately. However, a very simple introduction is beneficial so that the concept of heat transfer does not become too abstract and so that it can be related to the processes we examine. Heat transfer by conduction, convection (flow), and radiation is presented in terms of very simple models, emphasizing that it is driven by a temperature difference.

4.1 DEFINITION OF WORK

Work is usually defined as a force F acting through a displacement x, where the displacement is in the direction of the force. That is,

$$W = \int_1^2 F\,dx \tag{4.1}$$

This is a very useful relationship because it enables us to find the work required to raise a weight, to stretch a wire, or to move a charged particle through a magnetic field.

However, when treating thermodynamics from a macroscopic point of view, it is advantageous to link the definition of work with the concepts of systems, properties, and processes. We therefore define work as follows: Work is done by a system if the sole effect on the surroundings (everything external to the system) could be the raising of a weight. Notice that the raising of a weight is in effect a force acting through a distance. Notice also that our definition does not state that a weight was actually raised or that a force actually acted through a given distance, but only that the sole effect external to the system could be

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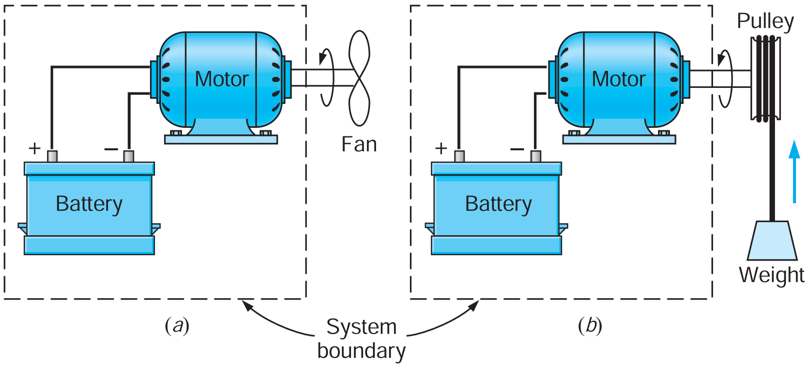

FIGURE 4.1 Example of work crossing the boundary of a system.

the raising of a weight. Work done *by* a system is considered positive and work done *on* a system is considered negative. The symbol W designates the work done by a system.

In general, work is a form of energy in transit, that is, energy being transferred across a system boundary. The concept of energy and energy storage or possession was discussed in detail in Section 2.6. Work is the form of energy that fulfills the definition given in the preceding paragraph.

Let us illustrate this definition of work with a few examples. Consider as a system the battery and motor of Fig. 4.1*a*, and let the motor drive a fan. Does work cross the boundary of the system? To answer this question using the definition of work given earlier, replace the fan with the pulley and weight arrangement shown in Fig. 4.1*b*. As the motor turns, the weight is raised, and the sole effect external to the system is the raising of a weight. Thus, for our original system of Fig. 4.1*a*, we conclude that work is crossing the boundary of the system, since the sole effect external to the system could be the raising of a weight.

Let the boundaries of the system be changed now to include only the battery shown in Fig. 4.2. Again we ask, does work cross the boundary of the system? To answer this question, we need to ask a more general question: Does the flow of electrical energy across the boundary of a system constitute work?

The only limiting factor when the sole external effect is the raising of a weight is the inefficiency of the motor. However, as we design a more efficient motor, with lower bearing and electrical losses, we recognize that we can approach a certain limit that meets the requirement of having the only external effect be the raising of a weight. Therefore, we can conclude that when there is a flow of electricity across the boundary of a system, as in Fig. 4.2, it is work.

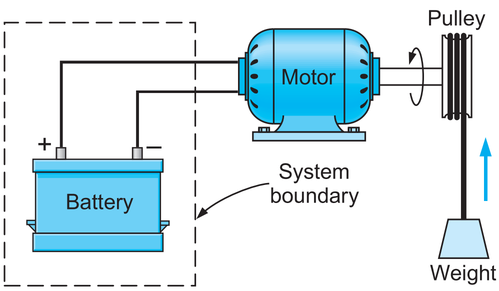

FIGURE 4.2 Example of work crossing the boundary of a system because of an electric current flow across the system boundary.

4.2 UNITS FOR WORK

As already noted, work done *by* a system, such as that done by a gas expanding against a piston, is positive, and work done *on* a system, such as that done by a piston compressing a gas, is negative. Thus, positive work means that energy leaves the system, and negative work means that energy is added to the system.

Our definition of work involves raising of a weight, that is, the product of a unit force (one newton) acting through a unit distance (one meter). This unit for work in SI units is called the joule (*J*).

$$1\ \text{J} = 1\ \text{N m}$$

Power is the time rate of doing work and is designated by the symbol $\dot{W}$:

$$\dot{W} \equiv \frac{\delta W}{dt}$$

The unit for power is a rate of work of one joule per second, which is a watt (W):

$$1\ \text{W} = 1\ \text{J/s}$$

A familiar unit for power in English units is the horsepower (hp), where

$$1\ \text{hp} = 550\ \text{ft lbf/s}$$

Note that the work crossing the boundary of the system in Fig. 4.1 is that associated with a rotating shaft. To derive the expression for power, we use the differential work from Eq. 4.1:

$$\delta W = F\,dx = Fr\,d\theta = T\,d\theta$$

that is, force acting through a distance dx or a torque ($T = Fr$) acting through an angle of rotation, as shown in Fig. 4.3. Now the power becomes

$$\dot{W} = \frac{\delta W}{dt} = F\frac{dx}{dt} = F\mathbf{V} = Fr\frac{d\theta}{dt} = T\omega \tag{4.2}$$

that is, force times rate of displacement (velocity) or torque times angular velocity.

It is often convenient to speak of the work per unit mass of the system, often termed *specific work*. This quantity is designated w and is defined as

$$w \equiv \frac{W}{m}$$

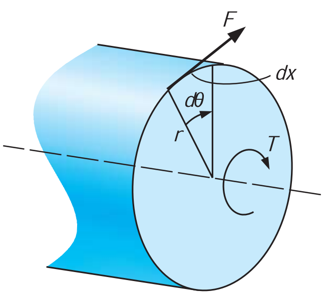

FIGURE 4.3 Force acting at radius r gives a torque $T = F_r$.

In-Text Concept Questions

a. The electric company charges the customers per kW-hour. What is that is SI units?

b. Torque, energy, and work have the same units (Nm). Explain the diference.

4.3 WORK DONE AT THE MOVING BOUNDARY OF A SIMPLE COMPRESSIBLE SYSTEM

We have already noted that there are a variety of ways in which work can be done on or by a system. These include work done by a rotating shaft, electrical work, and work done by the movement of the system boundary, such as the work done in moving the piston in a cylinder. In this section we will consider in some detail the work done at the moving boundary of a simple compressible system during a quasi-equilibrium process.

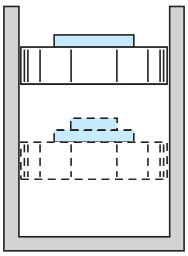

FIGURE 4.4 Example of work done at the moving boundary of a system in a quasi-equilibrium process.

Consider as a system the gas contained in a cylinder and piston, as in Fig. 4.4. Remove one of the small weights from the piston, which will cause the piston to move upward a distance dL. We can consider this quasi-equilibrium process and calculate the amount of work W done by the system during this process. The total force on the piston is PA, where P is the pressure of the gas and A is the area of the piston. Therefore, the work δW is

$$\delta W = PA\,dL$$

But $A\,dL = dV$, the change in volume of the gas. Therefore,

$$\delta W = P\,dV \tag{4.3}$$

The work done at the moving boundary during a given quasi-equilibrium process can be found by integrating Eq. 4.3. However, this integration can be performed only if we know the relationship between P and V during this process. This relationship may be expressed as an equation, or it may be shown as a graph.

Let us consider a graphical solution first. We use as an example a compression process such as occurs during the compression of air in a cylinder, Fig. 4.5. At the beginning of the process the piston is at position 1, and the pressure is relatively low. This state is represented

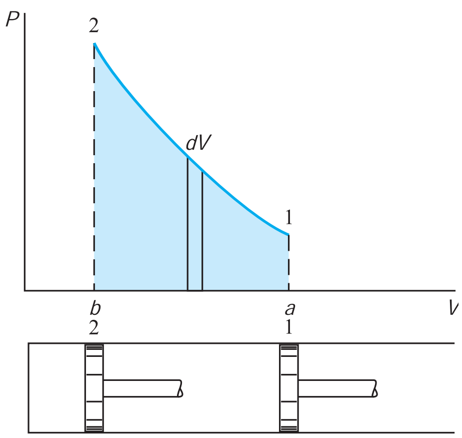

FIGURE 4.5 Use of a P–V diagram to show work done at the moving boundary of a system in a quasi-equilibrium process.

on a pressure–volume diagram (usually referred to as a *P–V diagram*). At the conclusion of the process the piston is in position 2, and the corresponding state of the gas is shown at point 2 on the *P–V* diagram. Let us assume that this compression was a quasi-equilibrium process and that during the process the system passed through the states shown by the line connecting states 1 and 2 on the *P–V* diagram. The assumption of a quasi-equilibrium process is essential here because each point on line 1–2 represents a definite state, and these states correspond to the actual state of the system only if the deviation from equilibrium is infinitesimal. The work done on the air during this compression process can be found by integrating Eq. 4.3:

$$ {}_1W_2 = \int_1^2 \delta W = \int_1^2 P\,dV \tag{4.4}$$

The symbol ${}_1W_2$ is to be interpreted as the work done during the process from state 1 to state 2. It is clear from the *P–V* diagram that the work done during this process,

$$\int_1^2 P\,dV$$

is represented by the area under curve 1–2, area *a*–1–2–*b*–*a*. In this example the volume decreased, and area *a*–1–2–*b*–*a* represents work done on the system. If the process had proceeded from state 2 to state 1 along the same path, the same area would represent work done by the system.

Further consideration of a *P–V* diagram, such as Fig. 4.6, leads to another important conclusion. It is possible to go from state 1 to state 2 along many different quasi-equilibrium paths, such as *A*, *B*, or *C*. Since the area under each curve represents the work for each process, the amount of work done during each process not only is a function of the end states of the process but also depends on the path followed in going from one state to another. For this reason, work is called a *path function* or, in mathematical parlance, δW is an inexact differential.

This concept leads to a brief consideration of point and path functions or, to use other terms, *exact* and *inexact differentials*. Thermodynamic properties are *point functions*, a name that comes from the fact that for a given point on a diagram (such as Fig. 4.6) or surface (such as Fig. 3.18) the state is fixed, and thus there is a definite value for each property corresponding to this point. The differentials of point functions are exact differentials, and the integration is simply

$$\int_1^2 dV = V_2 - V_1$$

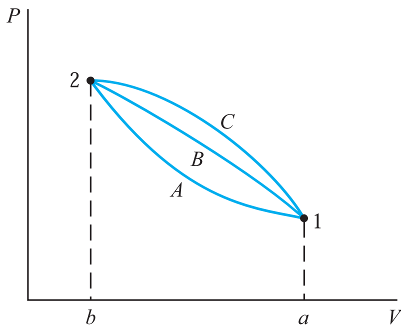

FIGURE 4.6 Various quasi-equilibrium processes between two given states, indicating that work is a path function.

Thus, we can speak of the volume in state 2 and the volume in state 1, and the change in volume depends only on the initial and final states.

Work, however, is a path function, for, as has been indicated, the work done in a quasi-equilibrium process between two given states depends on the path followed. The differentials of path functions are inexact differentials, and the symbol δ will be used in this book to designate inexact differentials (in contrast to d for exact differentials). Thus, for work, we write

$$\int_1^2 \delta W = {}_1W_2$$

It would be more precise to use the notation ${}_1W_{2A}$, which would indicate the work done during the change from state 1 to state 2 along path A. However, the notation ${}_1W_2$ indicates that the process between states 1 and 2 has been specified. Note that we never speak about the work in the system in state 1 or state 2, and thus we never write $W_2 - W_1$.

In evaluating the integral of Eq. 4.4, we should always keep in mind that we wish to determine the area under the curve in Fig. 4.6. In connection with this point, we identify the following two classes of problems:

1. The relationship between P and V is given in terms of experimental data or in graphical form (as, for example, the trace on an oscilloscope). Therefore, we may evaluate the integral, Eq. 4.4, by graphical or numerical integration.
2. The relationship between P and V makes it possible to fit an analytical relationship between them. We may then integrate directly.

One common example of this second type of functional relationship is a process called a polytropic process, one in which

$$PV^n = \text{constant}$$

throughout the process. The exponent n may be any value from $-\infty$ to $+\infty$, depending on the process. For this type of process, we can integrate Eq. 4.4 as follows:

$$PV^n = \text{constant} = P_1V_1^n = P_2V_2^n$$

$$P = \frac{\text{constant}}{V^n} = \frac{P_1V_1^n}{V^n} = \frac{P_2V_2^n}{V^n}$$

$$\int_1^2 P\,dV = \text{constant}\int_1^2 \frac{dV}{V^n} = \text{constant}\left(\frac{V^{-n+1}}{-n+1}\right)\Bigg|_1^2$$

$$\int_1^2 P\,dV = \frac{\text{constant}}{1-n}(V_2^{1-n} - V_1^{1-n}) = \frac{P_2V_2^nV_2^{1-n} - P_1V_1^nV_1^{1-n}}{1-n}$$

$$= \frac{P_2V_2 - P_1V_1}{1-n} \tag{4.5}$$

Note that the resulting equation, Eq. 4.5, is valid for any exponent n except $n = 1$. Where $n = 1$,

$$PV = \text{constant} = P_1V_1 = P_2V_2$$

and

$$\int_1^2 P\,dV = P_1 V_1 \int_1^2 \frac{dV}{V} = P_1 V_1 \ln \frac{V_2}{V_1} \tag{4.6}$$

Note that in Eqs. 4.5 and 4.6 we did not say that the work is equal to the expressions given in these equations. These expressions give us the value of a certain integral, that is, a mathematical result. Whether or not that integral equals the work in a particular process depends on the result of a thermodynamic analysis of that process. It is important to keep the mathematical result separate from the thermodynamic analysis, for there are many situations in which work is not given by Eq. 4.4.

The polytropic process as described demonstrates one special functional relationship between P and V during a process. There are many other possible relations, some of which will be examined in the problems at the end of this chapter.

EXAMPLE 4.1

Consider as a system the gas in the cylinder shown in Fig. 4.7; the cylinder is fitted with a piston on which a number of small weights are placed. The initial pressure is 200 kPa, and the initial volume of the gas is 0.04 m^3.

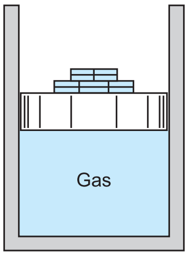

FIGURE 4.7 Sketch for Example 4.1.

a. Let a Bunsen burner be placed under the cylinder, and let the volume of the gas increase to 0.1 m^3 while the pressure remains constant. Calculate the work done by the system during this process.

$$_1W_2 = \int_1^2 P\,dV$$

Since the pressure is constant, we conclude from Eq. 4.4 that

$$_1W_2 = P \int_1^2 dV = P(V_2 - V_1)$$

$$_1W_2 = 200\text{ kPa} \times (0.1 - 0.04)\text{m}^3 = 12.0\text{ kJ}$$

b. Consider the same system and initial conditions, but at the same time that the Bunsen burner is under the cylinder and the piston is rising, remove weights from the piston at such a rate that, during the process, the temperature of the gas remains constant.

If we assume that the ideal-gas model is valid, then, from Eq. 3.5,

$$PV = mRT$$

We note that this is a polytropic process with exponent $n = 1$. From our analysis, we conclude that the work is given by Eq. 4.4 and that the integral in this equation is given by Eq. 4.6. Therefore,

$$_1W_2 = \int_1^2 P\,dV = P_1 V_1 \ln \frac{V_2}{V_1}$$

$$= 200\text{ kPa} \times 0.04\text{ m}^3 \times \ln \frac{0.10}{0.04} = 7.33\text{ kJ}$$