6 First-Law Analysis for a Control Volume

In the preceding chapter we developed the first-law analysis (energy balance) for a control mass going through a process. Many applications in thermodynamics do not readily lend themselves to a control mass approach but are conveniently handled by the more general control volume technique, as discussed in Chapter 2. This chapter is concerned with development of the control volume forms of the conservation of mass and energy in situations where flows of substance are present.

6.1 CONSERVATION OF MASS AND THE CONTROL VOLUME

A control volume is a volume in space of interest for a particular study or analysis. The surface of this control volume is referred to as *a control surface* and always consists of a closed surface. The size and shape of the control volume are completely arbitrary and are so defined as to best suit the analysis to be made. The surface may be fixed, or it may move so that it expands or contracts. However, the surface must be defined relative to some coordinate system. In some analyses it may be desirable to consider a rotating or moving coordinate system and to describe the position of the control surface relative to such a coordinate system.

Mass as well as heat and work can cross the control surface, and the mass in the control volume, as well as the properties of this mass, can change with time. Figure 6.1 shows a schematic diagram of a control volume that includes heat transfer, shaft work, moving boundary work, accumulation of mass within the control volume, and several mass flows. It is important to identify and label each flow of mass and energy and the parts of the control volume that can store (accumulate) mass.

Let us consider the conservation of mass law as it relates to the control volume. The physical law concerning mass, recalling Section 5.9, says that we cannot create or destroy mass. We will express this law in a mathematical statement about the mass in the control volume. To do this, we must consider all the mass flows into and out of the control volume and the net increase of mass within the control volume. As a somewhat simpler control volume, we consider a tank with a cylinder and piston and two pipes attached, as shown in Fig. 6.2. The rate of change of mass inside the control volume can be different from zero if we add or take a flow of mass out as

$$\text{Rate of change} = +\text{in} - \text{out}$$

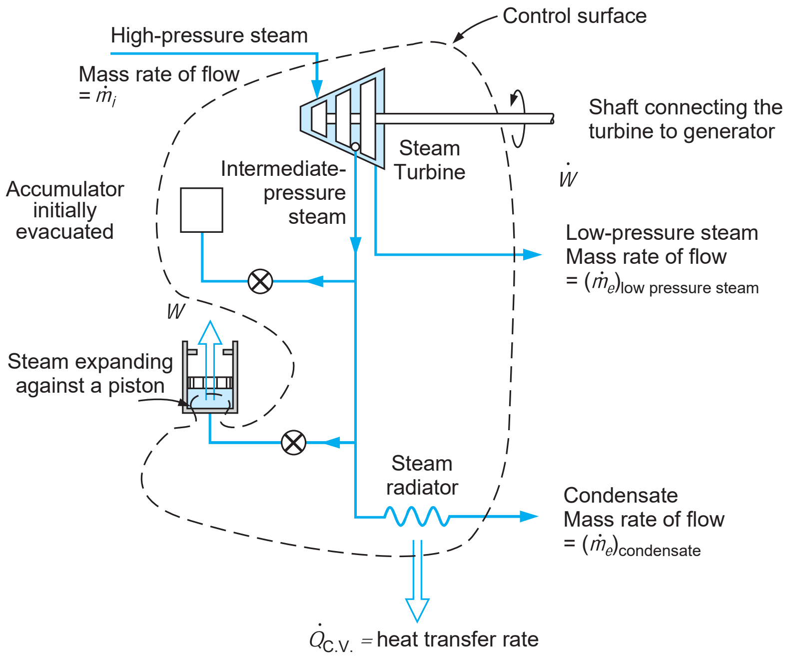

FIGURE 6.1 Schematic diagram of a control volume showing mass and energy transfers and accumulation.

With several possible flows this is written as

$$\frac{dm_{\text{C.V.}}}{\text{dt}} = \sum \dot{m}_i - \sum \dot{m}_e \tag{6.1}$$

which states that if the mass inside the control volume changes with time, it is because we add some mass or take some mass out. There are no other means by which the mass inside the control volume could change. Equation 6.1 expressing the conservation of mass is commonly termed the continuity equation. While this form of the equation is sufficient for the majority of applications in thermodynamics, it is frequently rewritten in terms of the local fluid properties in the study of fluid mechanics and heat transfer. In this book we are

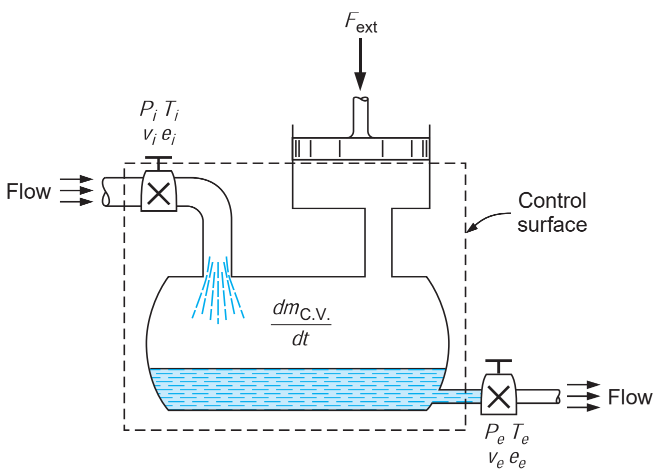

FIGURE 6.2 Schematic diagram of a control volume for the analysis of the continuity equation.

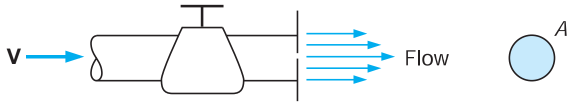

FIGURE 6.3 The flow across a control volume surface with a flow cross-sectional area of A. Average velocity is shown to the left of the valve and a distributed flow across the area is shown to the right of the valve.

mainly concerned with the overall mass balance and thus consider Eq. 6.1 as the general expression for the continuity equation.

Since Eq. 6.1 is written for the total mass (lumped form) inside the control volume, we may have to consider several contributions to the mass as

$$m_{\text{C.V.}} = \int \rho \, dV = \int (1/v) dV = m_A + m_B + m_C + \cdots$$

Such a summation is needed when the control volume has several accumulation units with different states of the mass.

Let us now consider the mass flow rates across the control volume surface in a little more detail. For simplicity we assume the fluid is flowing in a pipe or duct as illustrated in Fig. 6.3. We wish to relate the total flow rate that appears in Eq. 6.1 to the local properties of the fluid state. The flow across the control volume surface can be indicated with an average velocity shown to the left of the valve or with a distributed velocity over the cross section, as shown to the right of the valve.

The volume flow rate is

$$\dot{V} = \mathbf{V}A = \int \mathbf{V}_{\text{local}} \, dA \tag{6.2}$$

so the mass flow rate becomes

$$\dot{m} = \rho_{\text{avg}} \dot{V} = \dot{V}/v = \int (\mathbf{V}_{\text{local}}/v) \, dA = \mathbf{V}A/v \tag{6.3}$$

where often the average velocity is used. It should be noted that this result, Eq. 6.3, has been developed for a stationary control surface, and we tacitly assumed the flow was normal to the surface. This expression for the mass flow rate applies to any of the various flow streams entering or leaving the control volume, subject to the assumptions mentioned.

EXAMPLE 6.1 Air is flowing in a 0.2-m-diameter pipe at a uniform velocity of 0.1 m/s. The temperature is 25°C and the pressure is 150 kPa. Determine the mass flow rate.

Solution

From Eq. 6.3 the mass flow rate is

$$\dot{m} = \mathbf{V}A/v$$

For air, using R from Table A.5, we have

$$v = \frac{RT}{P} = \frac{0.287 \text{ kJ/kg K} \times 298.2 \text{ K}}{150 \text{ kPa}} = 0.5705 \text{ m}^3\text{/kg}$$

The cross-sectional area is

$$A = \frac{\pi}{4}(0.2)^2 = 0.0314\ \mathrm{m}^2$$

Therefore,

$$\dot{m} = \mathbf{V}A/v = 0.1\ \mathrm{m/s} \times 0.0314\ \mathrm{m}^2/0.5705\ \mathrm{m}^3/\mathrm{kg} = 0.0055\ \mathrm{kg/s}$$

In-Text Concept Question

a. A mass flow rate into a control volume requires a normal velocity component. Why?

6.2 THE FIRST LAW OF THERMODYNAMICS FOR A CONTROL VOLUME

We have already considered the first law of thermodynamics for a control mass, which consists of a fixed quantity of mass, and noted, in Eq. 5.5, that it may be written as

$$E_2 - E_1 = {}_1Q_2 - {}_1W_2$$

We have also noted that this may be written as an instantaneous rate equation as

$$\frac{dE_{\text{C.M.}}}{dt} = \dot{Q} - \dot{W} \tag{6.4}$$

To write the first law as a rate equation for a control volume, we proceed in a manner analogous to that used in developing a rate equation for the law of conservation of mass. For this purpose, a control volume is shown in Fig. 6.4 that involves the rate of heat transfer, rates of work, and mass flows. The fundamental physical law states that we cannot create or destroy energy such that any rate of change of energy must be caused by rates of energy into or out of the control volume. We have already included rates of heat transfer and work in Eq. 6.4, so the additional explanations we need are associated with the mass flow rates.

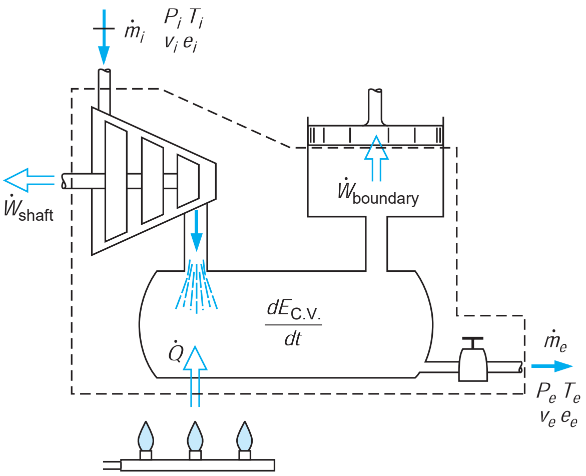

FIGURE 6.4 Schematic diagram illustrating terms in the energy equation for a general control volume.

The fluid flowing across the control surface enters or leaves with an amount of energy per unit mass as

$$e = u + \frac{1}{2}\mathbf{V}^2 + gZ$$

relating to the state and position of the fluid. Whenever a fluid mass enters a control volume at state i or exits at state e, there is a boundary movement work associated with that process.

To explain this in more detail, consider an amount of mass flowing into the control volume. As this mass flows in there is a pressure at its back surface, so as this mass moves into the control volume it is being pushed by the mass behind it, which is the surroundings. The net effect is that after the mass has entered the control volume, the surroundings have pushed it in against the local pressure with a velocity giving it a rate of work in the process. Similarly, a fluid exiting the control volume at state e must push the surrounding fluid ahead of it, doing work on it, which is work leaving the control volume. The velocity and the area correspond to a certain volume per unit time entering the control volume, enabling us to relate that to the mass flow rate and the specific volume at the state of the mass going in. Now we are able to express the rate of flow work as

$$\dot{W}_{\text{flow}} = F\mathbf{V} = \int P\mathbf{V}\,dA = P\dot{V} = Pv\dot{m} \tag{6.5}$$

For the flow that leaves the control volume, work is being done by the control volume, $P_e v_e \dot{m}_e$, and for the mass that enters, the surroundings do the rate of work, $P_i v_i \dot{m}_i$. The flow work per unit mass is then Pv, and the total energy associated with the flow of mass is

$$e + Pv = u + Pv + \frac{1}{2}\mathbf{V}^2 + gZ = h + \frac{1}{2}\mathbf{V}^2 + gZ \tag{6.6}$$

In this equation we have used the definition of the thermodynamic property enthalpy, and it is the appearance of the combination $(u + Pv)$ for the energy in connection with a mass flow that is the primary reason for the definition of the property enthalpy. Its introduction earlier in conjunction with the constant-pressure process was done to facilitate use of the tables of thermodynamic properties at that time.

EXAMPLE 6.2

Assume we are standing next to the local city's main water line. The liquid water inside flows at a pressure of 600 kPa (6 atm) with a temperature of about 10°C. We want to add a smaller amount, 1 kg, of liquid to the line through a side pipe and valve mounted on the main line. How much work will be involved in this process?

If the 1 kg of liquid water is in a bucket and we open the valve to the water main in an attempt to pour it down into the pipe opening, we realize that the water flows the other way. The water flows from a higher to a lower pressure, that is, from inside the main line to the atmosphere (from 600 kPa to 101 kPa).

We must take the 1 kg of liquid water and put it into a piston/cylinder (like a hand-held pump) and attach the cylinder to the water pipe. Now we can press on the piston until the water pressure inside is 600 kPa and then open the valve to the main line and slowly squeeze the 1 kg of water in. The work done at the piston surface to the water is

$$W = \int P\,dV = P_{\text{water}}\,mv = 600\,\text{kPa} \times 1\,\text{kg} \times 0.001\,\text{m}^3/\text{kg} = 0.6\,\text{kJ}$$

and this is the necessary flow work for adding the 1 kg of liquid.

The extension of the first law of thermodynamics from Eq. 6.4 becomes

$$\frac{dE_{\text{C.V.}}}{dt} = \dot{Q}_{\text{C.V.}} - \dot{W}_{\text{C.V.}} + \dot{m}_i e_i - \dot{m}_e e_e + \dot{W}_{\text{flow in}} - \dot{W}_{\text{flow out}}$$

and the substitution of Eq. 6.5 gives

$$\begin{aligned}\frac{dE_{\text{C.V.}}}{dt} &= \dot{Q}_{\text{C.V.}} - \dot{W}_{\text{C.V.}} + \dot{m}_i(e_i + P_i v_i) - \dot{m}_e(e_e + P_e v_e)\\ &= \dot{Q}_{\text{C.V.}} - \dot{W}_{\text{C.V.}} + \dot{m}_i\left(h_i + \frac{1}{2}\mathbf{V}_i^2 + gZ_i\right) - \dot{m}_e\left(h_e + \frac{1}{2}\mathbf{V}_e^2 + gZ_e\right)\end{aligned}$$

In this form of the energy equation the rate of work term is the sum of all shaft work terms and boundary work terms and any other types of work given out by the control volume; however, the flow work is now listed separately and included with the mass flow rate terms.

For the general control volume we may have several entering or leaving mass flow rates, so a summation over those terms is often needed. The final form of the first law of thermodynamics then becomes

$$\frac{dE_{\text{C.V.}}}{dt} = \dot{Q}_{\text{C.V.}} - \dot{W}_{\text{C.V.}} + \sum \dot{m}_i\left(h_i + \frac{1}{2}\mathbf{V}_i^2 + gZ_i\right) - \sum \dot{m}_e\left(h_e + \frac{1}{2}\mathbf{V}_e^2 + gZ_e\right) \tag{6.7}$$

expressing that the rate of change of energy inside the control volume is due to a net rate of heat transfer, a net rate of work (measured positive out), and the summation of energy fluxes due to mass flows into and out of the control volume. As with the conservation of mass, this equation can be written for the total control volume and can therefore be put in the lumped or integral form where

$$E_{\text{C.V.}} = \int \rho e \, dV = me = m_A e_A + m_B e_B + m_C e_C + \cdots$$

As the kinetic and potential energy terms per unit mass appear together with the enthalpy in all the flow terms, a shorter notation is often used:

$$h_{\text{tot}} = h + \frac{1}{2}\mathbf{V}^2 + gZ$$

$$h_{\text{stag}} = h + \frac{1}{2}\mathbf{V}^2$$

defining the total enthalpy and the stagnation enthalpy (used in fluid mechanics). The shorter equation then becomes

$$\frac{dE_{\text{C.V.}}}{dt} = \dot{Q}_{\text{C.V.}} - \dot{W}_{\text{C.V.}} + \sum \dot{m}_i h_{\text{tot},i} - \sum \dot{m}_e h_{\text{tot},e} \tag{6.8}$$

giving the general energy equation on a rate form. All applications of the energy equation start with the form in Eq. 6.8, and for special cases this will result in a slightly simpler form, as shown in the subsequent sections.

6.3 THE STEADY-STATE PROCESS

Our first application of the control volume equations will be to develop a suitable analytical model for the long-term steady operation of devices such as turbines, compressors, nozzles, boilers, and condensers—a very large class of problems of interest in thermodynamic analysis. This model will not include the short-term transient startup or shutdown of such devices, but only the steady operating period of time.

Let us consider a certain set of assumptions (beyond those leading to Eqs. 6. 1 and 6.7) that lead to a reasonable model for this type of process, which we refer to as the steady-state process.

1. The control volume does not move relative to the coordinate frame.
2. The state of the mass at each point in the control volume does not vary with time.
3. As for the mass that flows across the control surface, the mass flux and the state of this mass at each discrete area of flow on the control surface do not vary with time. The rates at which heat and work cross the control surface remain constant.

As an example of a steady-state process, consider a centrifugal air compressor that operates with a constant mass rate of flow into and out of the compressor, constant properties at each point across the inlet and exit ducts, a constant rate of heat transfer to the surroundings, and a constant power input. At each point in the compressor the properties are constant with time, even though the properties of a given elemental mass of air vary as it flows through the compressor. Often, such a process is referred to as a *steady-flow process*, since we are concerned primarily with the properties of the fluid entering and leaving the control volume. However, in the analysis of certain heat transfer problems in which the same assumptions apply, we are primarily interested in the spatial distribution of properties, particularly temperature, and such a process is referred to as a *steady-state process*. Since this is an introductory book, we will use the term *steady-state process* for both. The student should realize that the terms *steady-state process* and *steady-flow process* are both used extensively in the literature.

Let us now consider the significance of each of these assumptions for the steady-state process.

1. The assumption that the control volume does not move relative to the coordinate frame means that all velocities measured relative to the coordinate frame are also velocities relative to the control surface, and there is no work associated with the acceleration of the control volume.
2. The assumption that the state of the mass at each point in the control volume does not vary with time requires that

$$\frac{dm_{\text{C.V.}}}{dt} = 0 \qquad \text{and} \qquad \frac{dE_{\text{C.V.}}}{dt} = 0$$

Therefore, we conclude that for the steady-state process we can write, from Eqs. 6.1 and 6.7,

Continuity equation:
$$\sum \dot{m}_i = \sum \dot{m}_e \tag{6.9}$$

First law:
$$\dot{Q}_{\text{C.V.}} + \sum \dot{m}_i\left(h_i + \frac{\mathbf{V}_i^2}{2} + gZ_i\right) = \sum \dot{m}_e\left(h_e + \frac{\mathbf{V}_e^2}{2} + gZ_e\right) + \dot{W}_{\text{C.V.}} \tag{6.10}$$

3. The assumption that the various mass flows, states, and rates at which heat and work cross the control surface remain constant requires that every quantity in Eqs. 6.9 and

6.10 be steady with time. This means that application of Eqs. 6.9 and 6.10 to the operation of some device is independent of time.

Many of the applications of the steady-state model are such that there is only one flow stream entering and one leaving the control volume. For this type of process, we can write

Continuity equation: $$\dot{m}_i = \dot{m}_e = \dot{m} \tag{6.11}$$

First law: $$\dot{Q}_{\text{C.V.}} + \dot{m}\left(h_i + \frac{\mathbf{V}_i^2}{2} + gZ_i\right) = \dot{m}\left(h_e + \frac{\mathbf{V}_e^2}{2} + gZ_e\right) + \dot{W}_{\text{C.V.}} \tag{6.12}$$

Rearranging this equation, we have

$$q + h_i + \frac{\mathbf{V}_i^2}{2} + gZ_i = h_e + \frac{\mathbf{V}_e^2}{2} + gZ_e + w \tag{6.13}$$

where, by definition,

$$q = \frac{\dot{Q}_{\text{C.V.}}}{\dot{m}} \quad \text{and} \quad w = \frac{\dot{W}_{\text{C.V.}}}{\dot{m}} \tag{6.14}$$

Note that the units for q and w are kJ/kg. From their definition, q and w can be thought of as the heat transfer and work (other than flow work) per unit mass flowing into and out of the control volume for this particular steady-state process.

The symbols q and w are also used for the heat transfer and work per unit mass of a control mass. However, since it is always evident from the context whether it is a control mass (fixed mass) or control volume (involving a flow of mass) with which we are concerned, the significance of the symbols q and w will also be readily evident in each situation.

The steady-state process is often used in the analysis of reciprocating machines, such as reciprocating compressors or engines. In this case the rate of flow, which may actually be pulsating, is considered to be the average rate of flow for an integral number of cycles. A similar assumption is made regarding the properties of the fluid flowing across the control surface and the heat transfer and work crossing the control surface. It is also assumed that for an integral number of cycles the reciprocating device undergoes, the energy and mass within the control volume do not change.

A number of examples are given in the next section to illustrate the analysis of steady-state processes.

In-Text Concept Questions

b. Can a steady-state device have boundary work?

c. What can you say about changes in $\dot{m}$ and $\dot{V}$ through a steady flow device?

d. In a multiple-device flow system, I want to determine a state property. Where should I look for information—upstream or downstream?

6.4 EXAMPLES OF STEADY-STATE PROCESSES

In this section, we consider a number of examples of steady-state processes in which there is one fluid stream entering and one leaving the control volume, such that the first law can